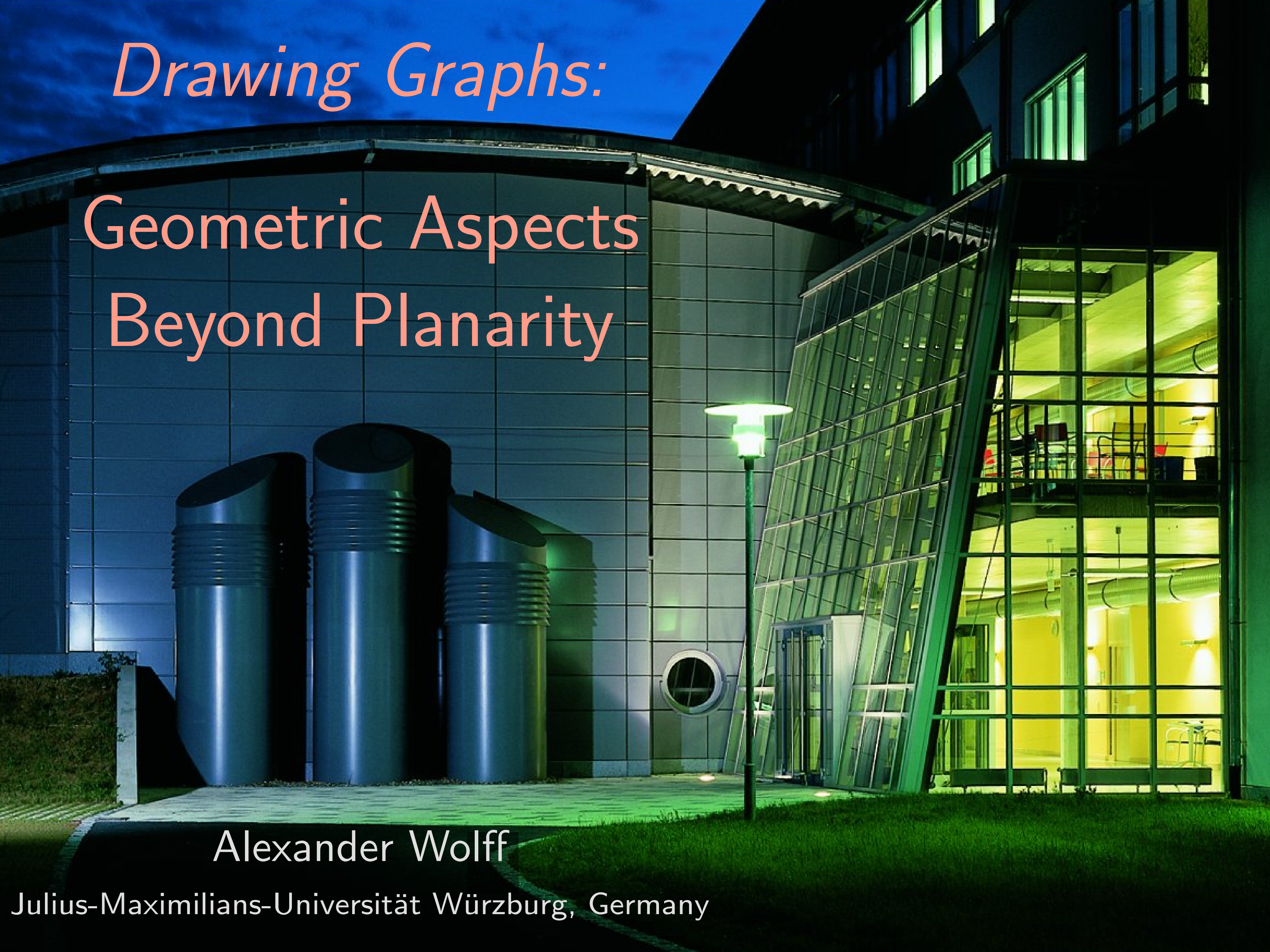

Drawing Graphs:

Geometric Aspects Beyond Planarity

Alexander Wolff

Julius-Maximilians-Universität Würzburg, Germany


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Political Map
D E N M A R K
SWEDEN
N
BALTIC SEA
NORTH SEA
Flensburg
Kiel
Schleswig-Holstein
Stralsund
Rostock
Lübeck
Mecklenburg-Western Pomerania
Schwerin
Hamburg
Hamburg
Bremerhaven
Wilhelmshaven
Emden
Bremen
Bremen
Oldenburg
Wittenberge
POLAND
Lower Saxony
Saxony-Anhalt
Berlin
Potsdam
Berlin
Wolfsburg
Hanover
Osnabrück
Frankfurt (Oder)
NETHERLANDS
Hildesheim
Braunschweig
Brandenburg
Salzgitter
Magdeburg
Münster
Bielefeld
Dessau
Cottbus
Duisburg
Dortmund
Göttingen
Halle (Saale)
Essen
Leipzig
Düsseldorf
Görlitz
Kassel
North Rhine-Westphalia
Dresden
Saxony
Erfurt
Cologne
Siegen
Bad Hersfeld
Eisenach
Chemnitz
Jena
Aachen
Gera
Bonn
Zwickau
BELGIUM
Hesse
Thuringia
Koblenz
Frankfurt
Hof
Wiesbaden
Offenbach
CZECH REPUBLIC
Bamberg
Mainz
Darmstadt
LUXEMBOURG
Rhineland-Palatinate
Ludwigshafen
Mannheim
Saarland
Nuremberg
Saarbrücken
Heidelberg
Heilbronn
Karlsruhe
Regensburg
Stuttgart
Passau
FRANCE
Baden-Württemberg
Bavaria
Ulm
Augsburg
Munich
Freiburg
International Boundary
State Boundary
National Capital
Province Capital
Other Cities
Constance
Garmisch-Partenkirchen
AUSTRIA
0 50 100 150Km
SWITZERLAND
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. . . baroque architecture!

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. . . good beer!

. . . baroque architecture!

. . . Franconian wines!

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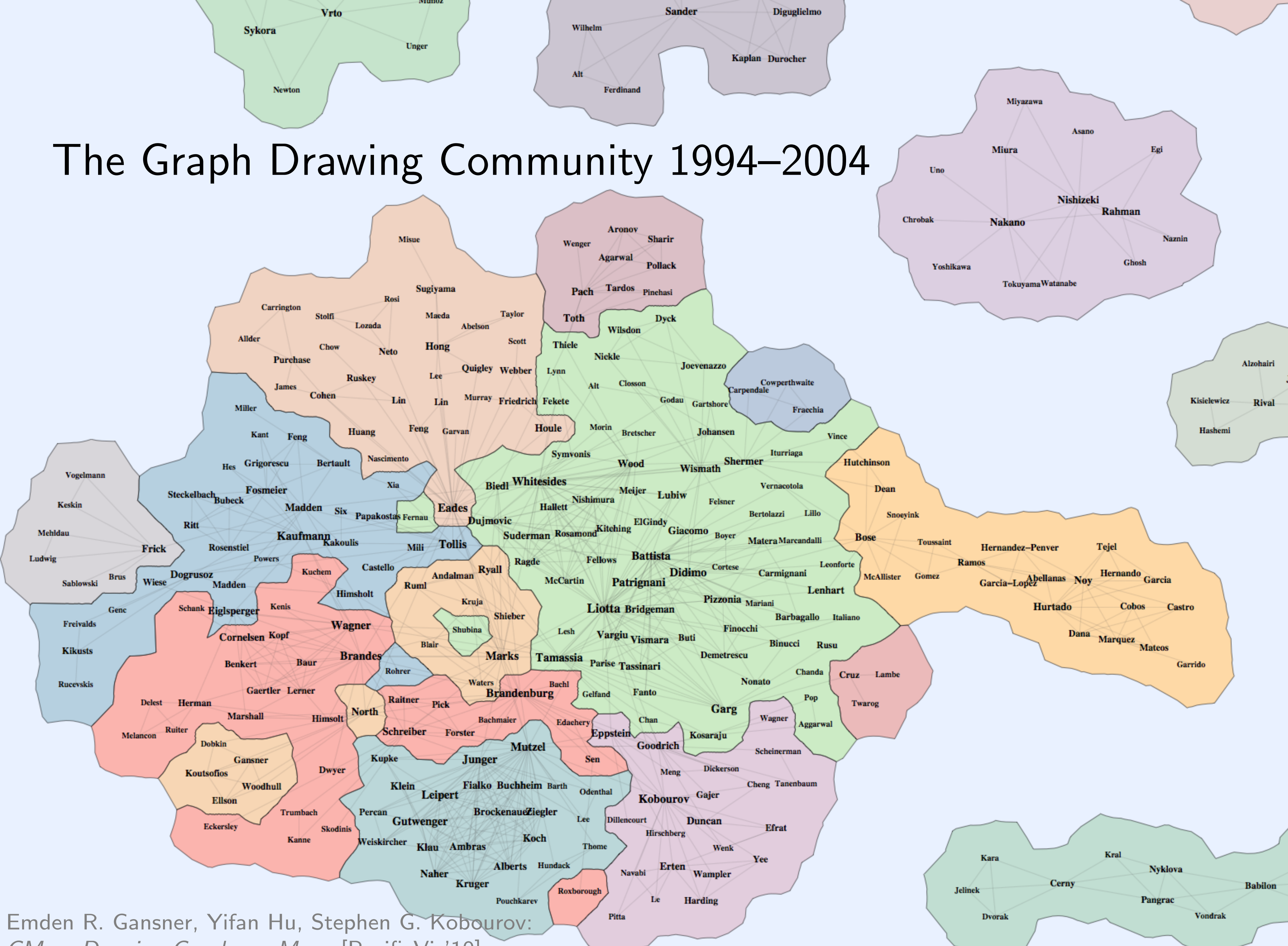
The Graph Drawing Community 1994–2004



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GMap: Drawing Graphs as Maps [PacificVis'10]

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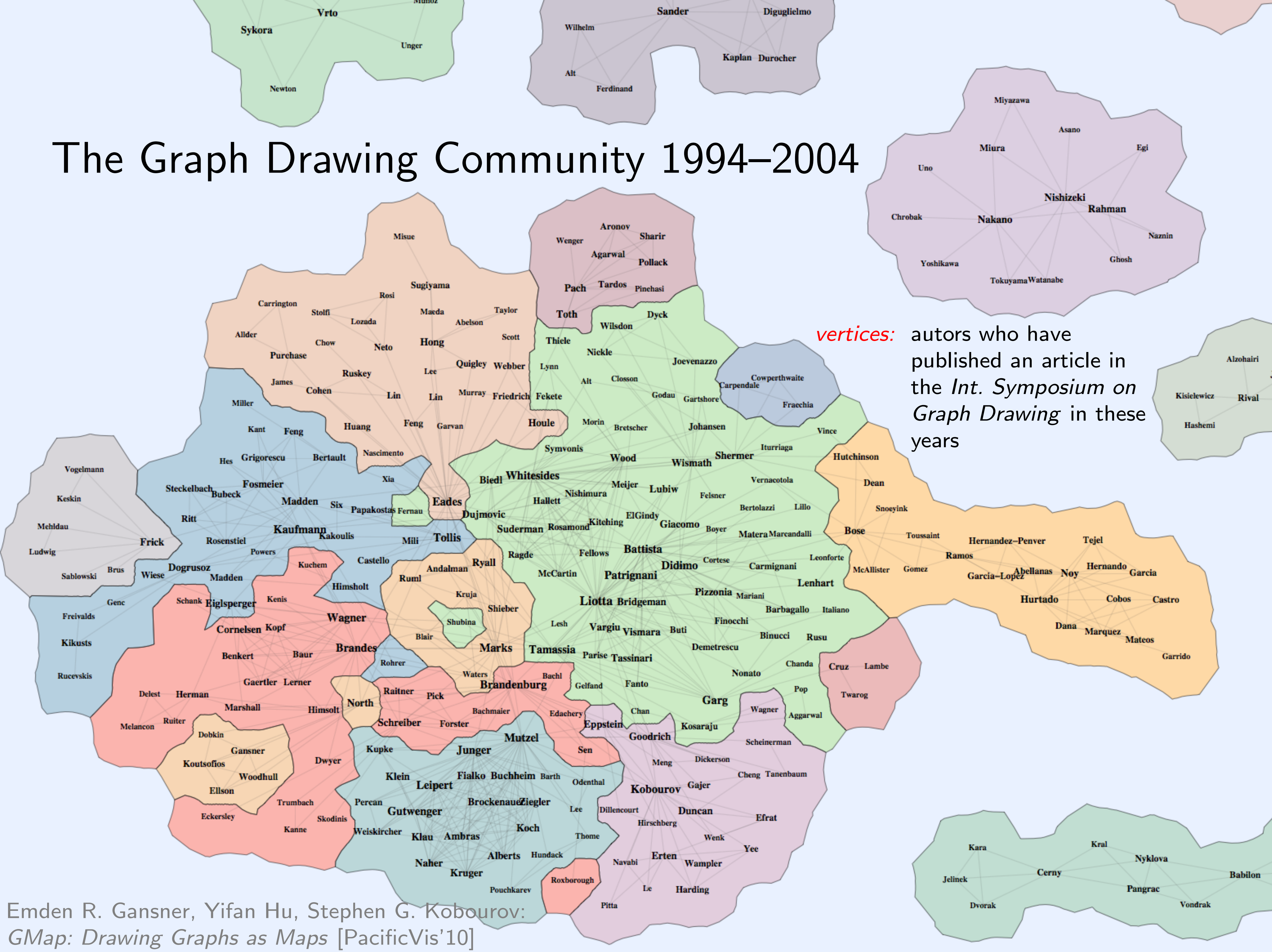

vertices: autors who have published an article in the *Int. Symposium on Graph Drawing* in these years

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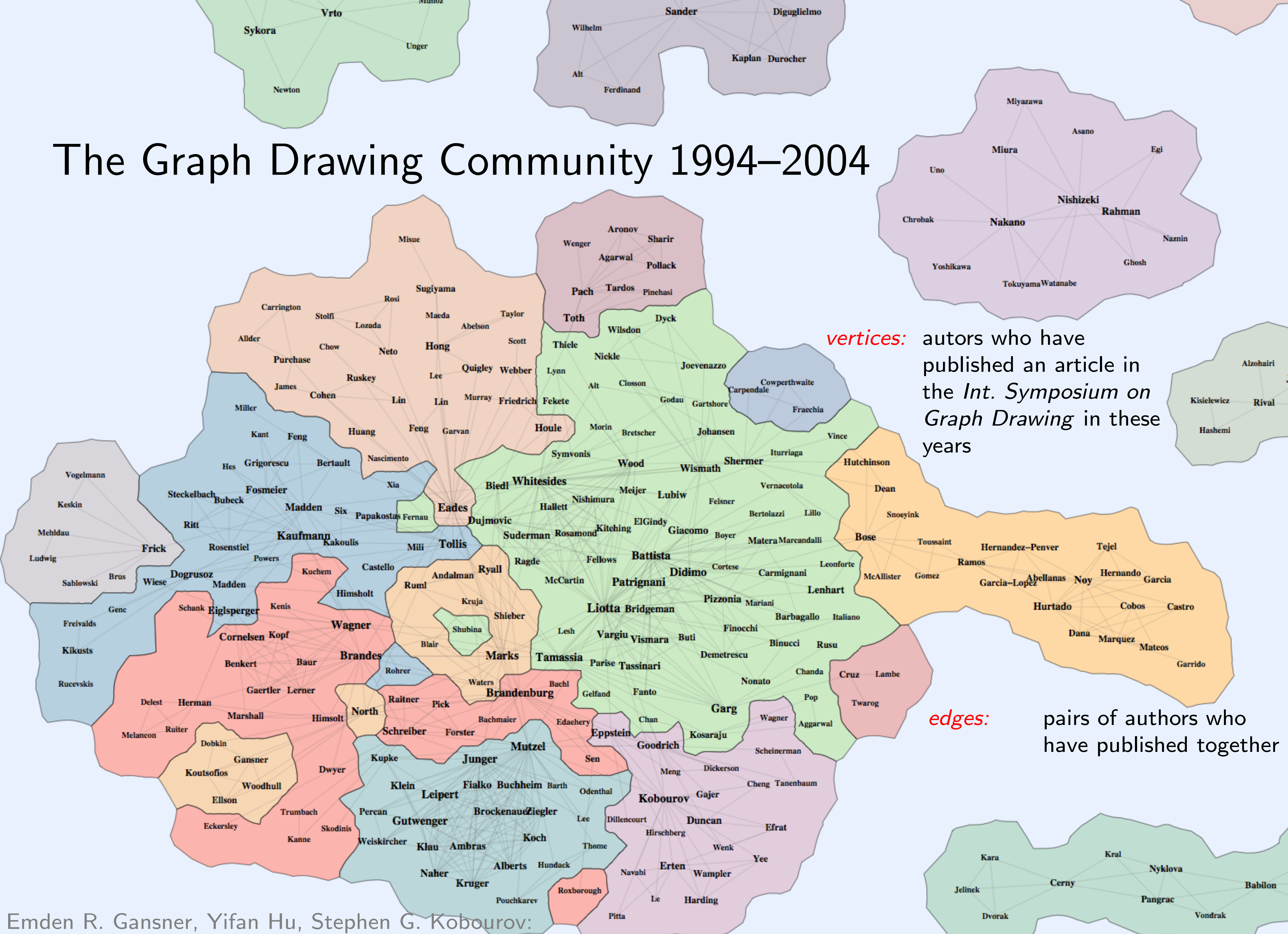

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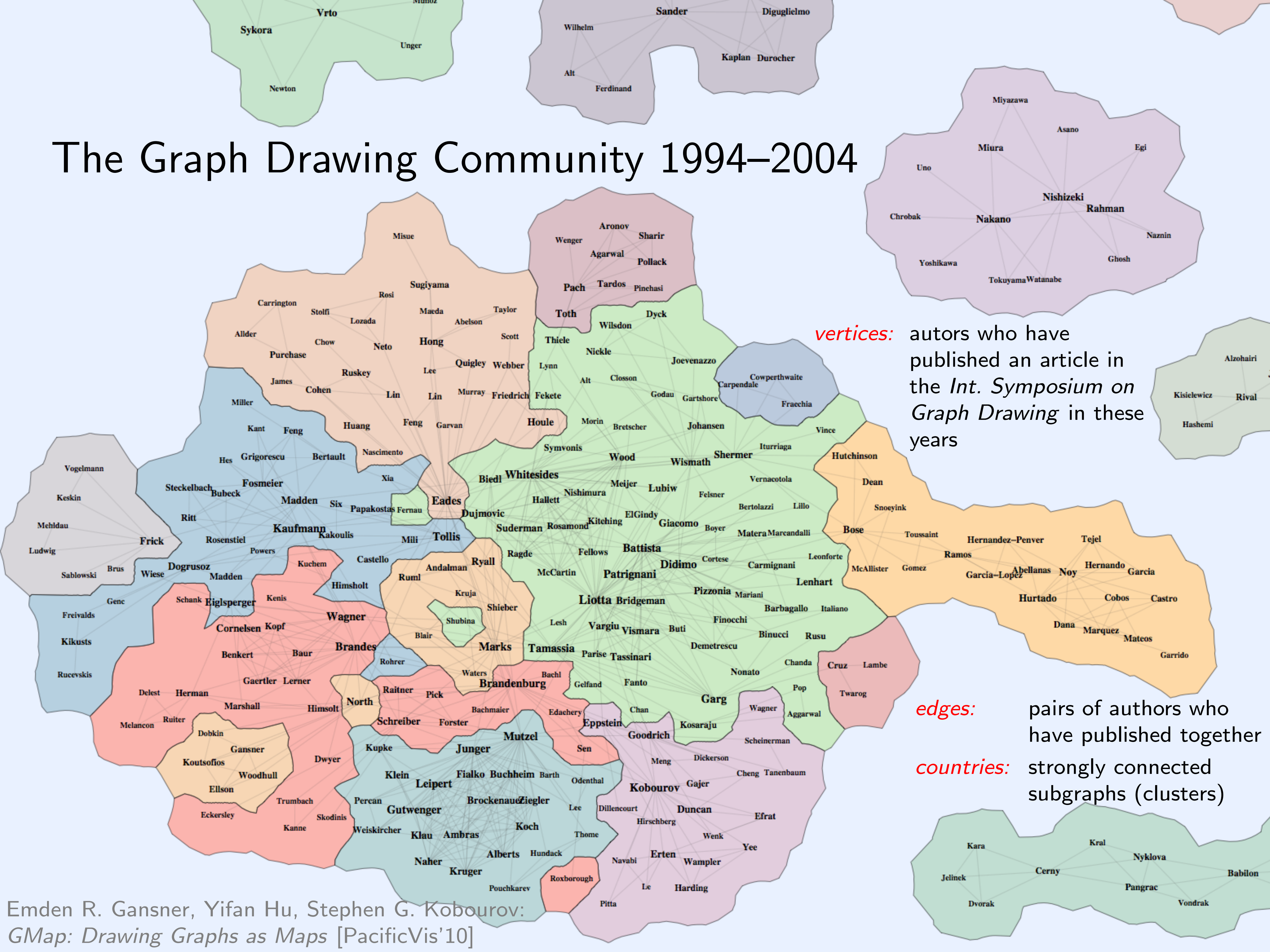

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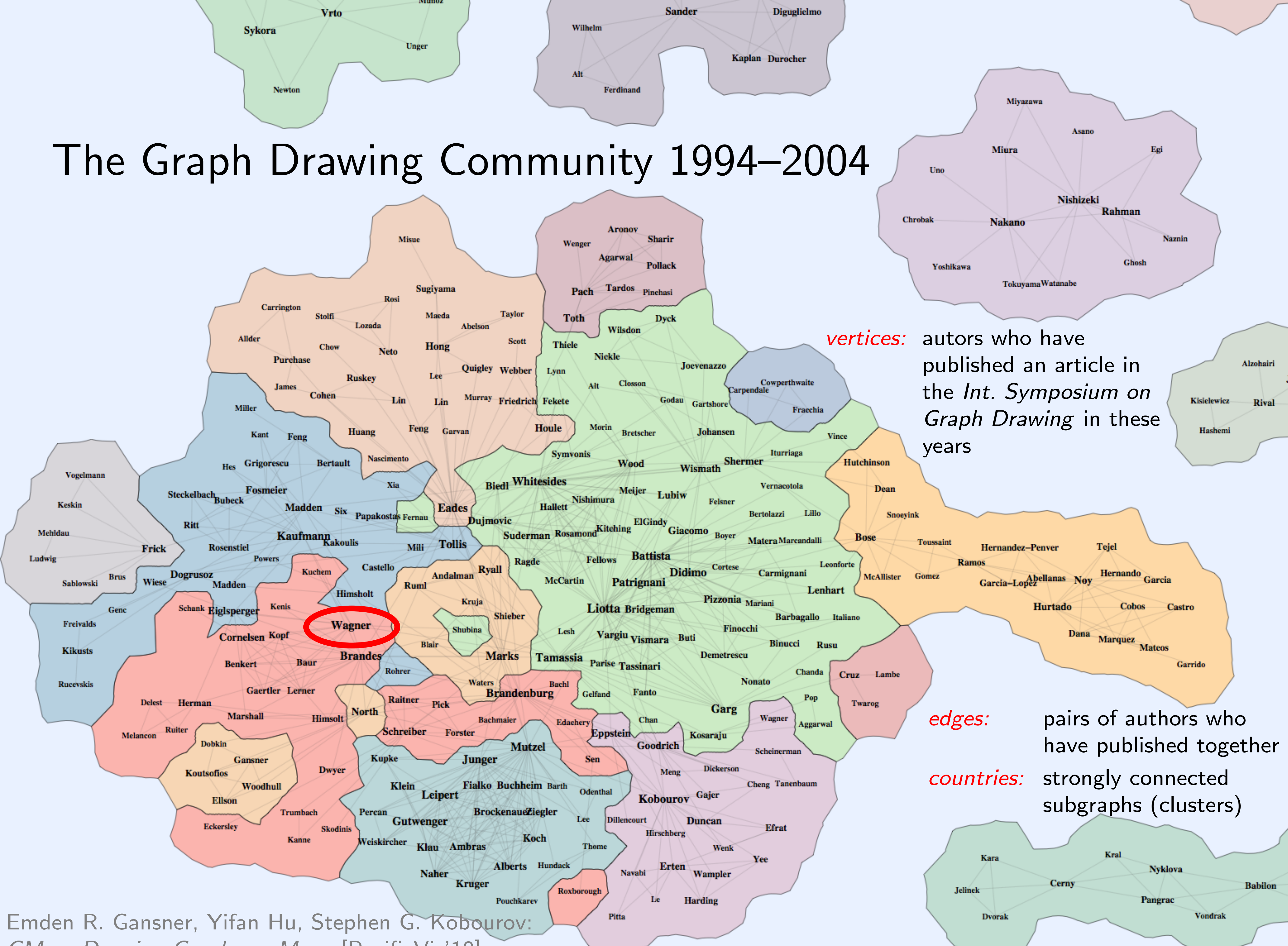

