All-Pairs Shortest Paths and Fine-Grained Complexity

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Abstract. The all-pairs shortest paths (APSP) problem is one of the most fundamental problems in algorithm design and fine-grained complexity. The problem for general weighted dense graphs is conjectured to require close to n^3 time. On the other hand, substantially subcubic algorithms are known in some important special cases via fast matrix multiplication; for example, for directed graphs that are unweighted (or have small integer weights), the current best algorithm due to Zwick (FOCS 1998) had running time near $n^{2.5}$ if the matrix multiplication exponent ω is equal to 2.

In this talk, I will survey the current landscape surrounding the complexity of APSP and its variants, and how the conjectured hardness of APSP in the general and unweighted cases have been used as the basis for establishing conditional lower bounds for other problems. In particular, I will describe recent joint work with Virginia Vassilevska Williams and Yinzhan Xu (ICALP 2021), showing that Zwick's algorithm is in some sense optimal for directed unweighted graphs.