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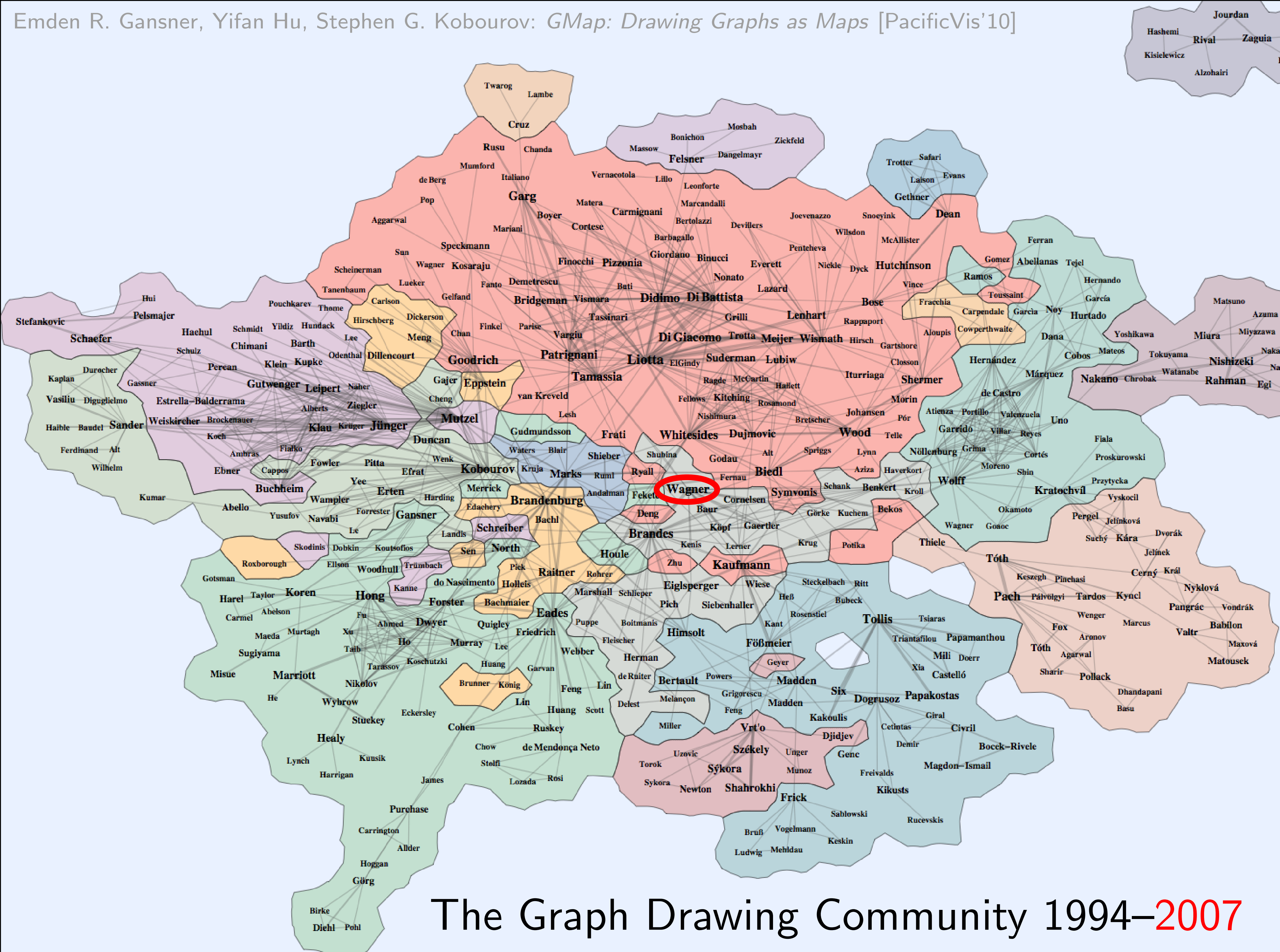

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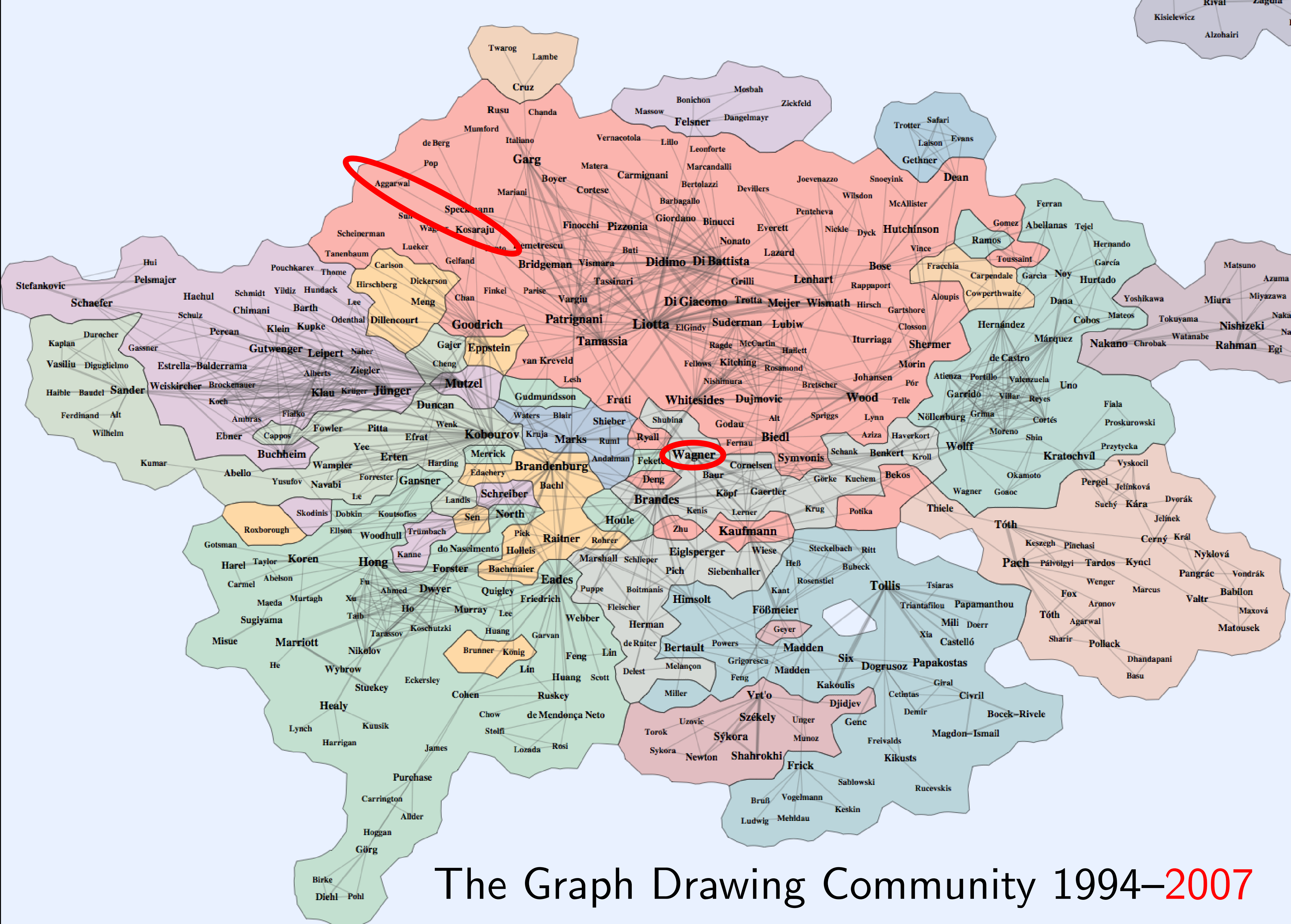

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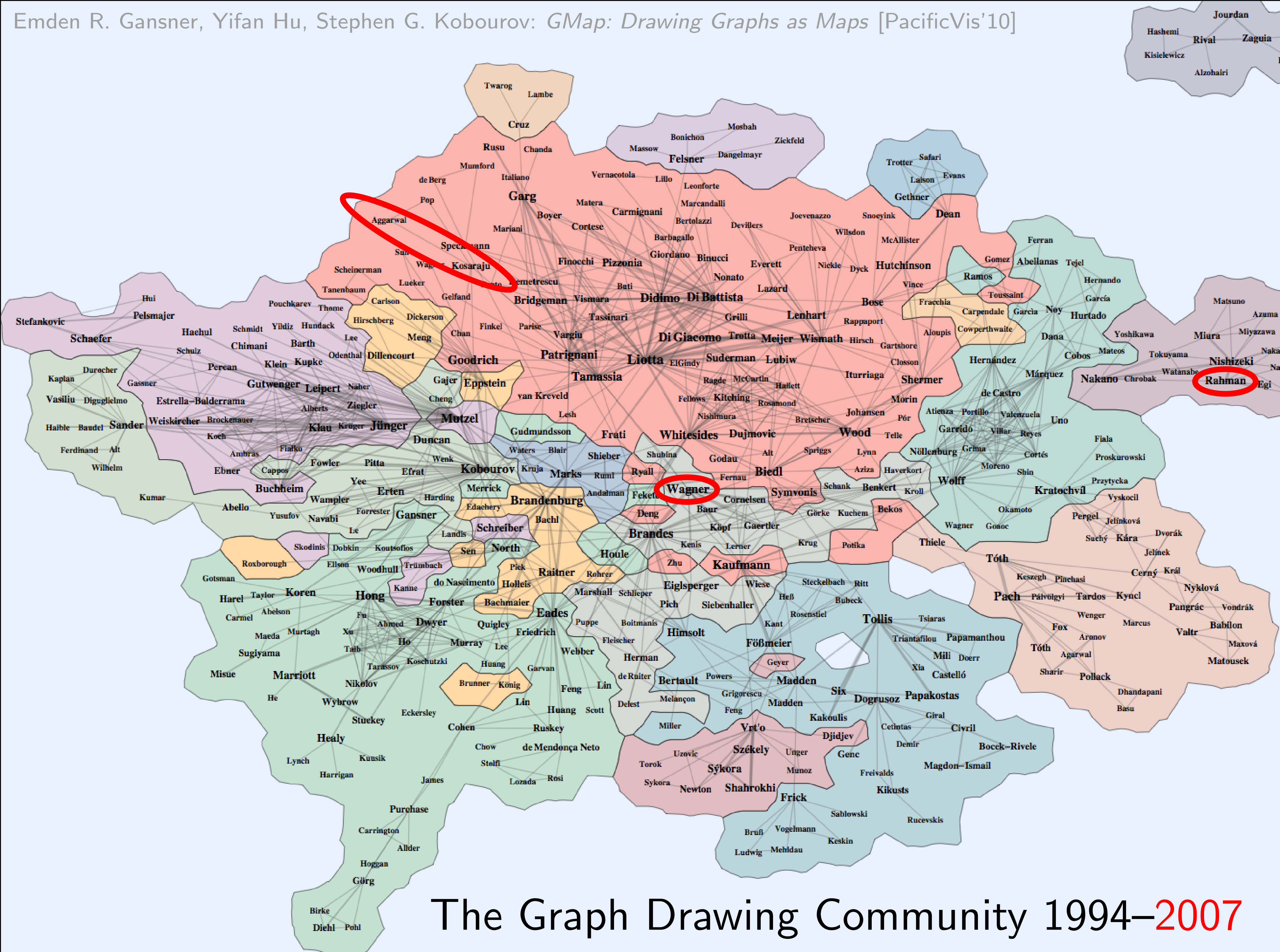

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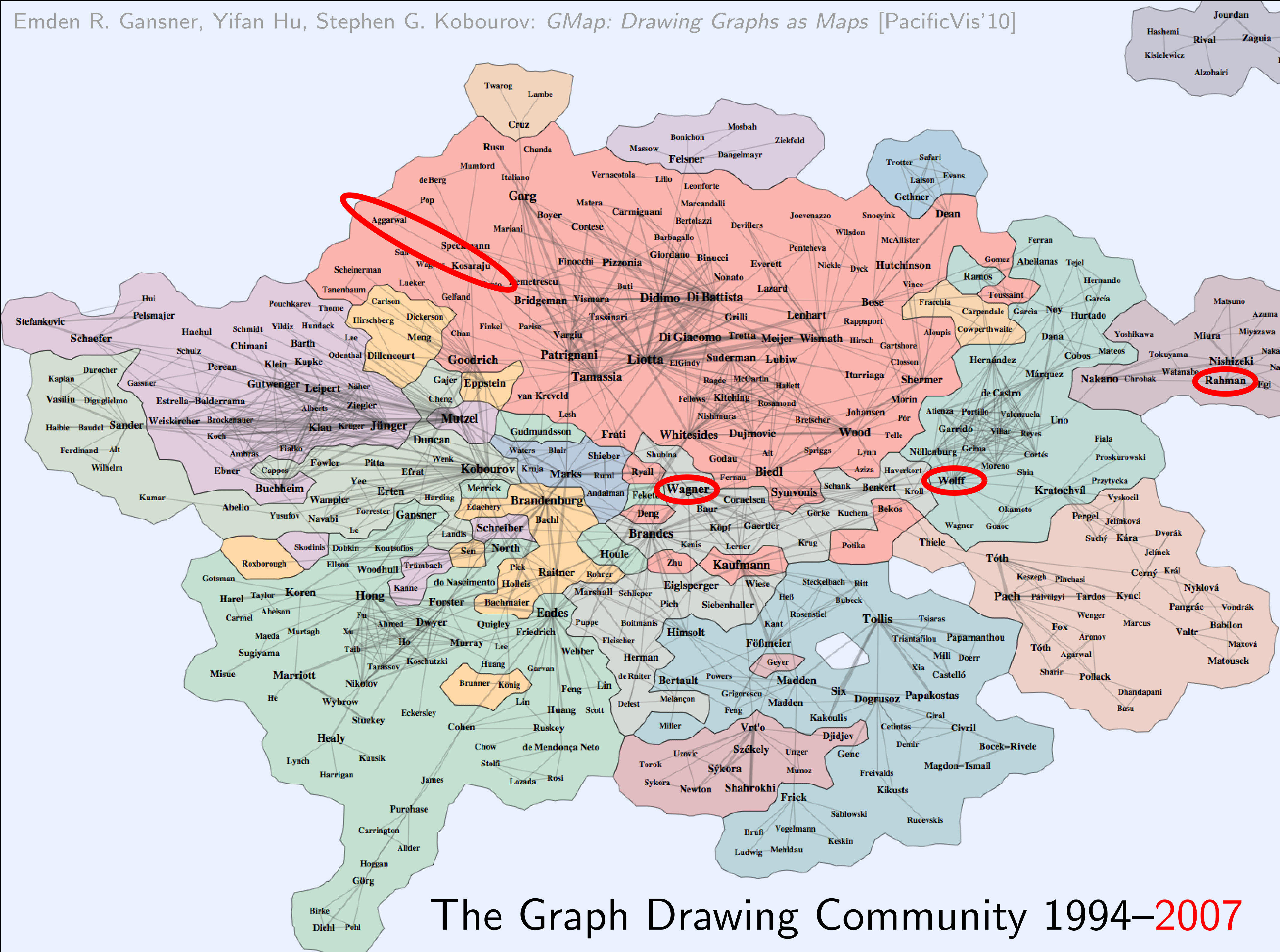

The Graph Drawing Community 1994–2007

Outline

Drawing graphs with low visual complexity

- slope number
- segment / arc number
- line / plane cover number

Geometric restrictions

- point-set embeddability
- tracks

Making crossings nicer

- RAC / LAC drawings
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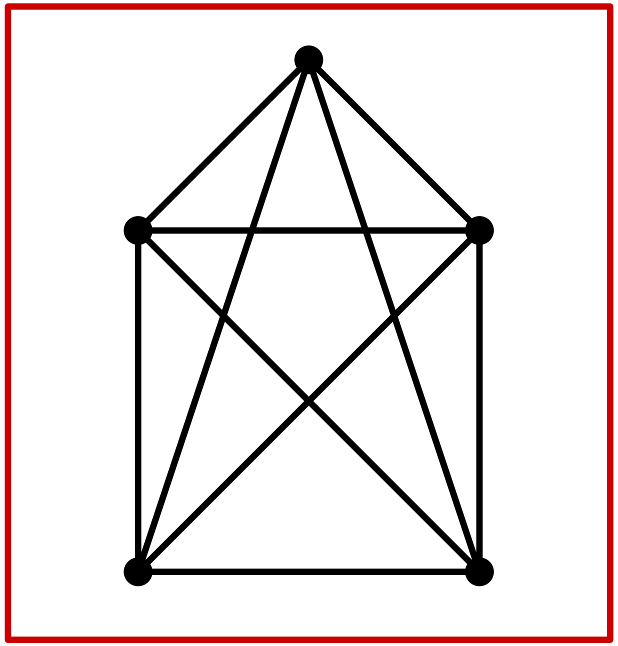

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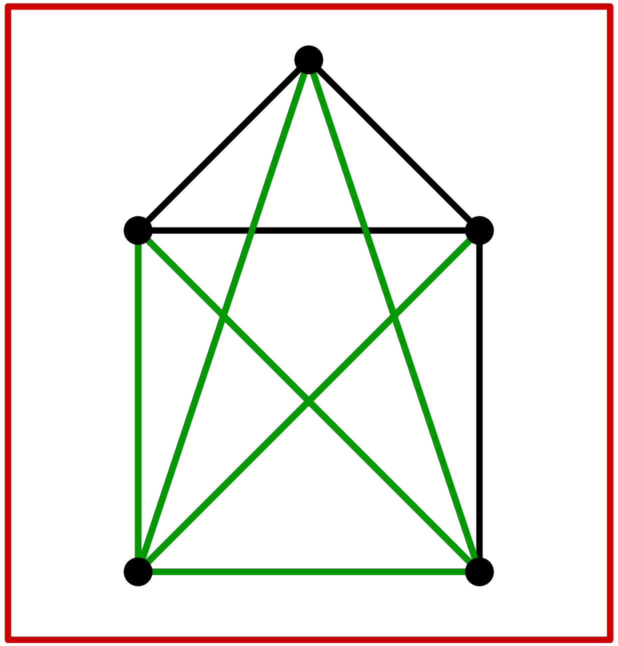

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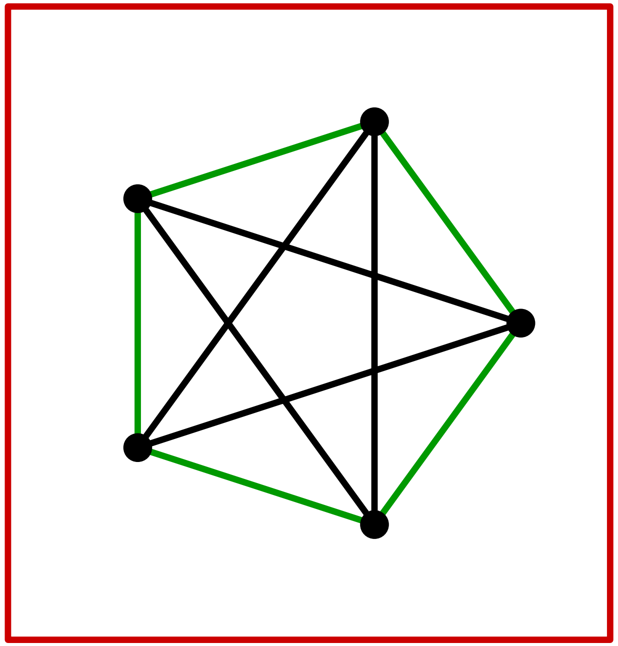

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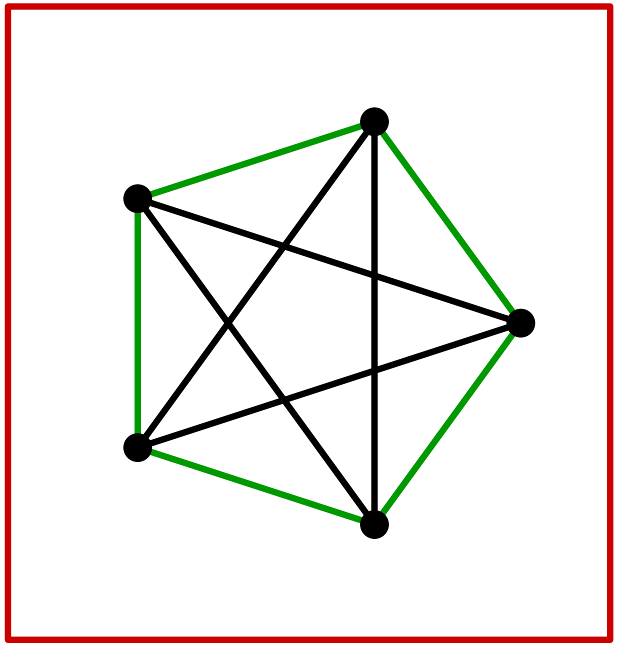

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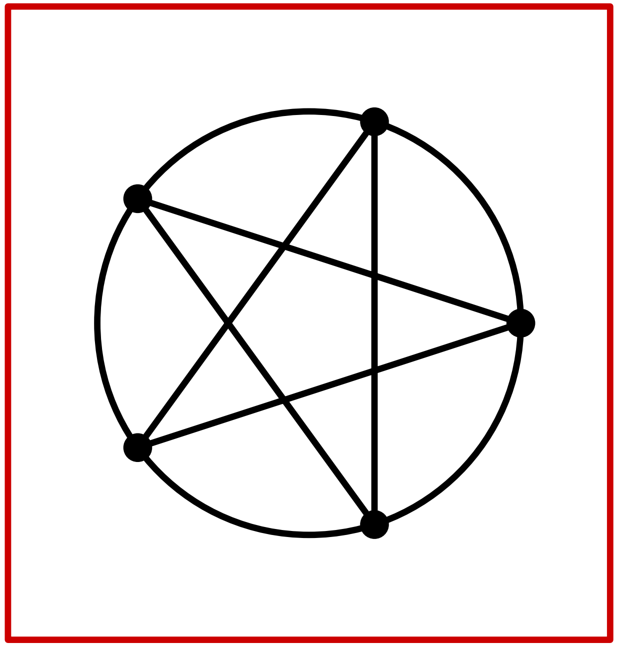

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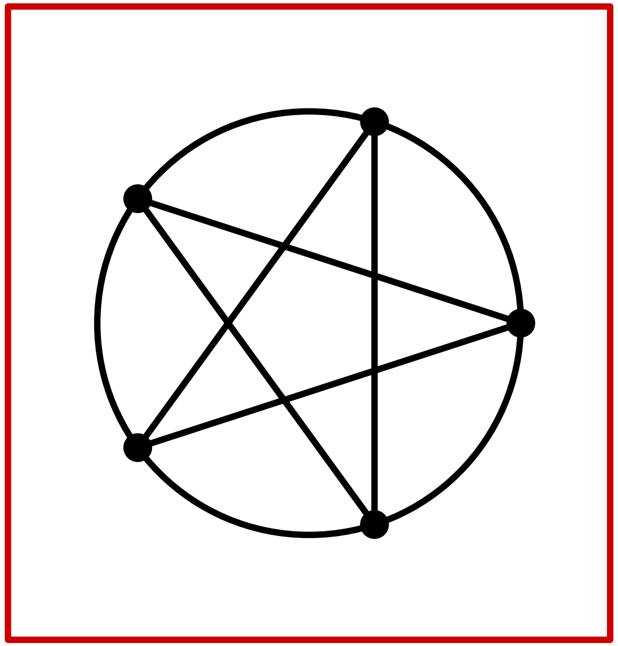

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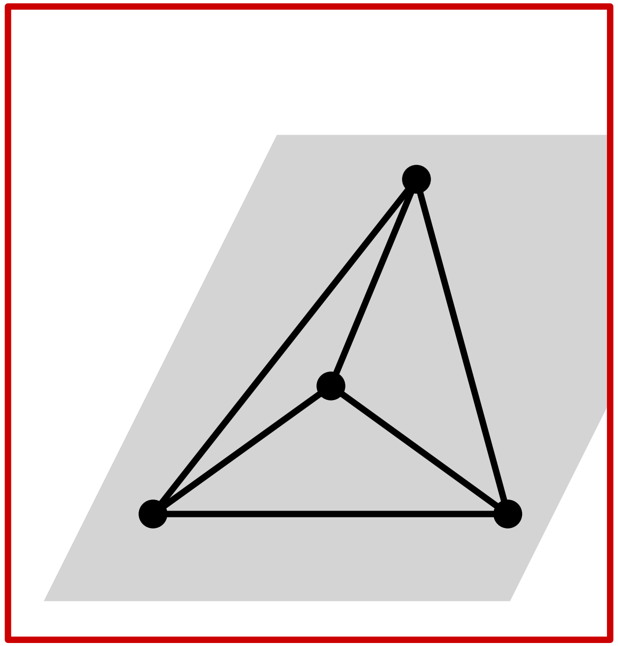

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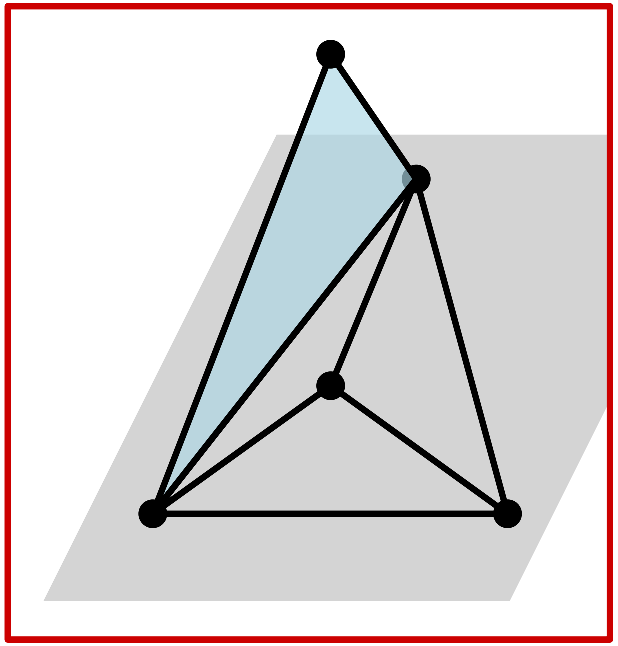

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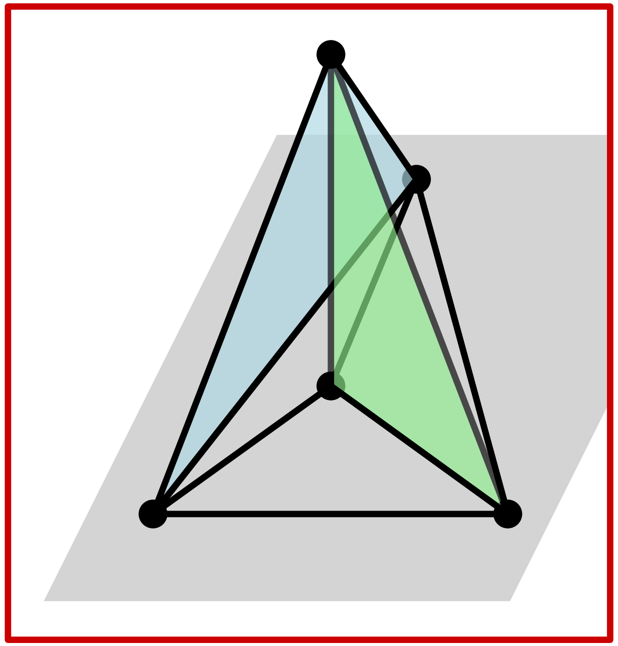

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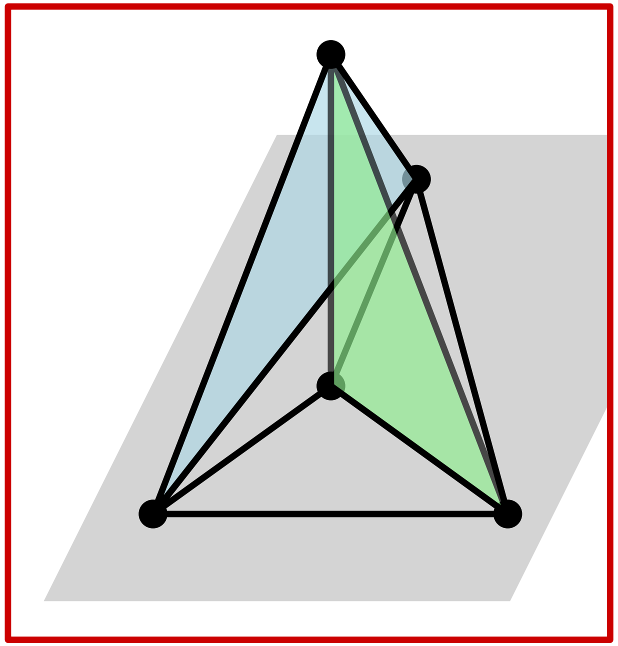

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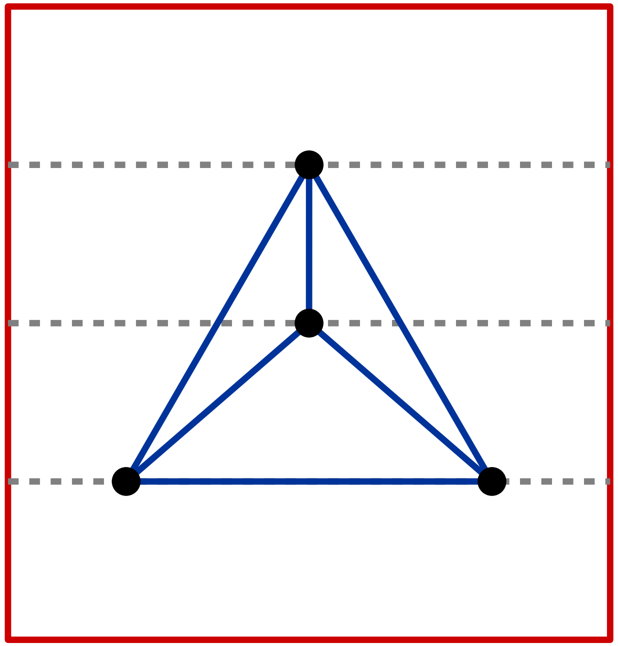

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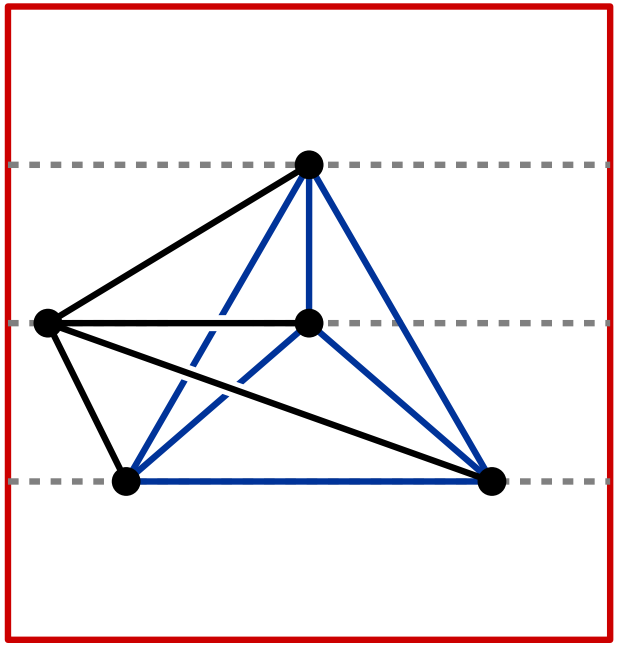

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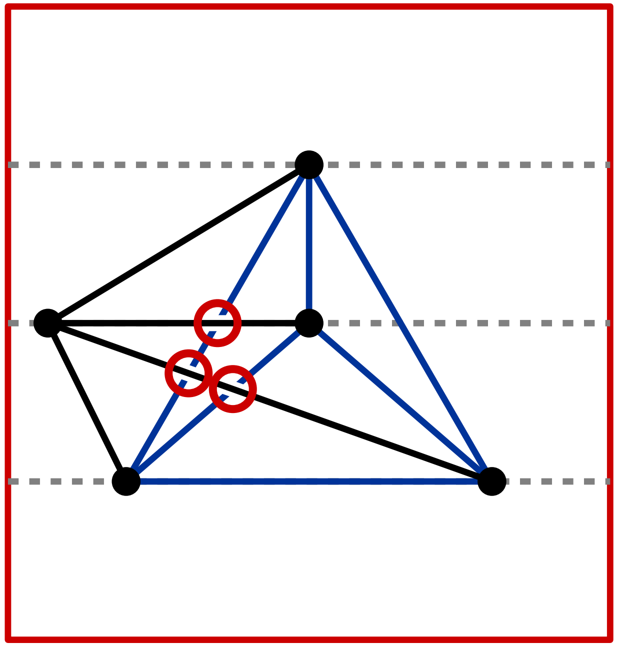

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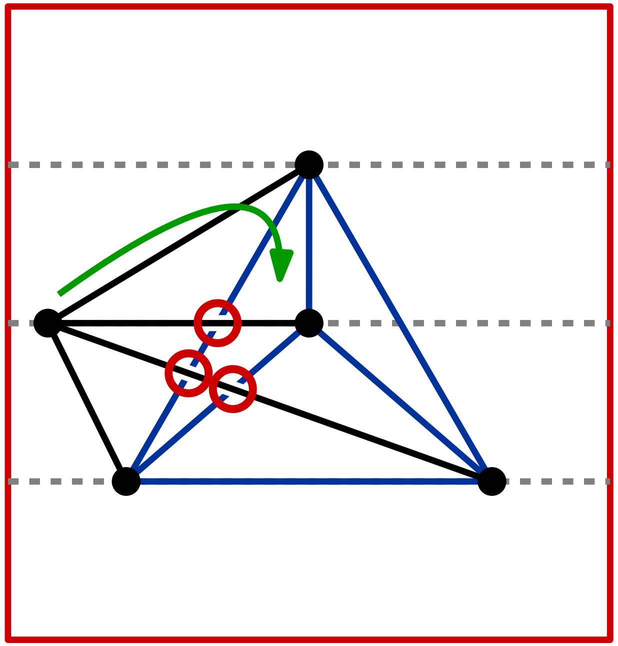

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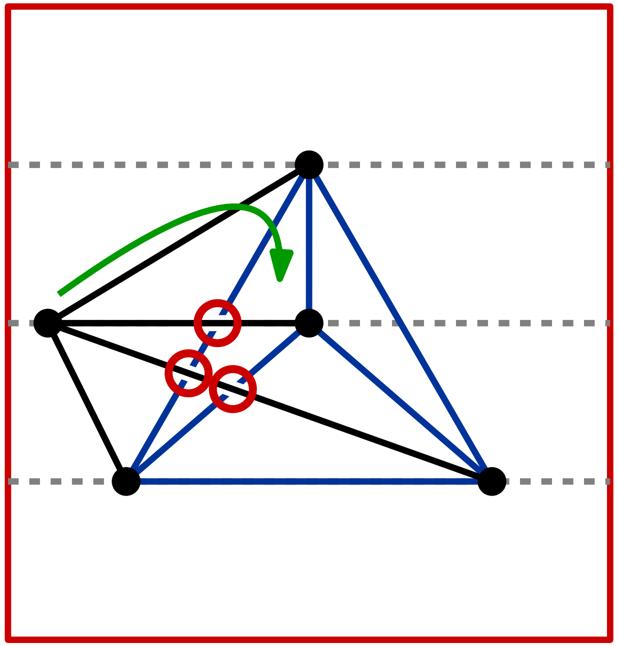

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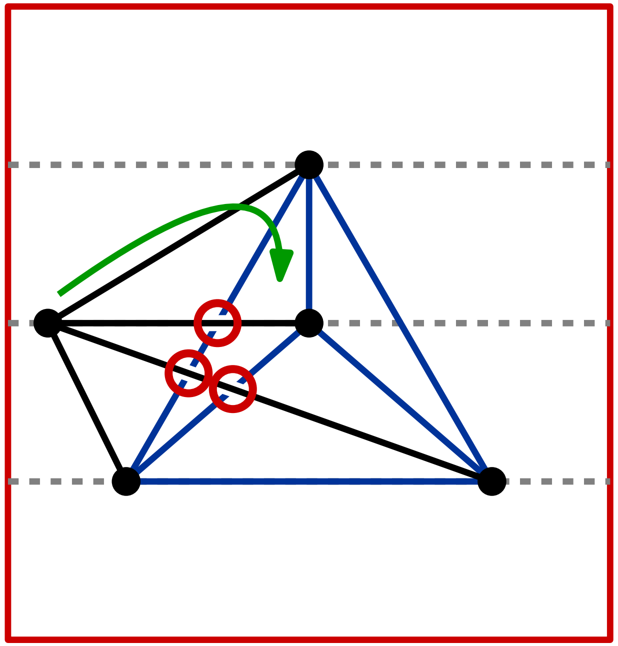

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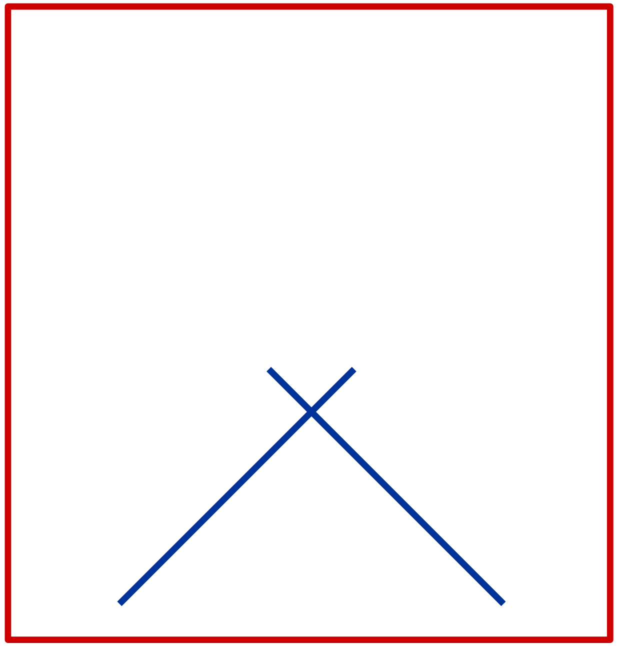

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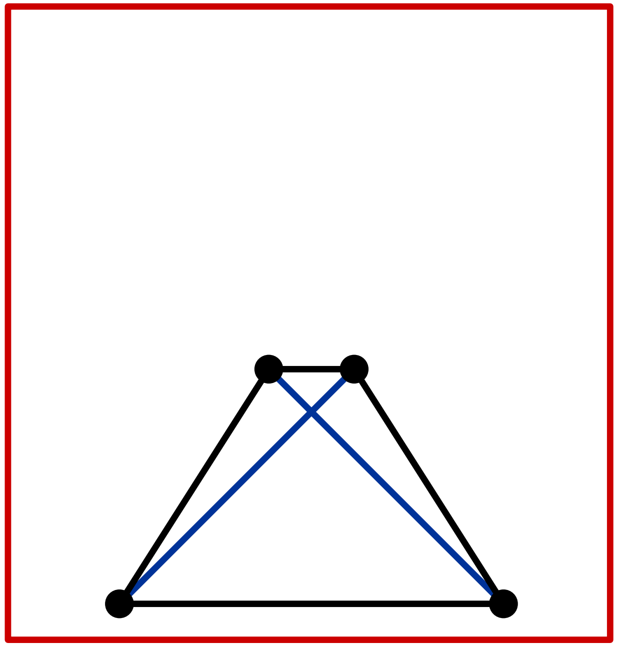

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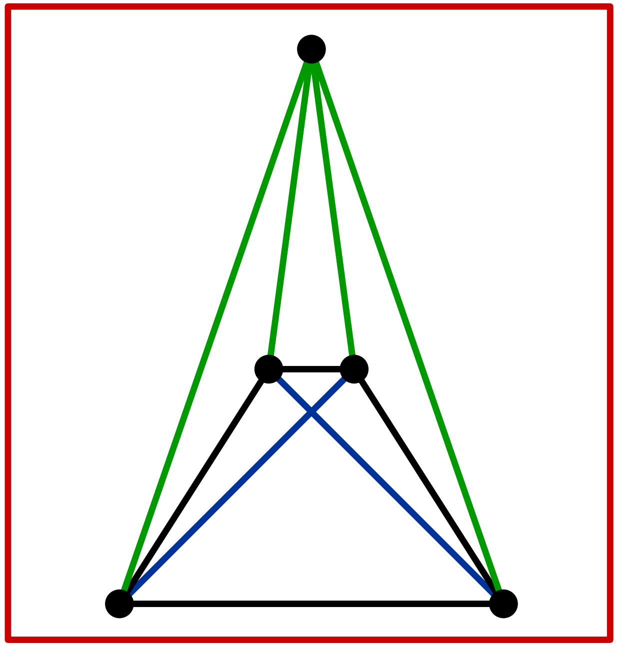

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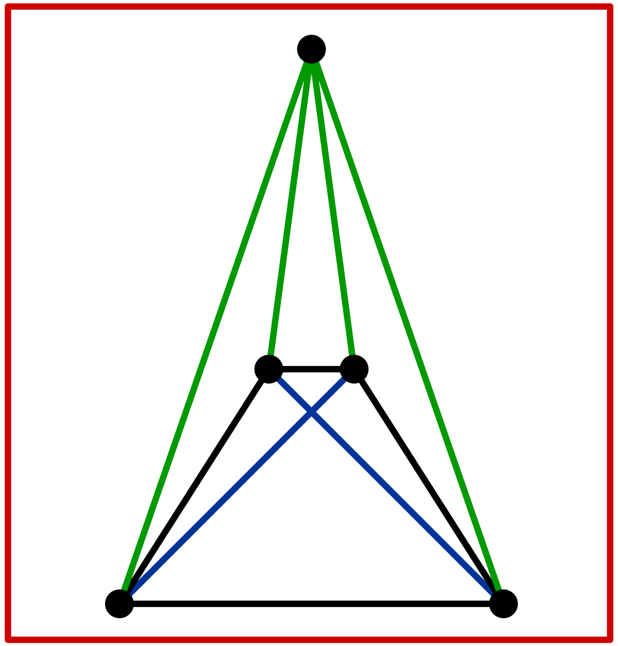

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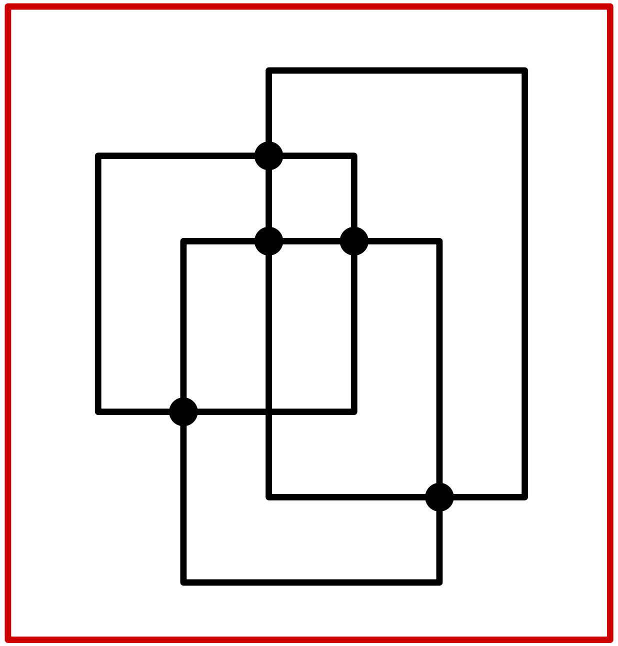

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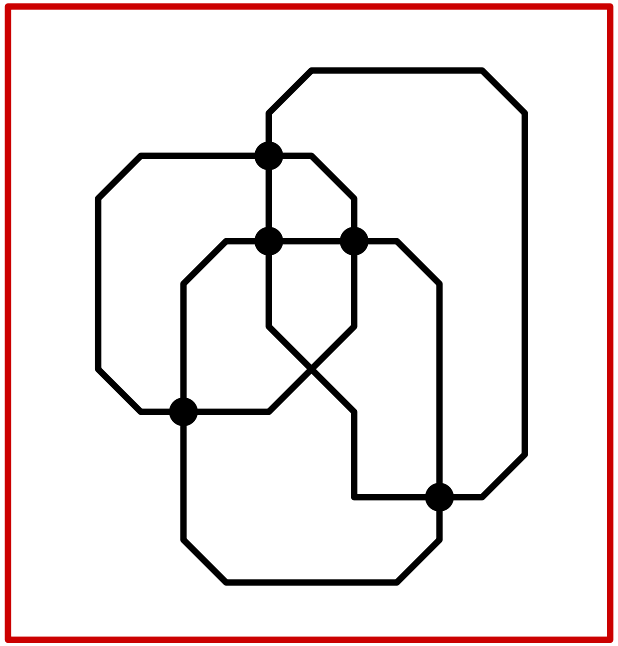

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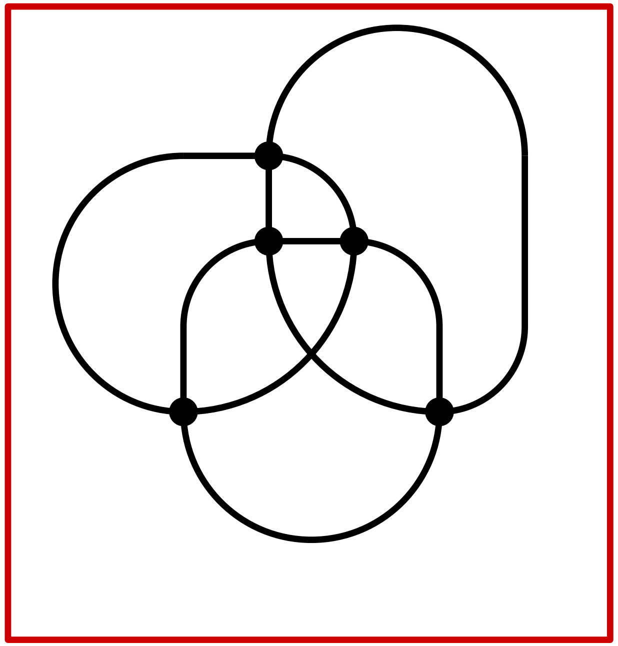

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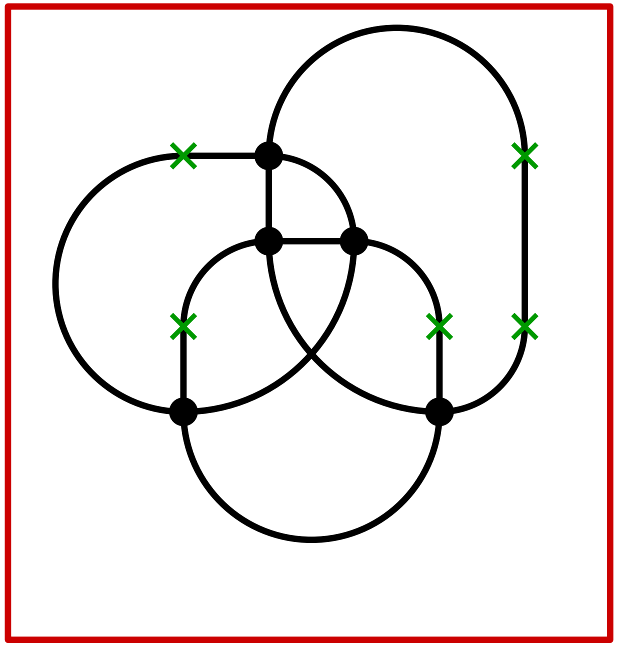

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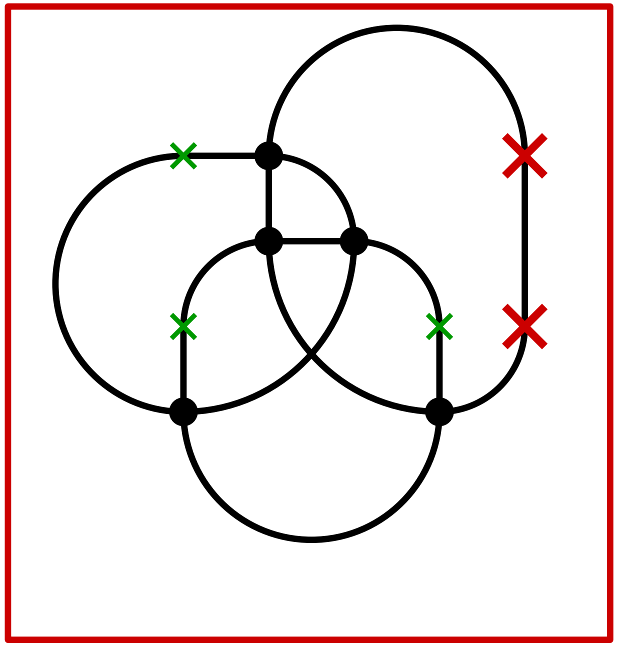

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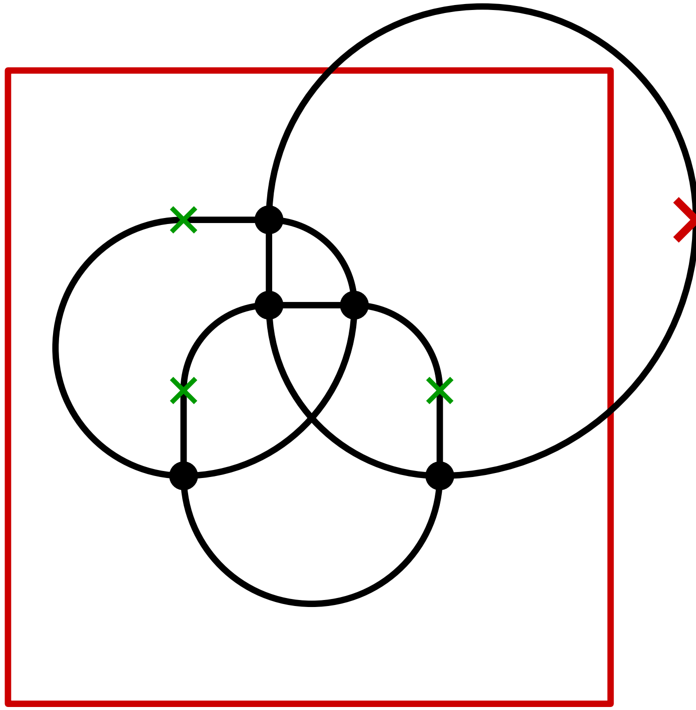

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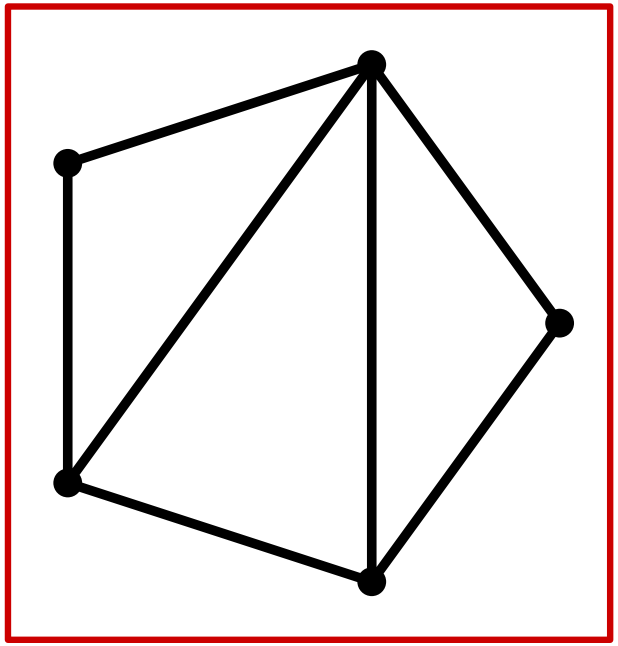

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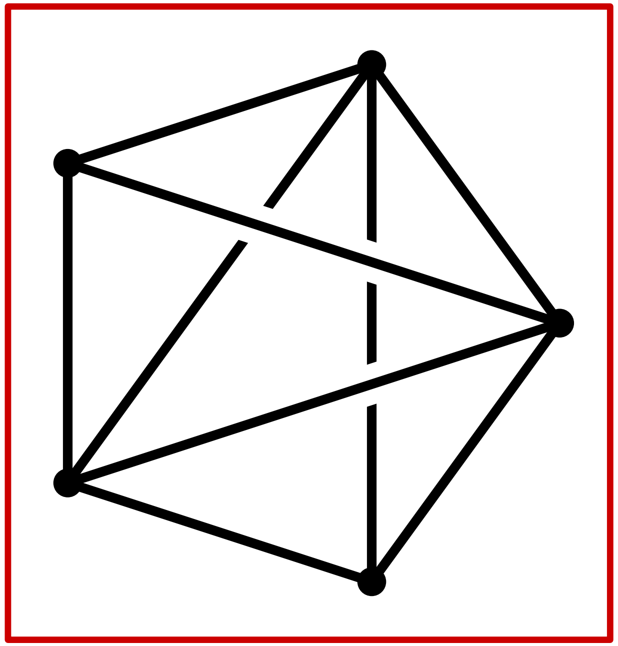

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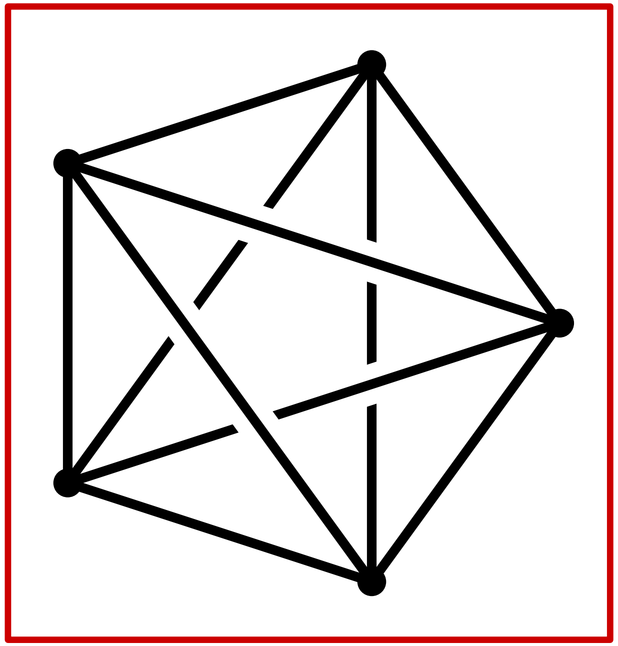

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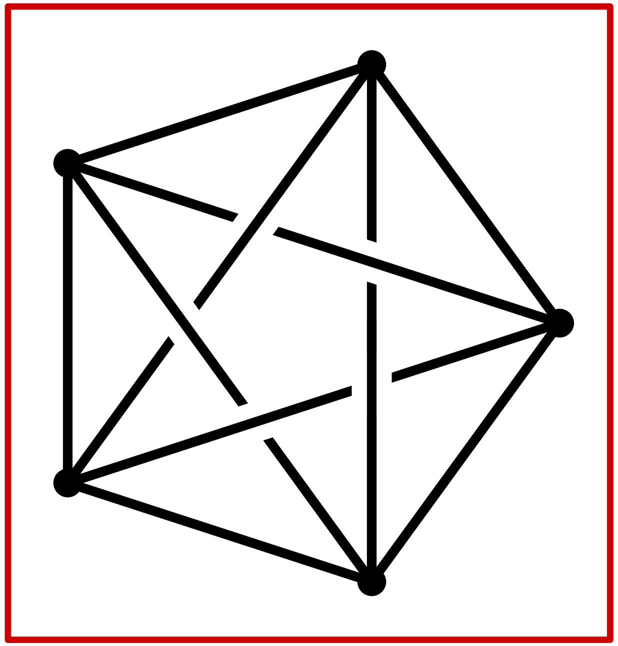

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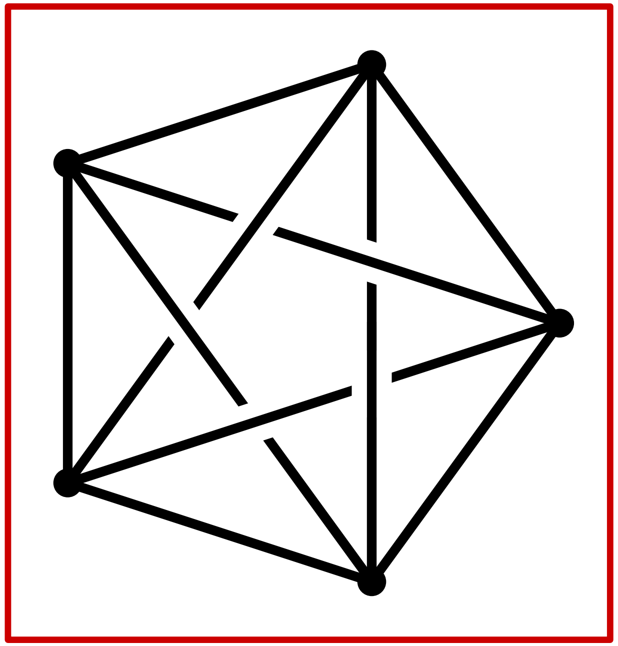

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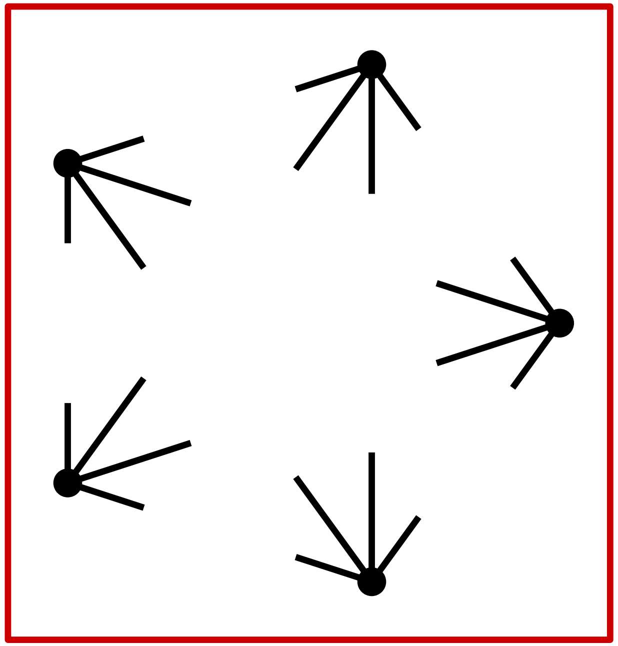

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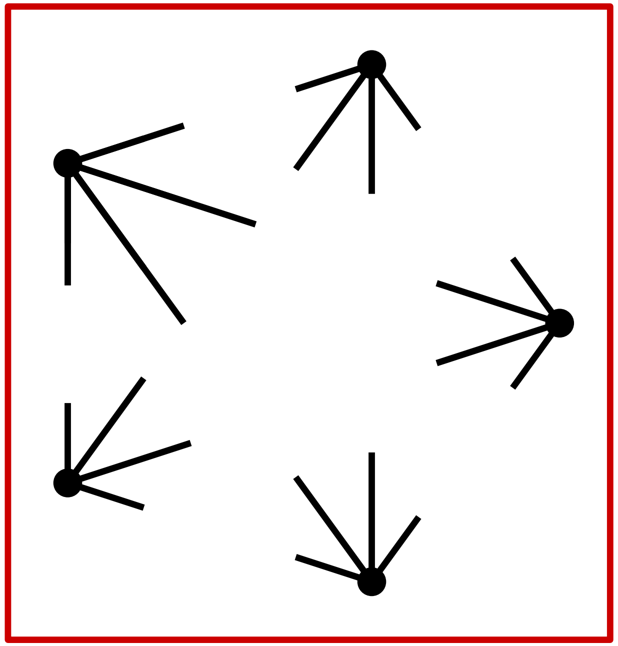

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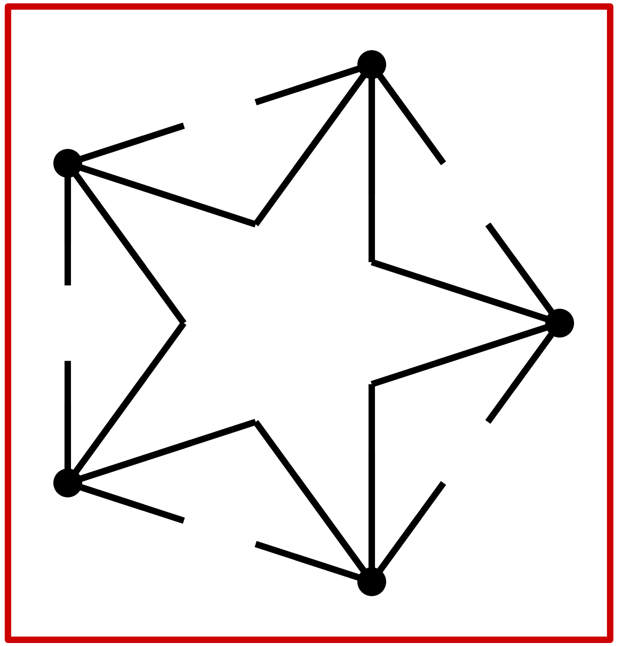

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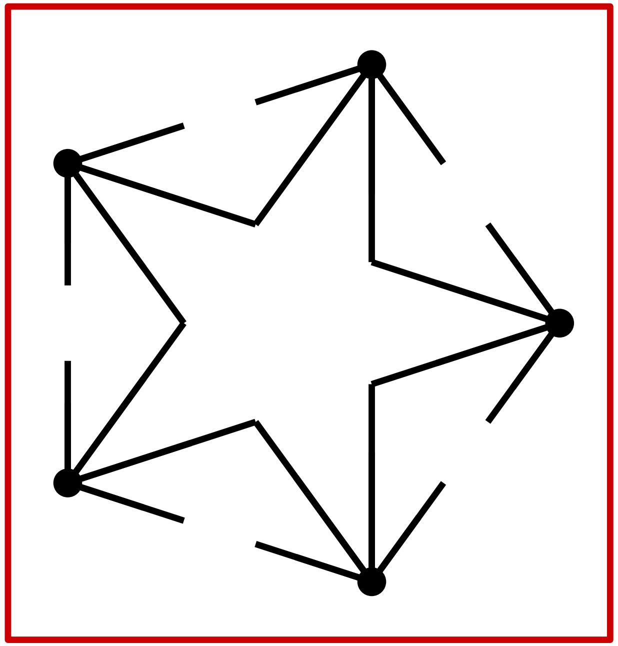

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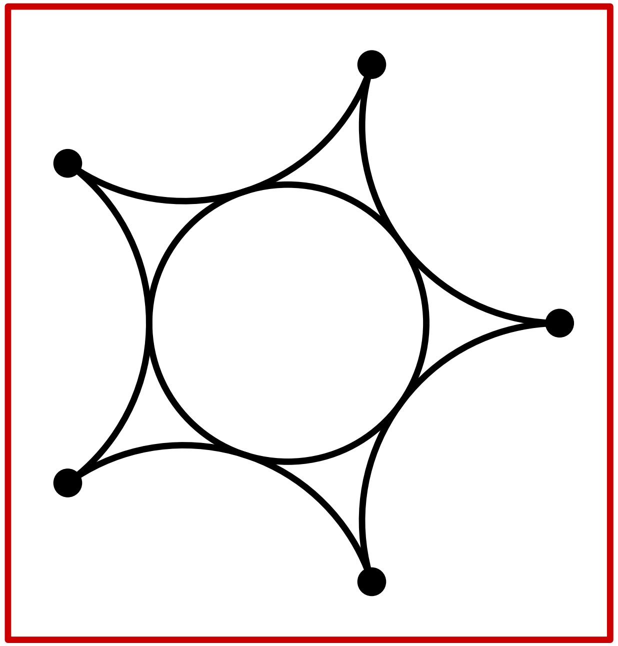

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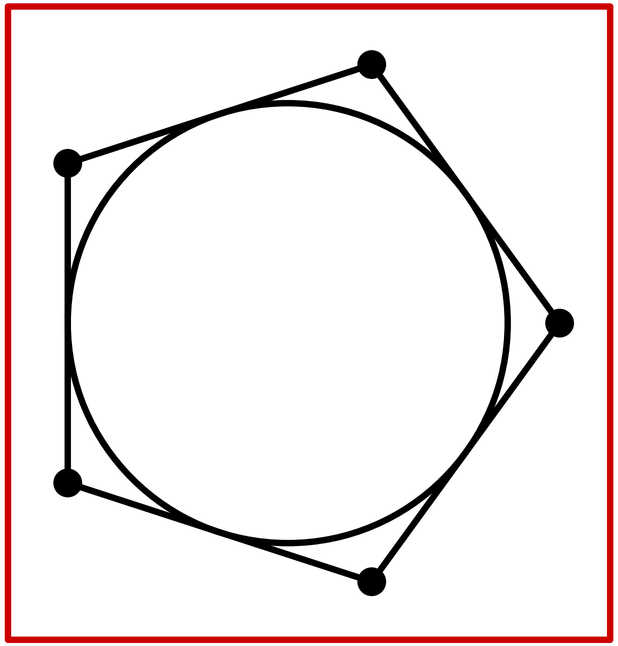

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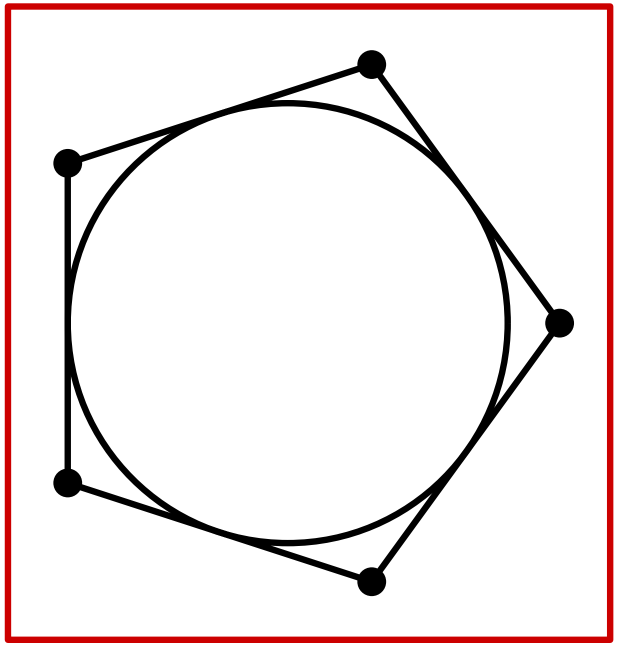

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What I Won't Talk About

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- intersection representations
- contact representations (e.g., balls in 3D)
- visibility representations (e.g., 1-bar)
- map graphs (countries are adjacent even if they touch just in a point)

Topological stuff

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Yes: for planar partial 3-trees, $f(\Delta) = \Delta^5$. [Jelínek et al. GC'13]

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Thm. For any tree T, $\mathrm{seg}(T) =$

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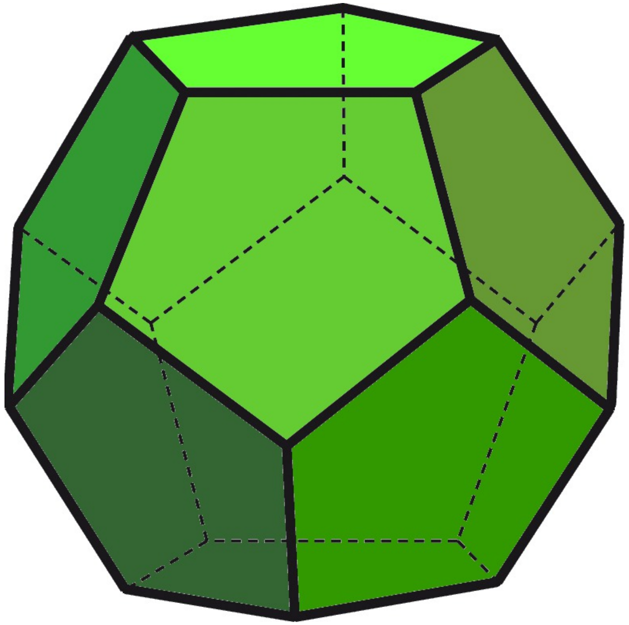
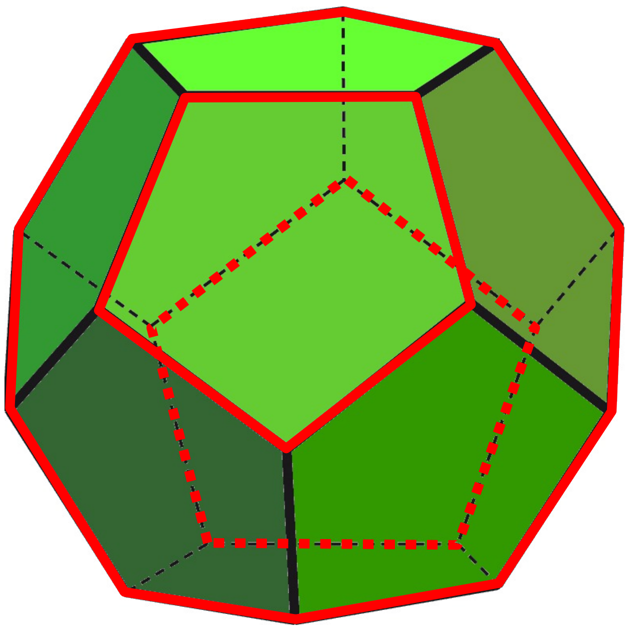
Example



12 planes in 3D

Example



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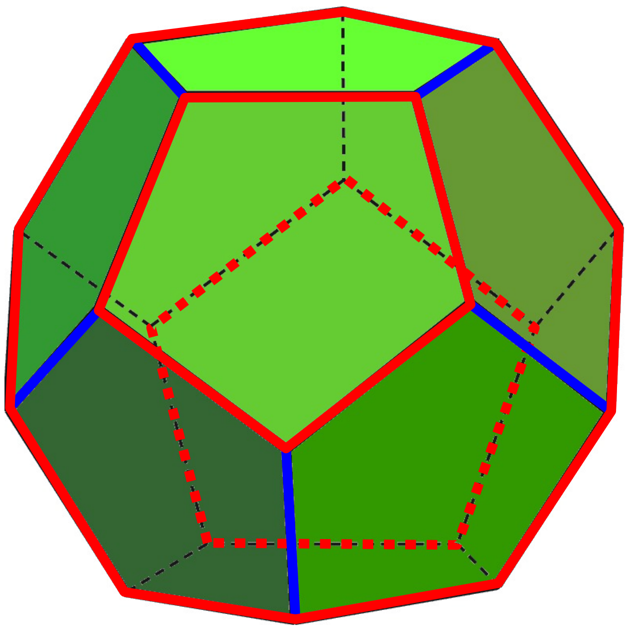

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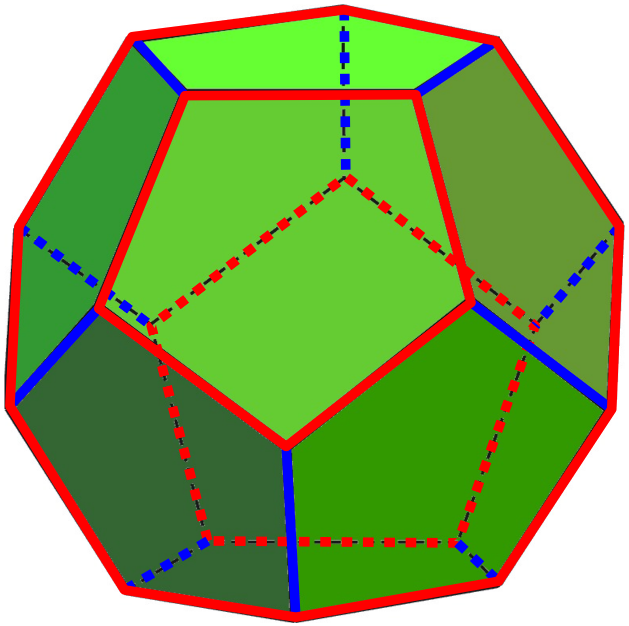

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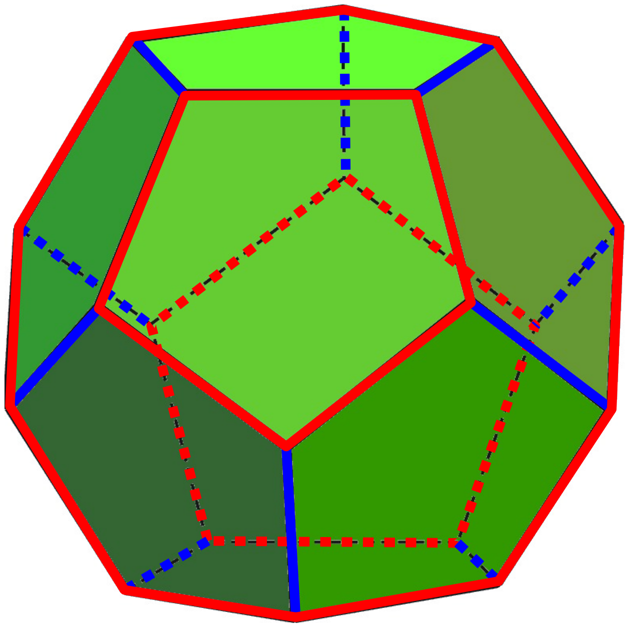

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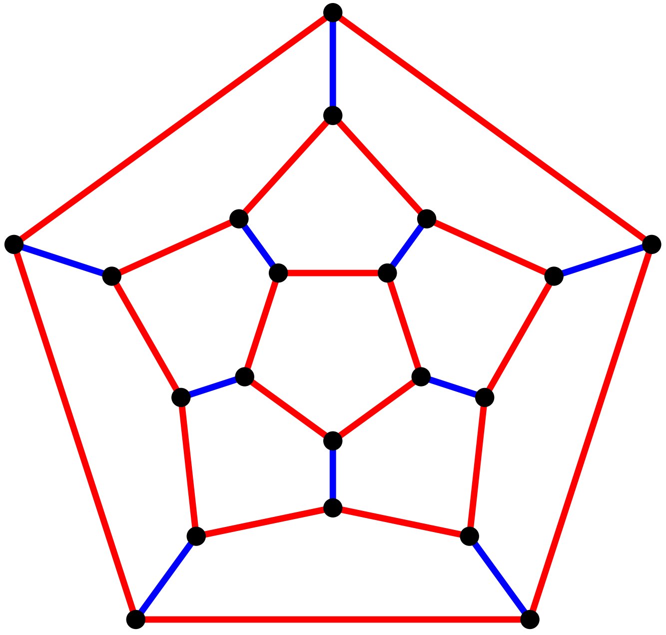
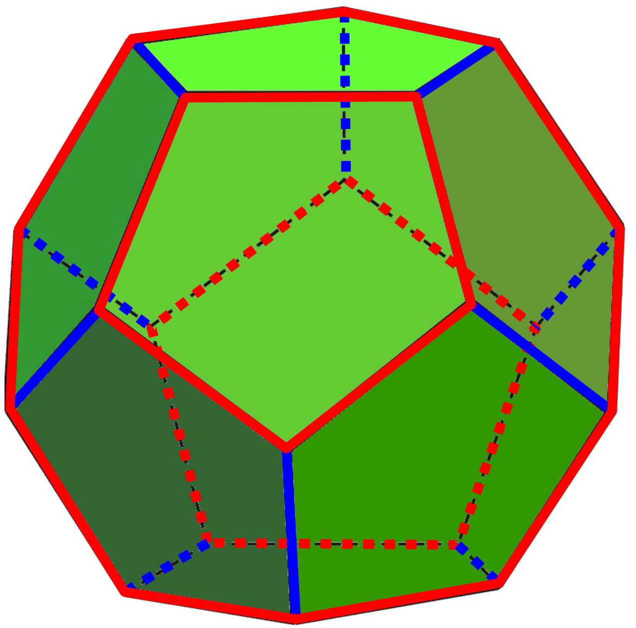


Schlegel diagram

Example



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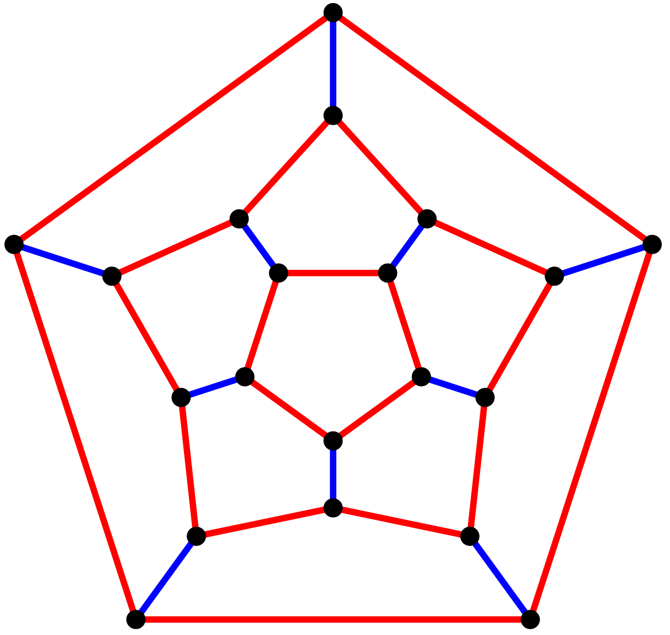

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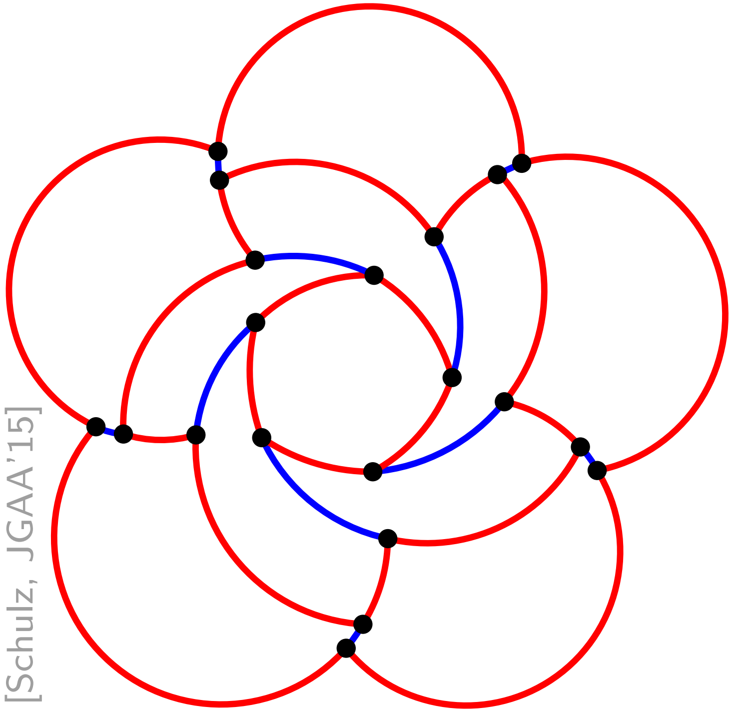
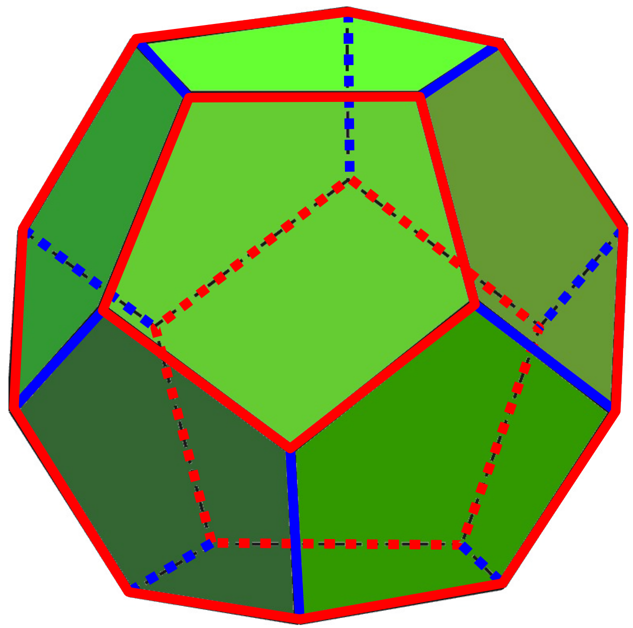


10 arcs

[Schulz, JGAA'15]

Example



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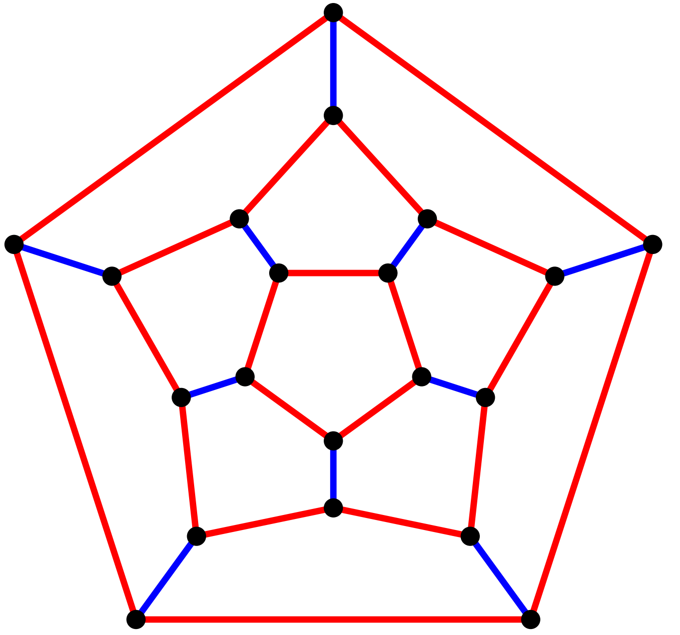

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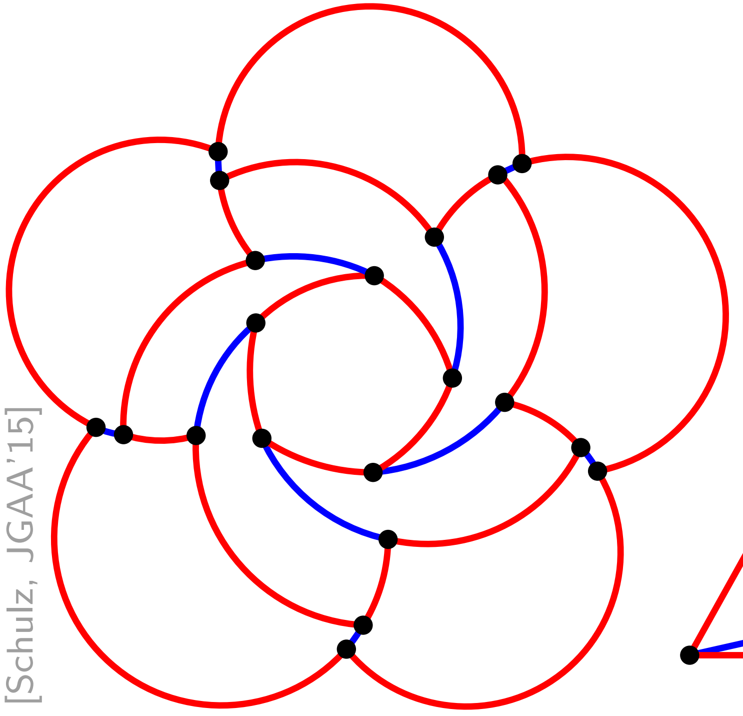

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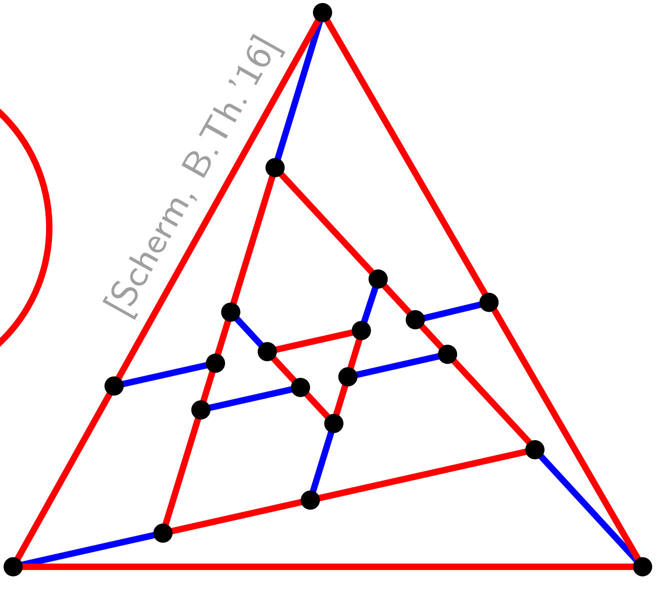
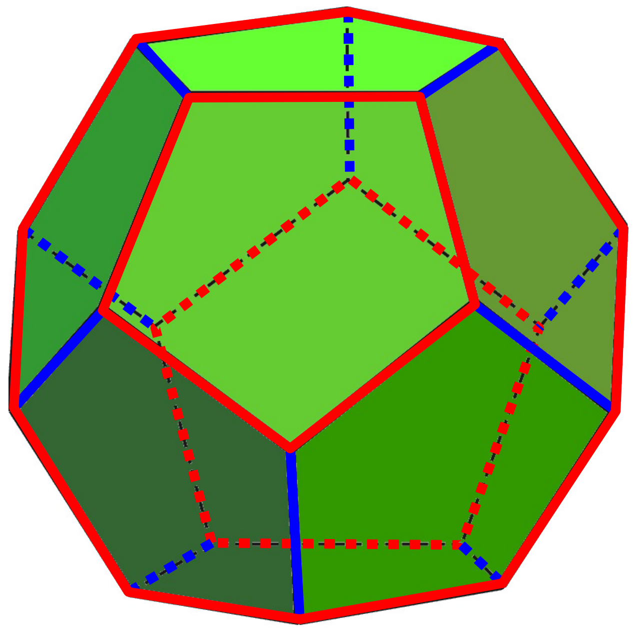


10 lines; 6 slopes; 13 seg.

Example



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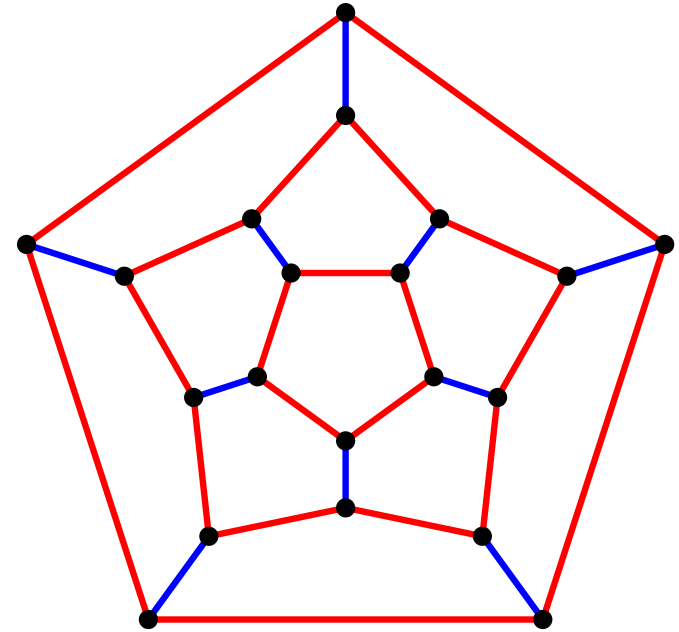

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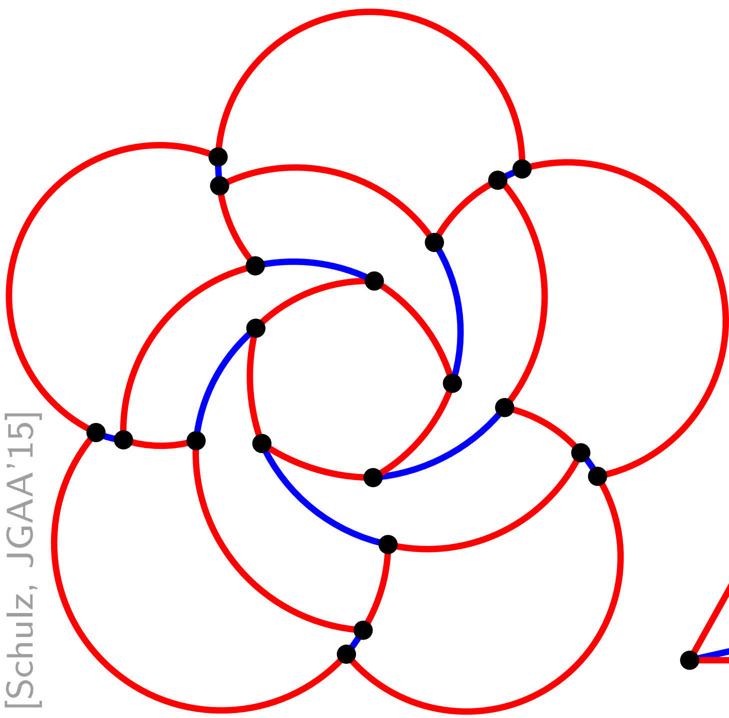

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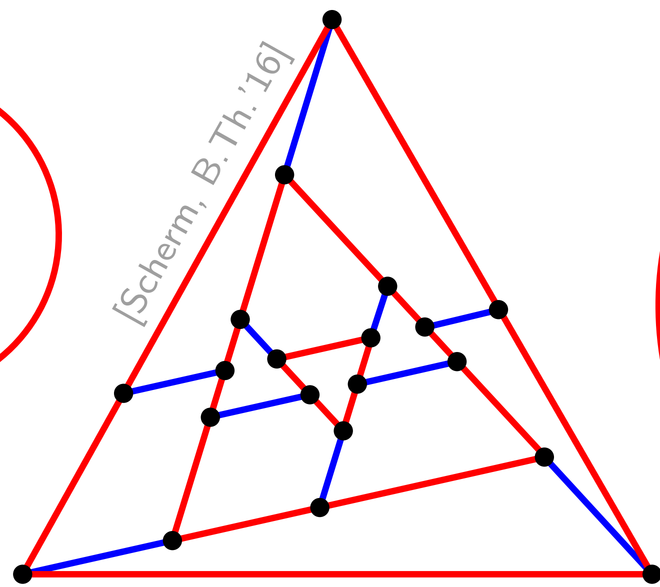

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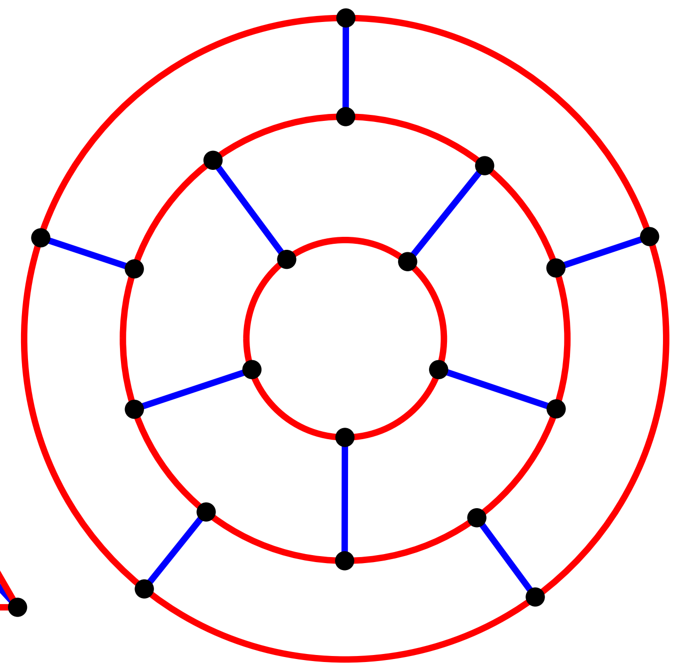

3 circles + 5 lines

[Scherm, B.Th.'16]

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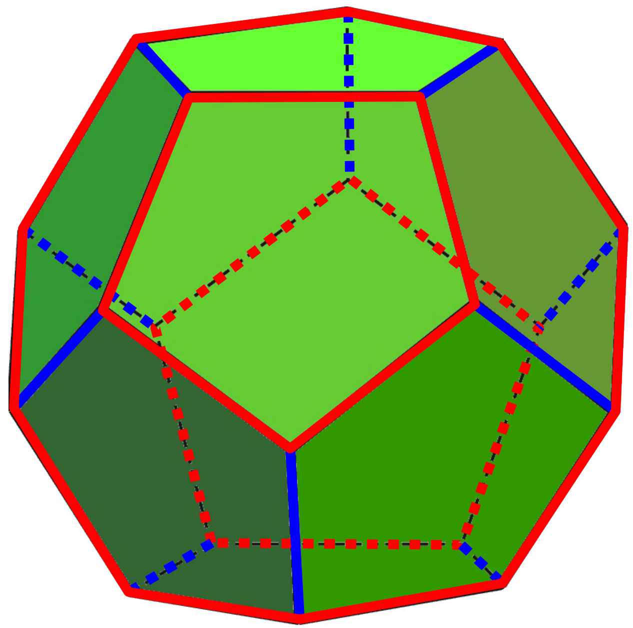

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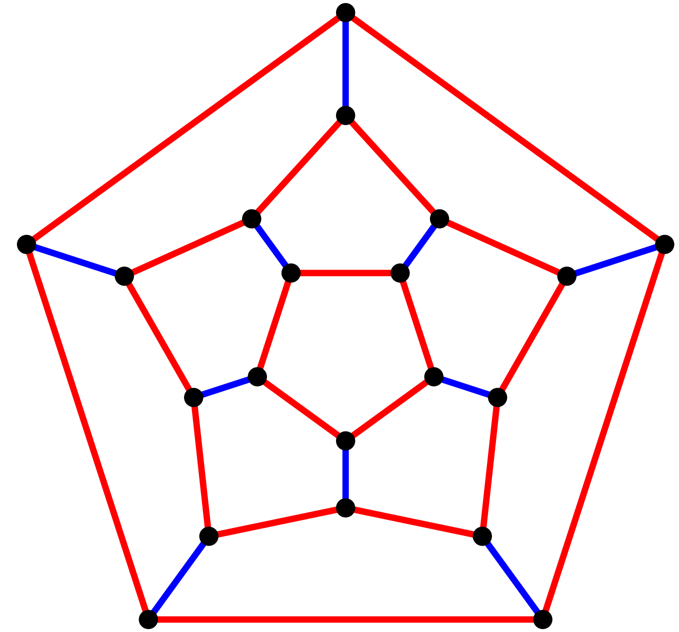
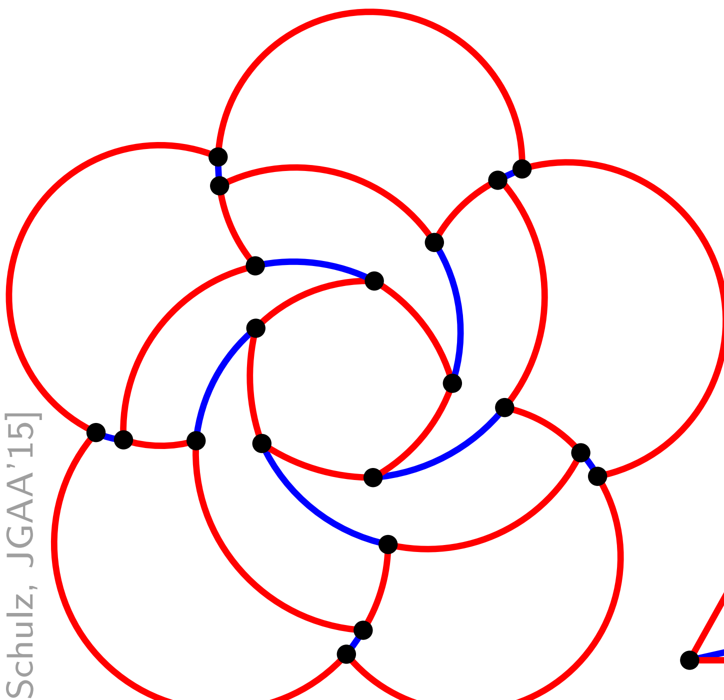


Schlegel diagram

Study this model!

[Schulz, JGAA'15]



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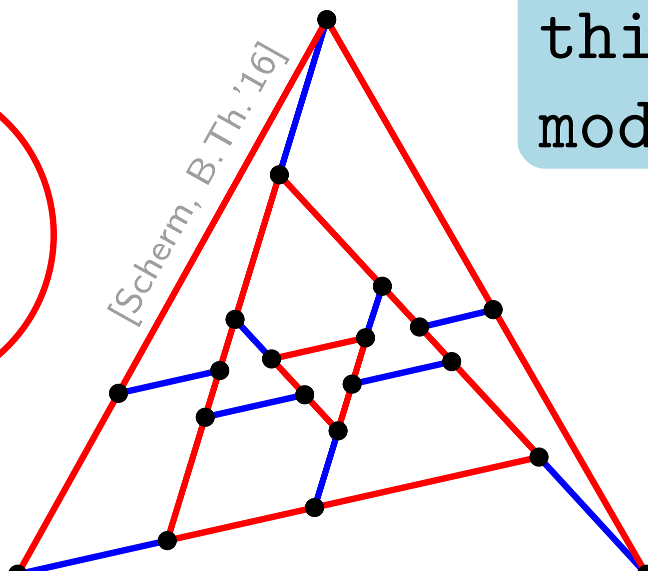

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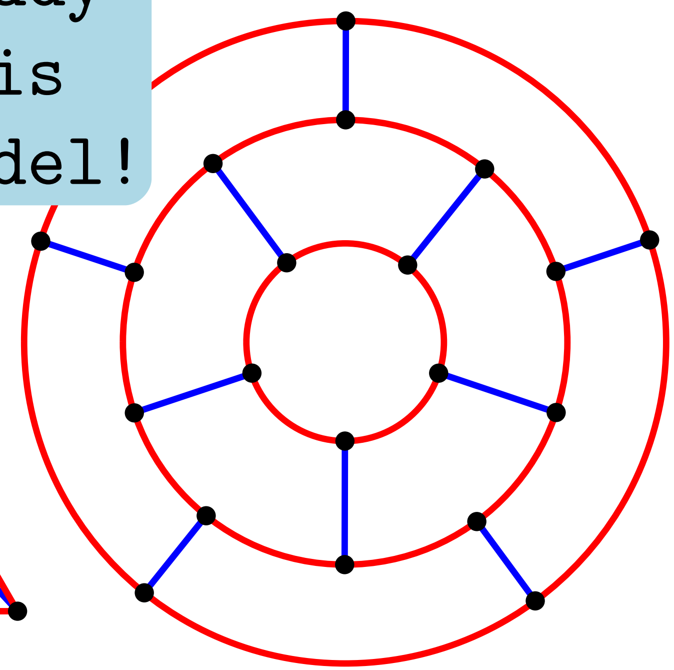

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Geometric restrictions

- point-set embeddability
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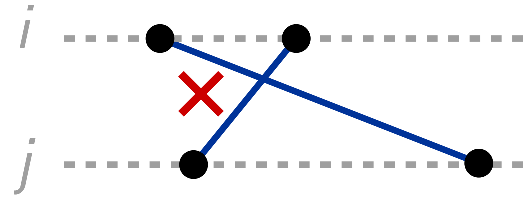
How can we combine this concept with beyond-planarity?

Track Number

[Dujmović, Pór, Wood DMTCS'04]

Def. A *t-track layout* consists of a map $\tau\colon V \to \{1, \ldots, t\}$ and orders $(<_i)_{1 \le i \le t}$ on $V_i := \tau^{-1}(i)$ such that
– V_i is independent and
– there are no X-crossings:



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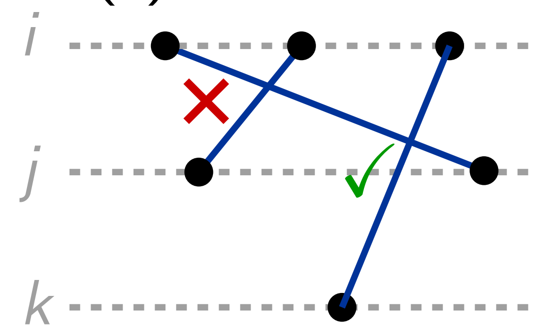

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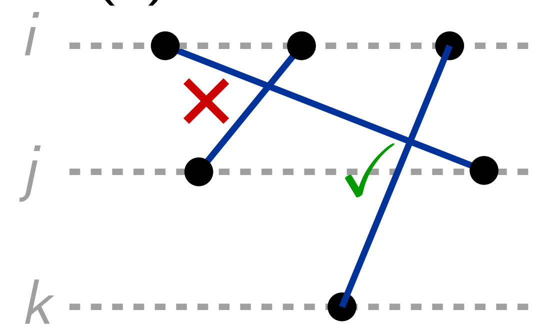

The *track number* tn(G) of G is the smallest t such that G has a t-track layout.

Thm. Any t-track G has a s-l drawing in $O(t) \times O(t) \times O(n)$.

[Dujmović, Morin, Wood SIAM.J.Comp.]

Track Number – Results

graph family	track-number	reference
n vertices	n	trivial
m edges	$15m^{2/3}$	Dujmović and Wood [28]
m edges, max. degree Δ	$14\sqrt{\Delta m}$	Dujmović and Wood [28]
no K_h-minor	$O(h^{3/2}n^{1/2})$	Dujmović and Wood [28]
genus γ	$O(\gamma^{1/2}n^{1/2})$	Dujmović and Wood [28]
tree-width w	$3^w \cdot 6^{(4^w-3w-1)/9}$	Dujmović *et al*. [26]
tree-width w, max. degree Δ	$72\Delta w$	Dujmović *et al*. [26]
queue-number k, acyclic chromatic number c	$c(2k)^{c-1}$	Dujmović *et al*. [26]; see Theorem 2
queue-number k	$4k \cdot 4^{k(2k-1)(4k-1)}$	Theorem 8
path-width p	$p+1$	Dujmović *et al*. [26]
band-width b	$b+1$	Lemma 17
series-parallel graphs	15	Di Giacomo *et al*. [21]
Halin	$8^\dagger$	Di Giacomo and Meijer [23]
X-trees	$6^\dagger$	Di Giacomo and Meijer [23]
outerplanar	$5^\dagger$	Lemma 22
1-queue graphs	4	Theorem 11
trees	3	Felsner *et al*. [32]

Making crossings nicer

- RAC / LAC drawings
- (smooth / slanted) orthogonal drawings
- edge casings

Right- / Large-Angle Crossings

[Didimo, Eades, Liotta TCS'11]

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All graphs can be RAC_3-drawn on an $O(n+m)^2$-grid.

[DEL'11, Arikushi, Fulek, Keszegh, Morić, Tóth CGTA'12]

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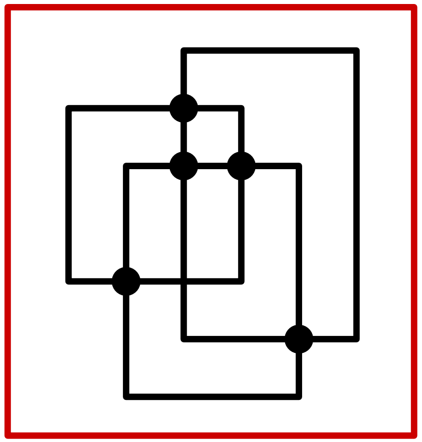
Orthogonal Drawings



Slanted Orthogonal Drawings

Def. In a *slanted* orthogonal drawing,
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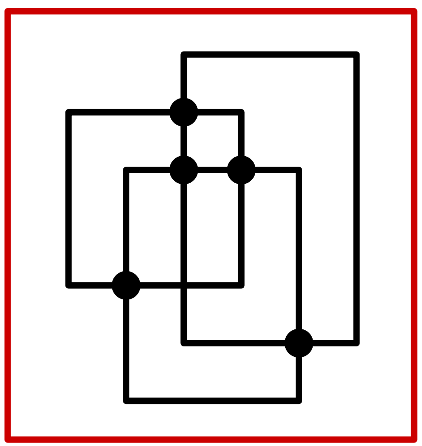
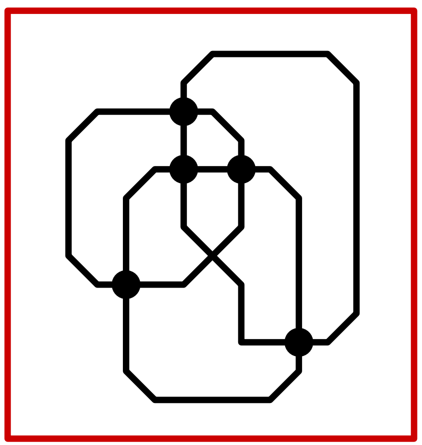

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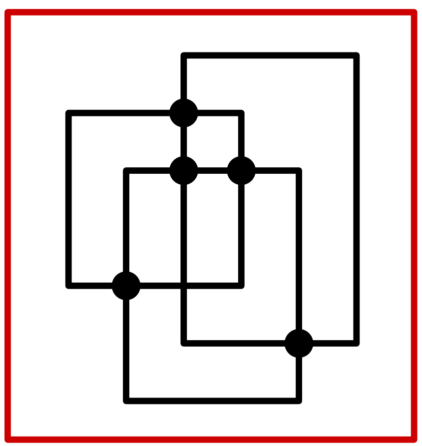

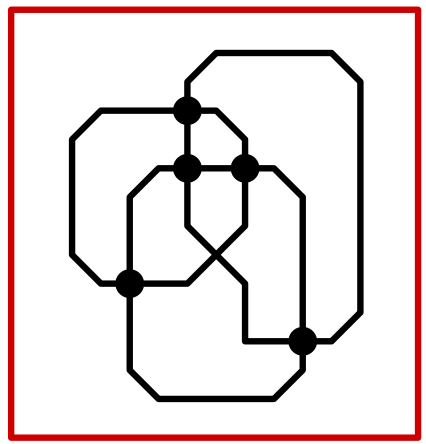

(Slanted / Smooth) Orthogonal Drawings

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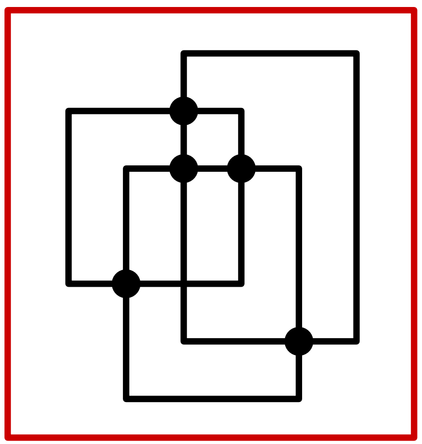

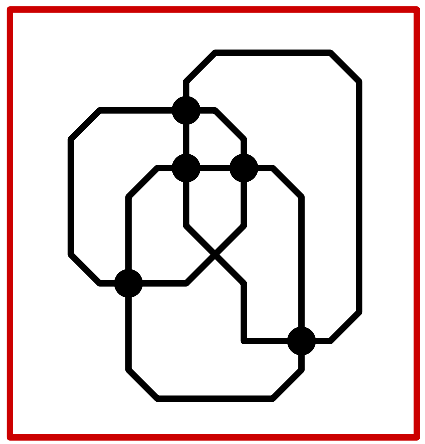

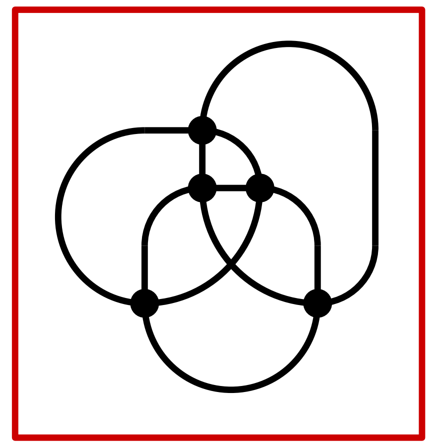

Def. In a *smooth* orthogonal drawing,
- edges enter vertices only in the four ports N, E, S, W,
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- consecutive arcs meet in horizontal or vert. tangents.

(Slanted / Smooth) Orthogonal Drawings

Def. In a *slanted* orthogonal drawing,
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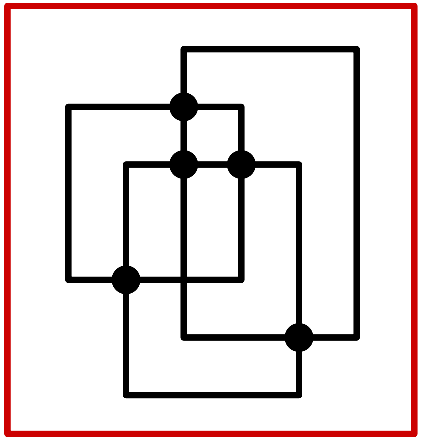

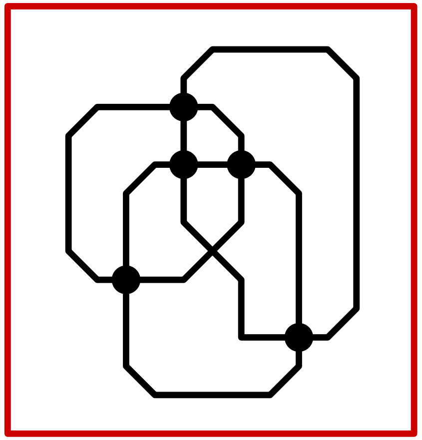

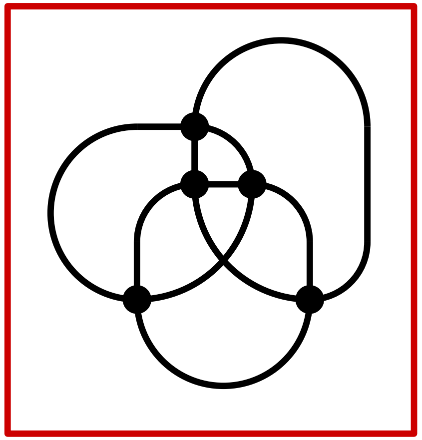

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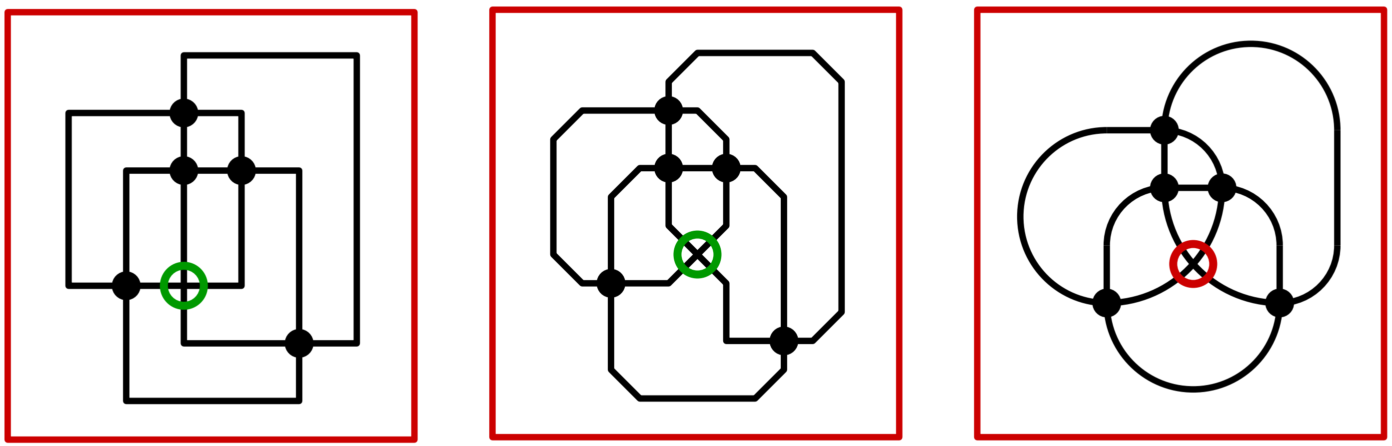

Study: Which crossin do we allow?

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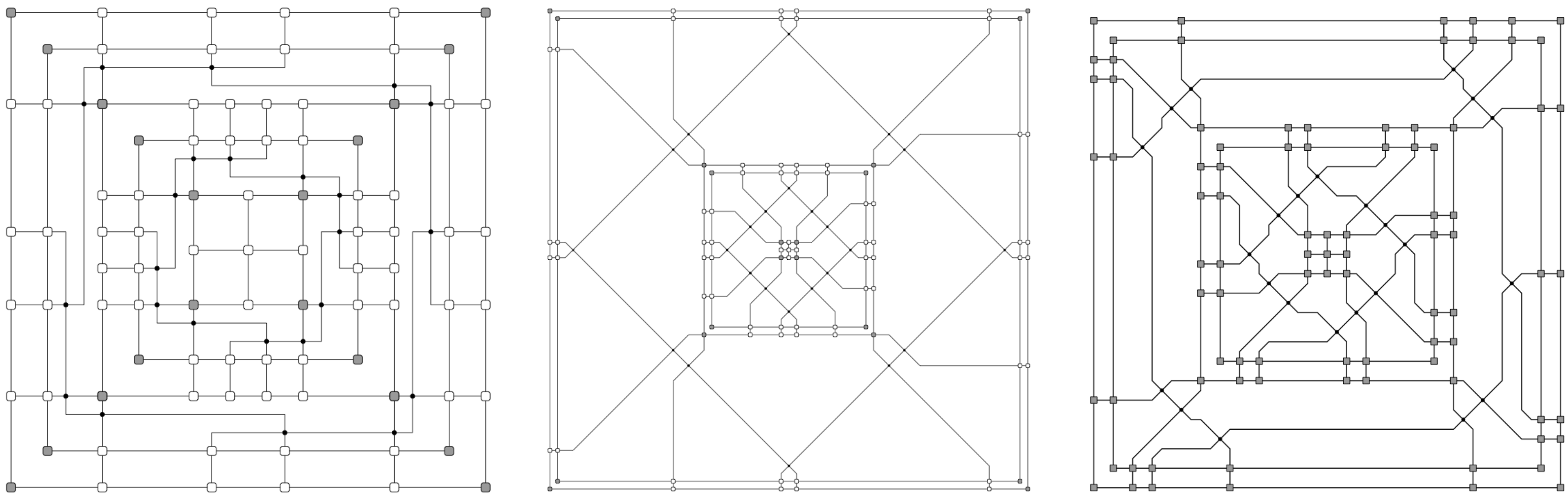
Slanted Orthogonal (slog) Drawings

[Bekos, Kaufmann, Krug, Ludwig, Näher, Roselli JGAA’14]



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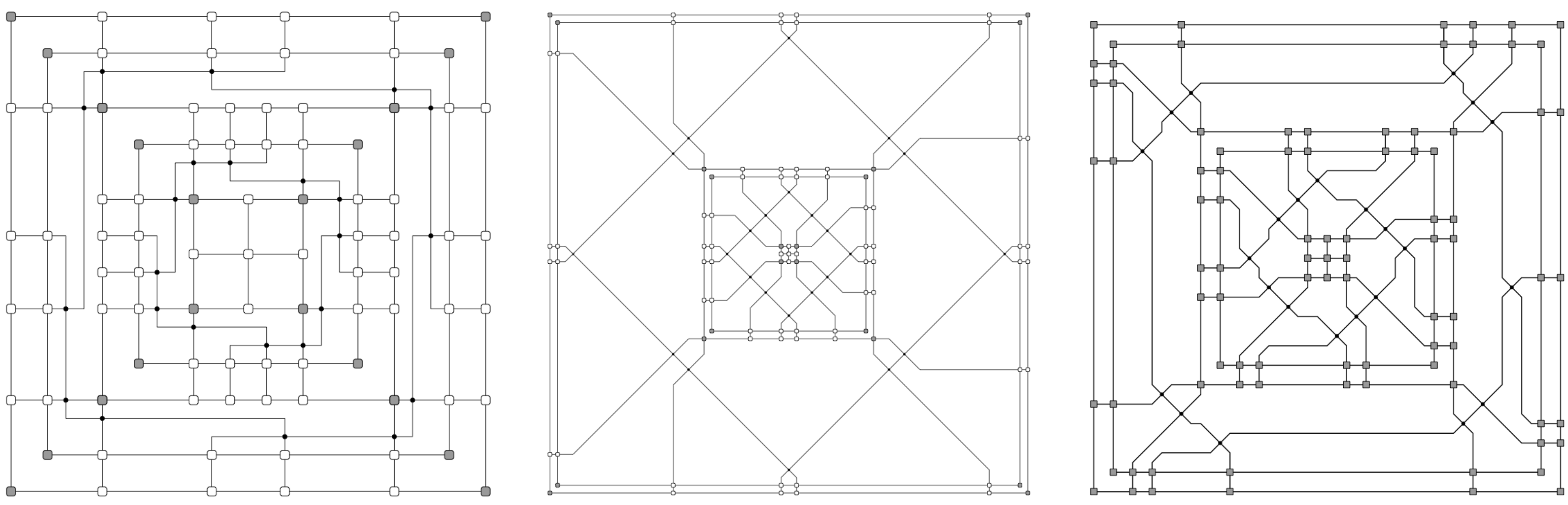

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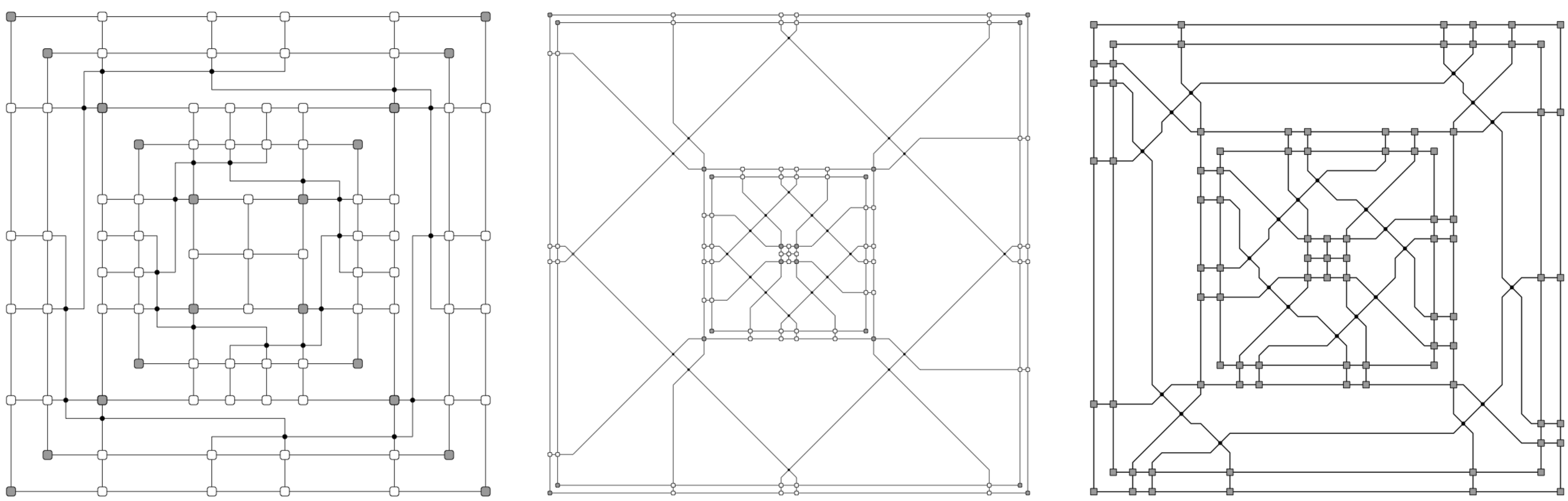

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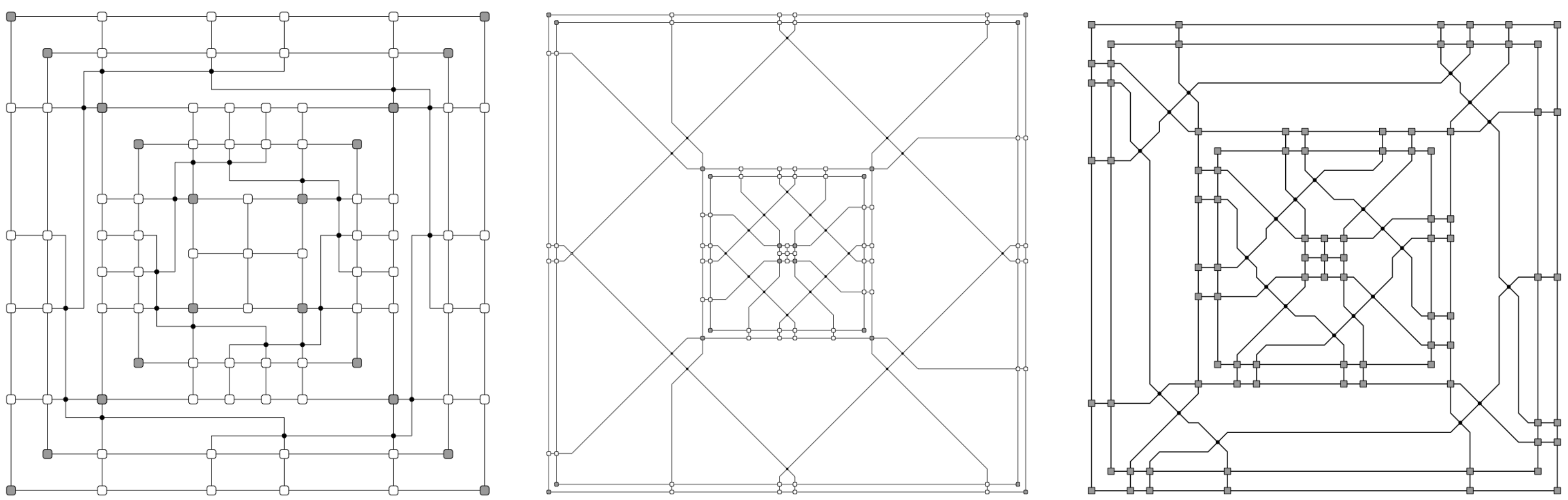

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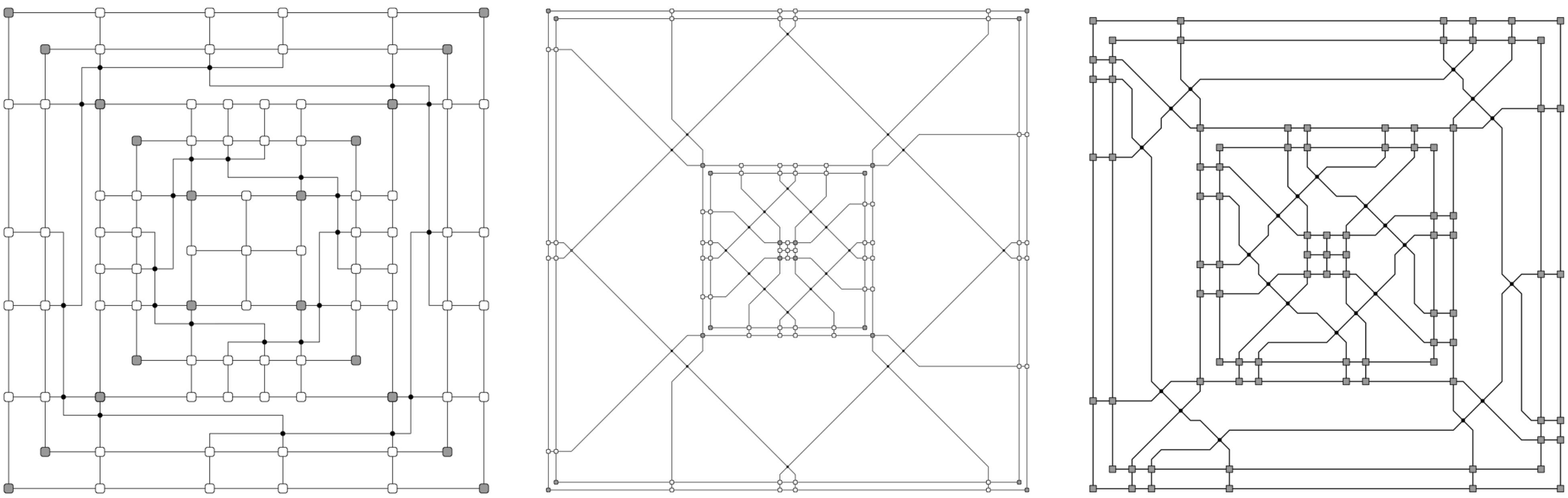
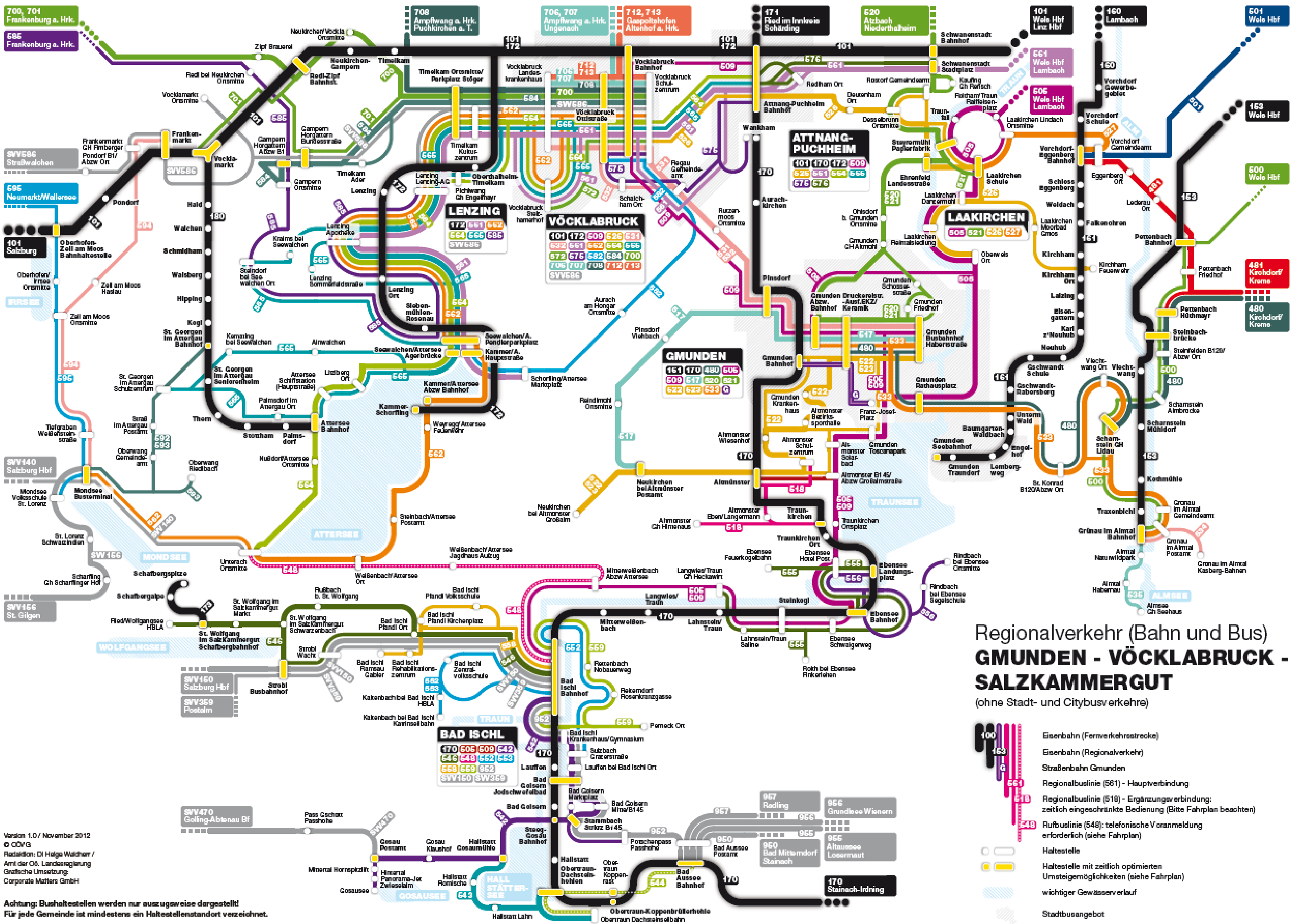
- Bend-optimal drawings may require exponential area.



Smooth Orthogonal Drawings



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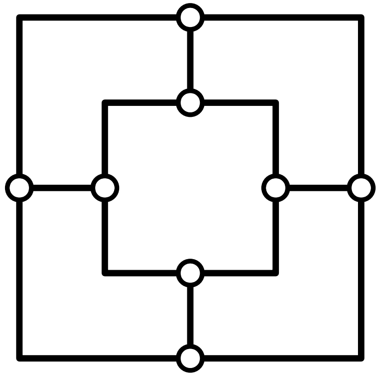
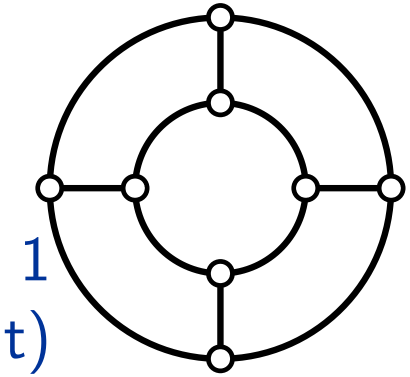


orthogonal complexity 2
(OC_2-layout)

smooth complexity 1
(SC_1-layout)



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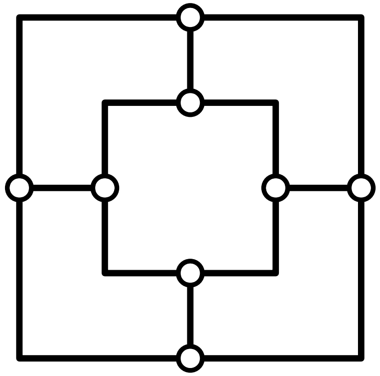

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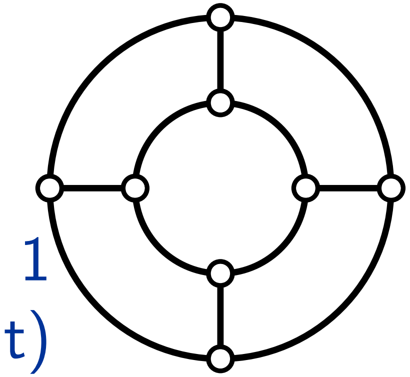

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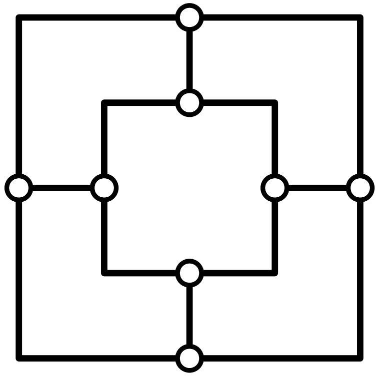

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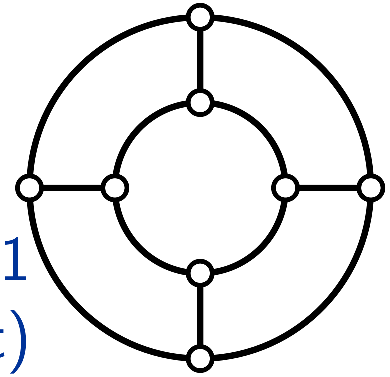

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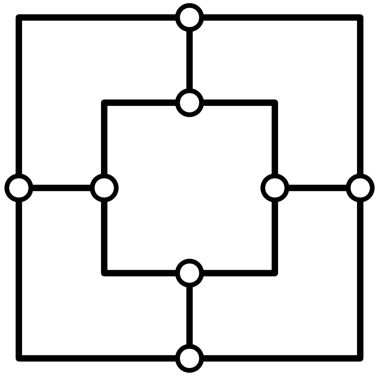

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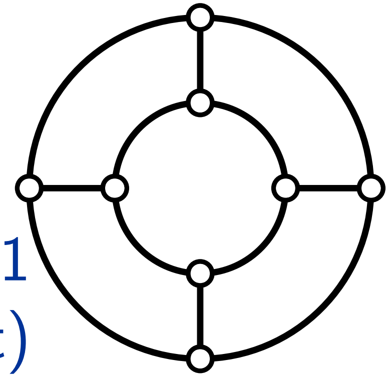

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Smooth Orthogonal Drawings

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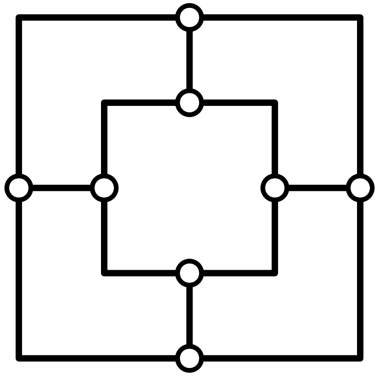

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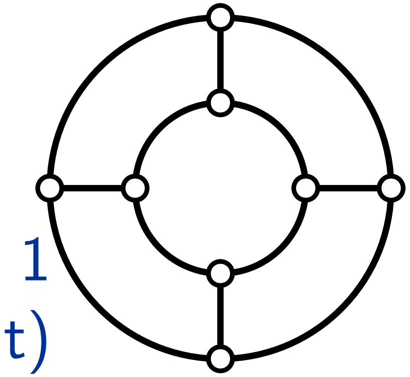

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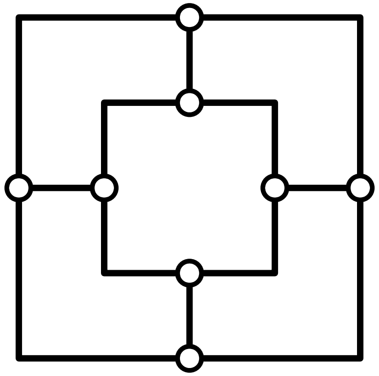

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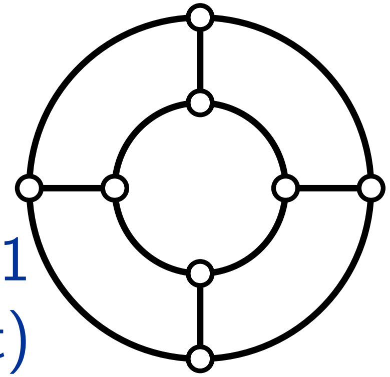

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Thm. Any 4-planar graph admits an SC_2-layout.

Thm. Any biconn. 4-outerplane graph admits an SC_1-layout.

Thm. Some graphs require exponential area if drawn with SC_1.

- Do all 4-planar graphs admit an SC_2-layout in *polynomial* area?
- Do all 4-outerplanar graphs admit an SC_1-layout?
- Do all 3-planar graphs admit an SC_1-layout?
- NP-hard to decide whether a 4-planar graph admits an SC_1-layout?

Edge casing – Edges & Switches, Tunnels & Bridges

[Eppstein, van Kreveld, Mumford, Speckmann CGTA'09]

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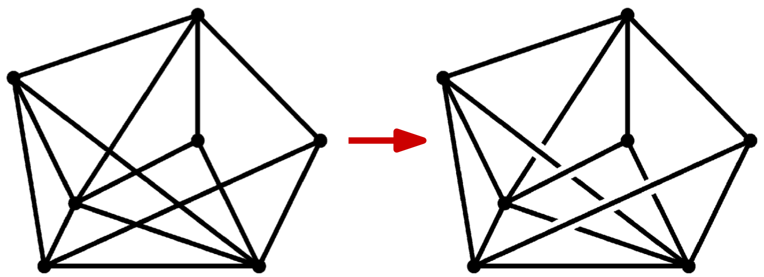

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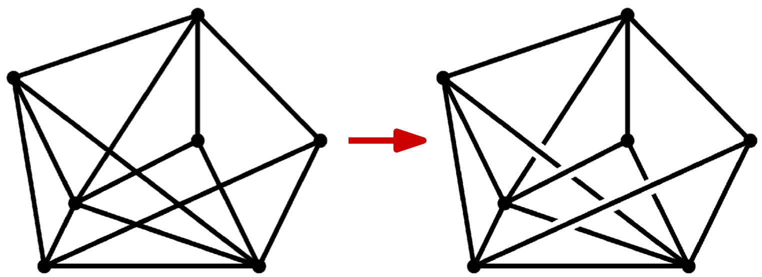

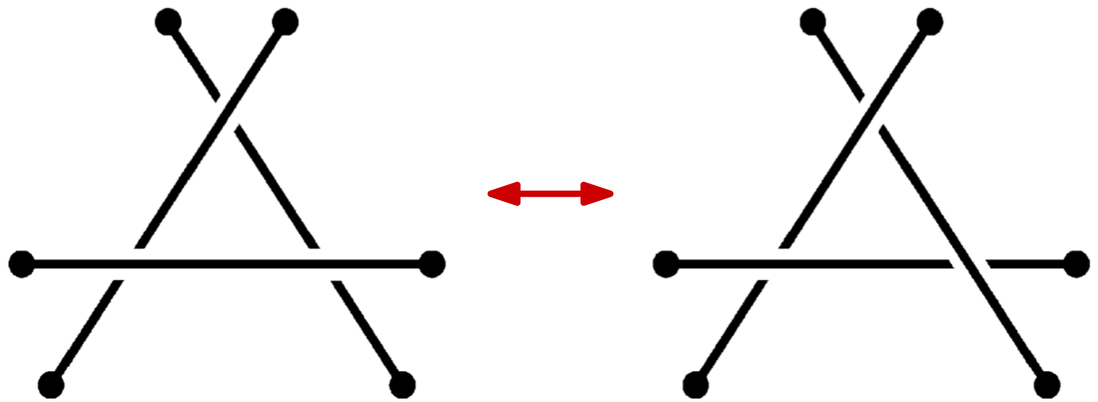

stacking vs. weaving

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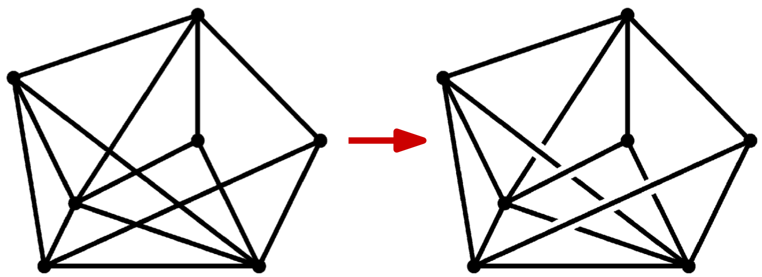

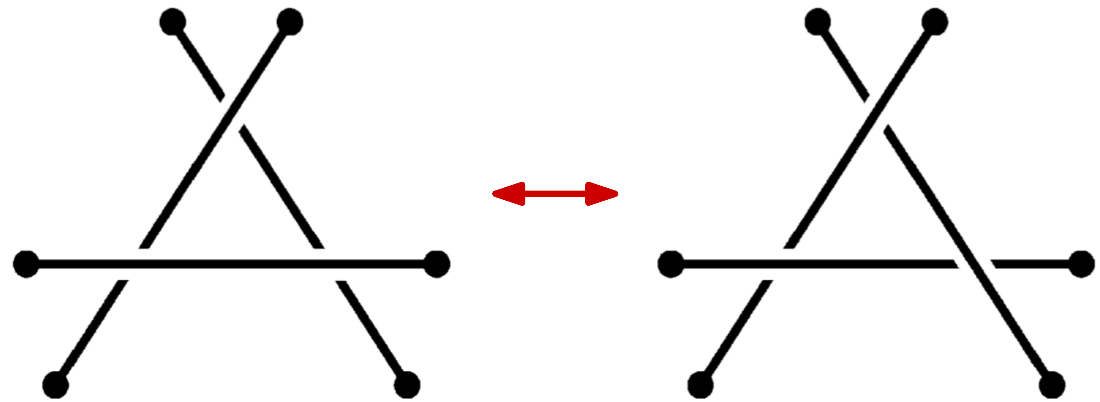

stacking vs. weaving

	stacking	weaving
MinTotalSwitches	open	$O(k^{5/2} \log^{3/2} k)$
MaxTotalSwitches	open	$O(k^{5/2} \log^{3/2} k)$
MinMaxSwitches	open	open
MinMaxTunnels	$O(m \log m + k)$ exp.	$O(m^4)$
MinMaxTunnelLength	$O(m \log m + k)$ exp.	NP-hard
MaxMinTunnelDistance	$O(m \log m + k \log m)$ exp.	$O(m^3 \log m)$ exp.

$k = O(m^2)$ – number of crossings (I simplified some runtimes.)

Eliminating crossings

- confluent drawings
- partial edge drawings

Confluent Drawings

[Dickerson, Eppstein, Goodrich, Meng JGAA'05]

Def. G is *confluent* if G admits a planar drawing s.t.
– vertices correspond to points,
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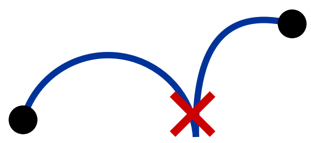

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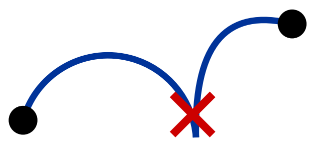
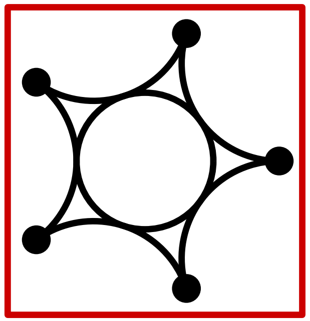


Obs. Any complete graph is confluent.



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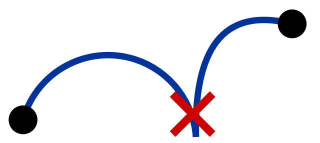

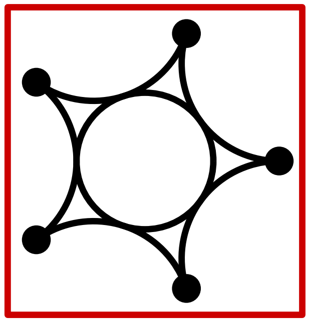

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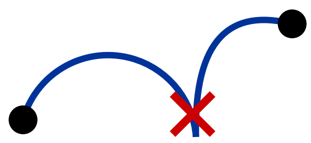

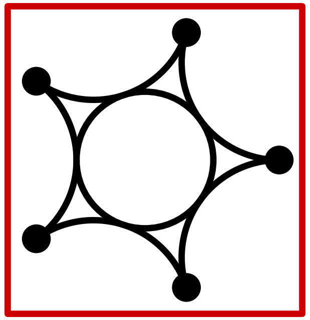

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Thm. Any cograph, any complement of a tree or cycle, and any interval graph is confluent –

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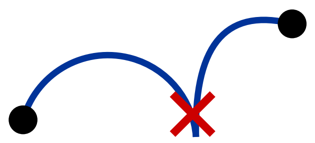

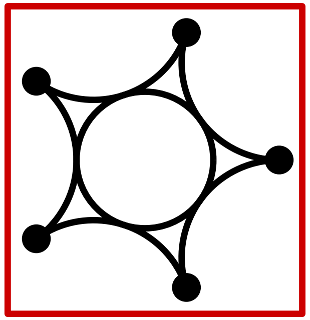

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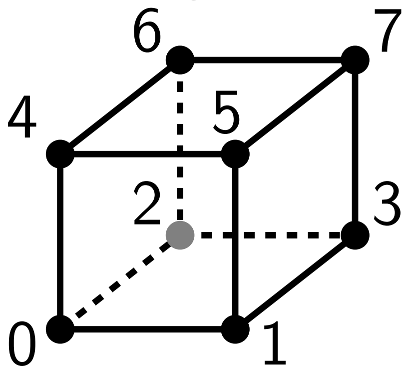

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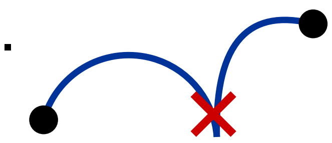

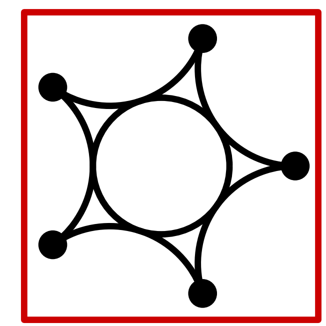

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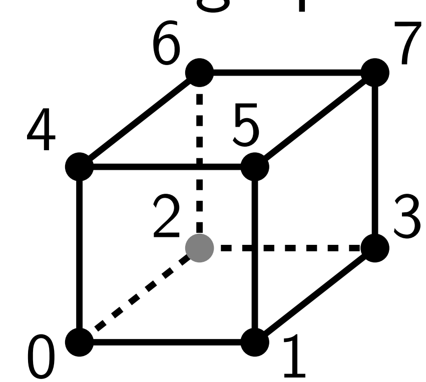

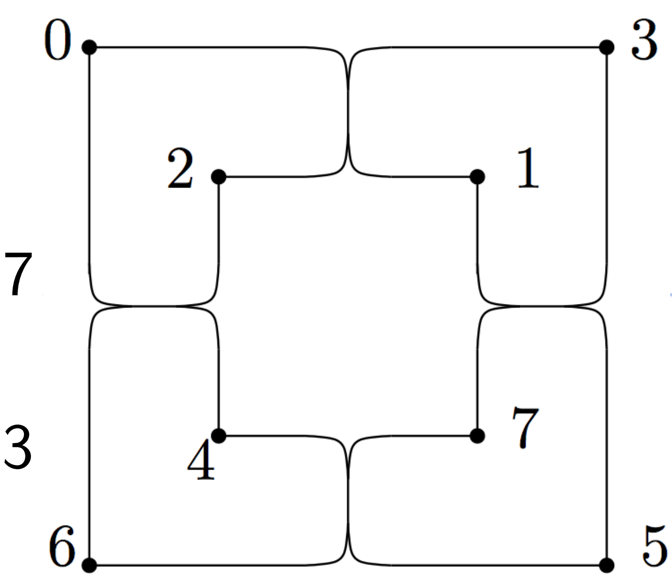

[DEGM'05]

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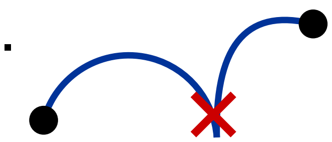

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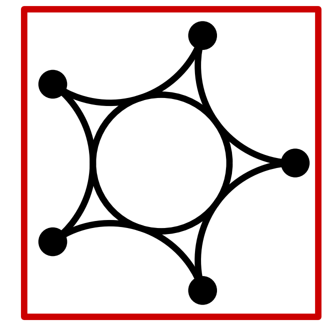

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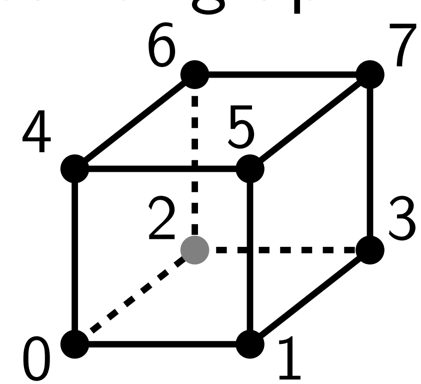

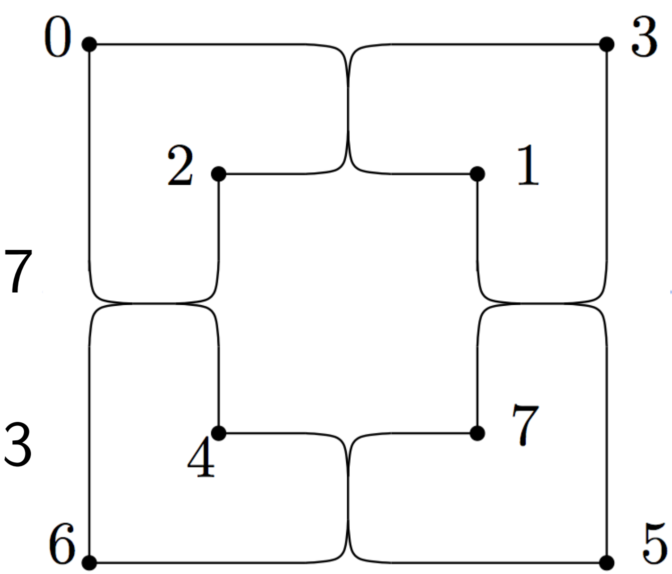

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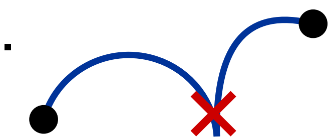
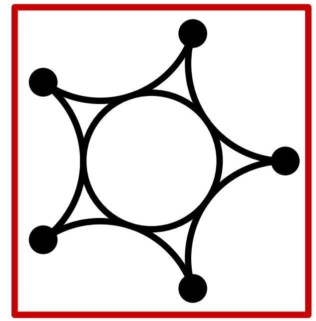
– edges correspond 1 : 1 to *locally-monotone* curves ("train tracks").

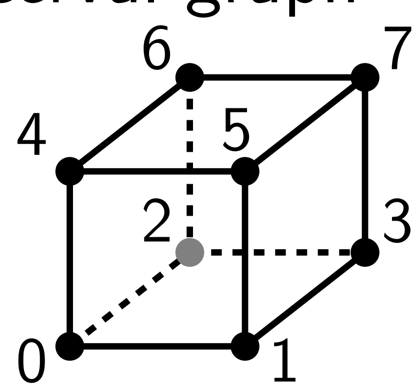


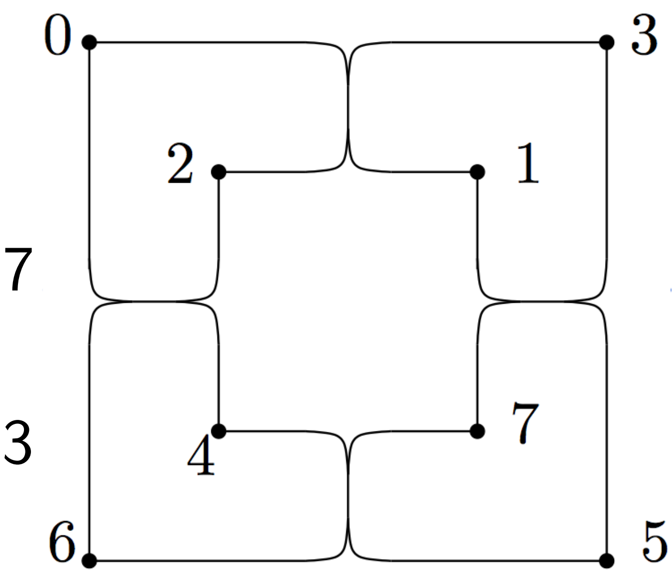
Obs. Any complete graph is confluent.



Def. *Cographs* can be obtained from a single node by performing a finite number of unions and complementations.

Thm. Any cograph, any complement of a tree or cycle, and any interval graph is confluent – but the *4D* hypercube isn't.





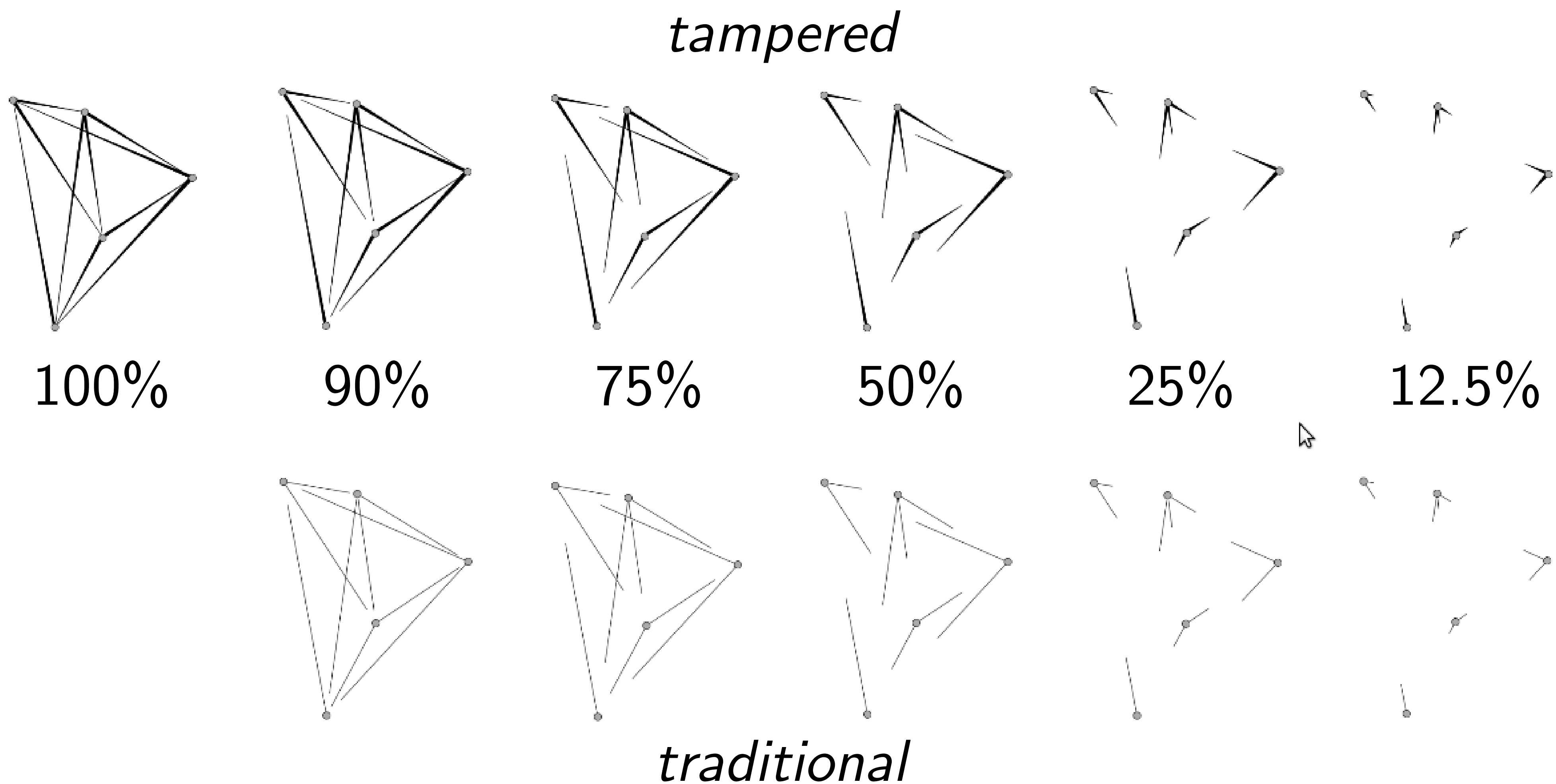
[DEGM'05]

More: layered-con., Δ-con., . . .

Partial Edge Drawings: A User Study

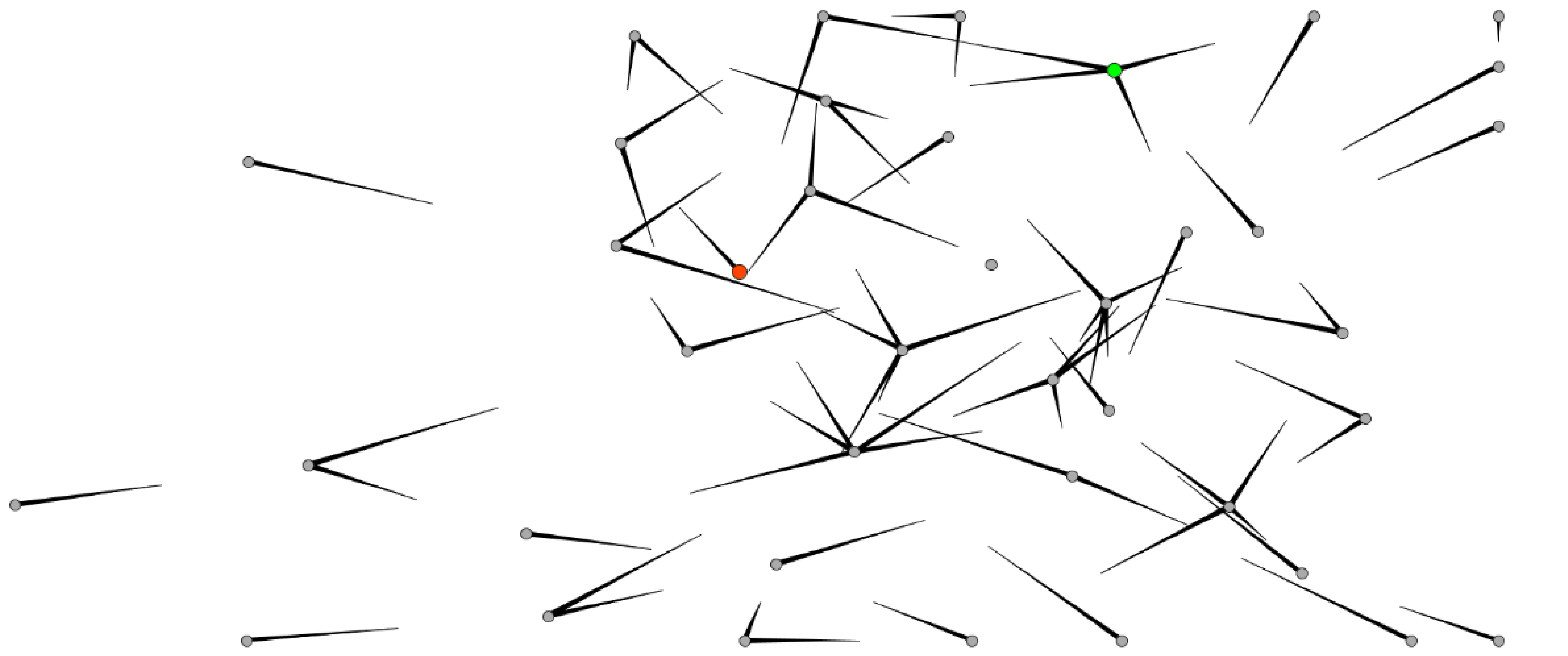
Evaluating Partially Drawn Links for Directed Graph Edges

[Burch et al., GD'11]



Questionnaire

Is it possible to get from the green to the red node via exactly one intermediate node?



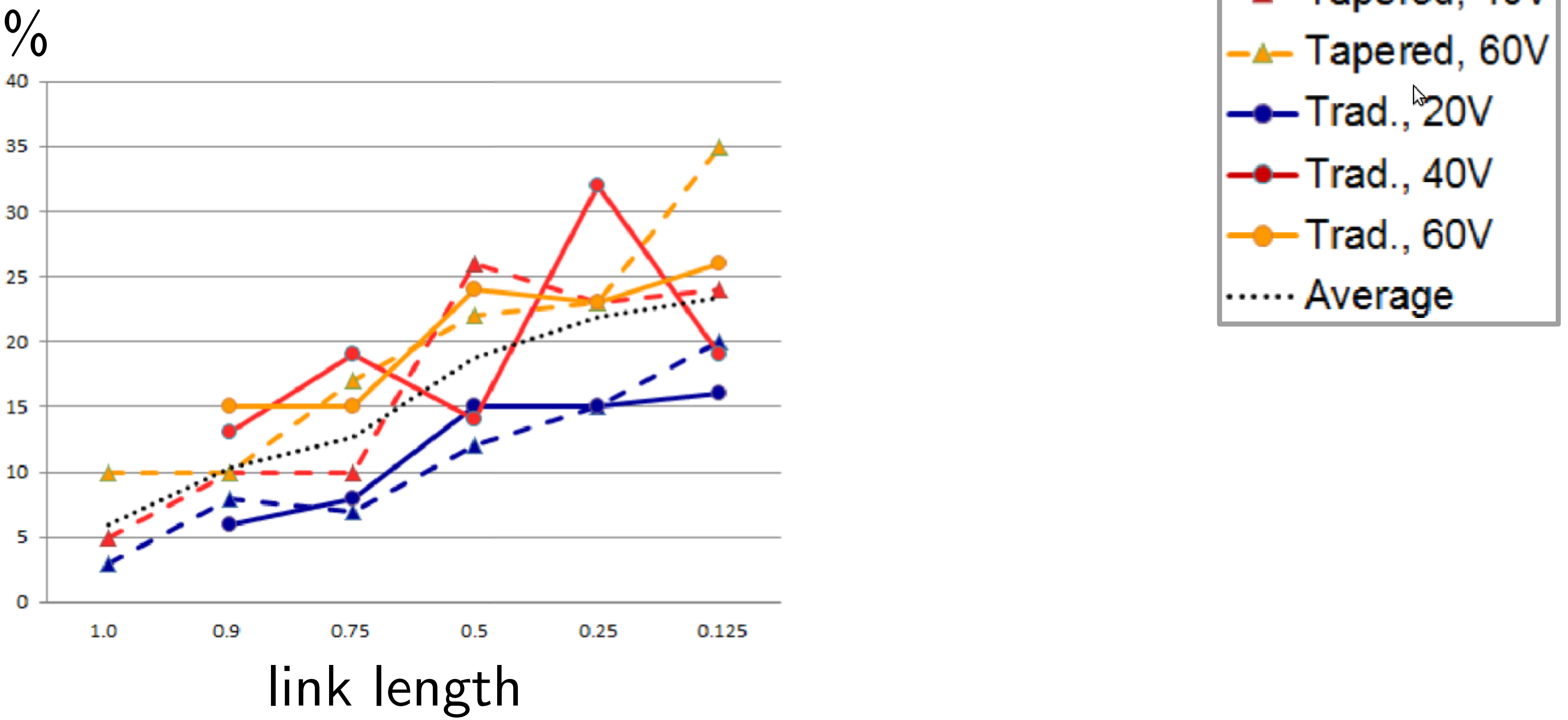
YES

NO

Results – Error Rates

Is it possible to go from the green node to the red node via exactly one other node?

(42 participants, 2 repetitions, avg. age 24.0 yrs.)

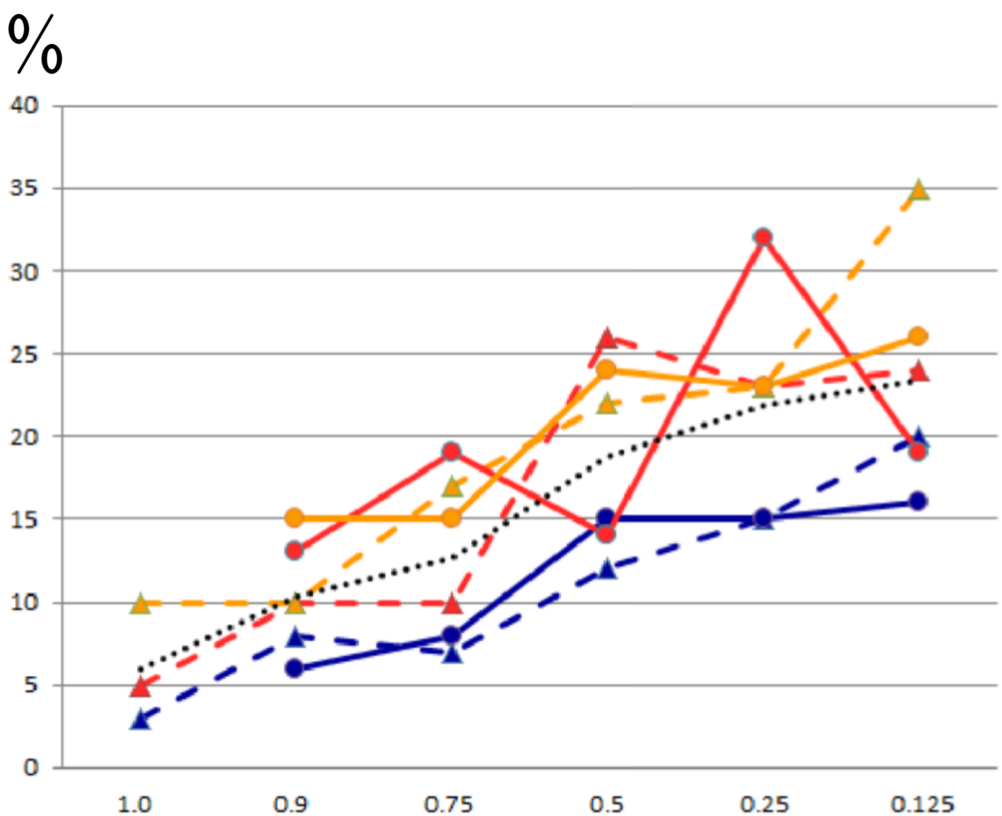


Results – Error Rates

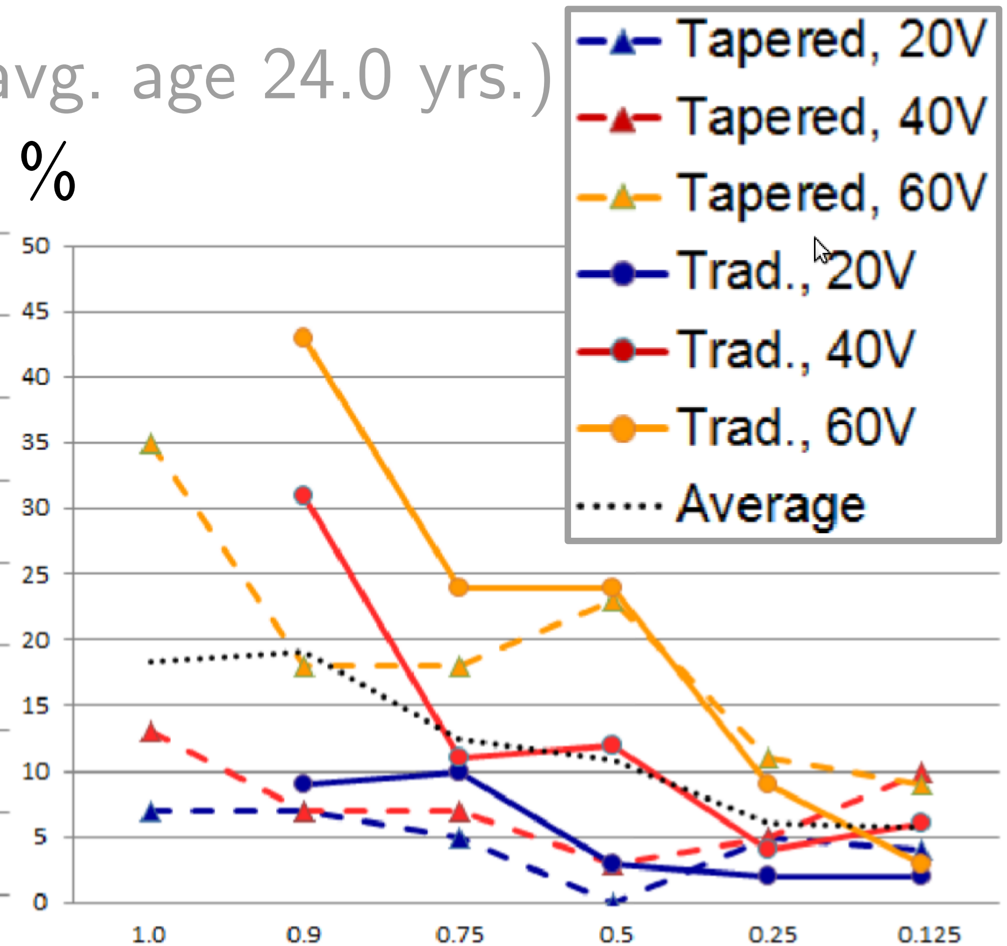
Is it possible to go from the green node to the red node via exactly one other node?

Which node has the highest number of outgoing edges?

(42 participants, 2 repetitions, avg. age 24.0 yrs.)



link length



link length

A Classification and Some Theory

PED = Partial Edge Drawing

A Classification and Some Theory

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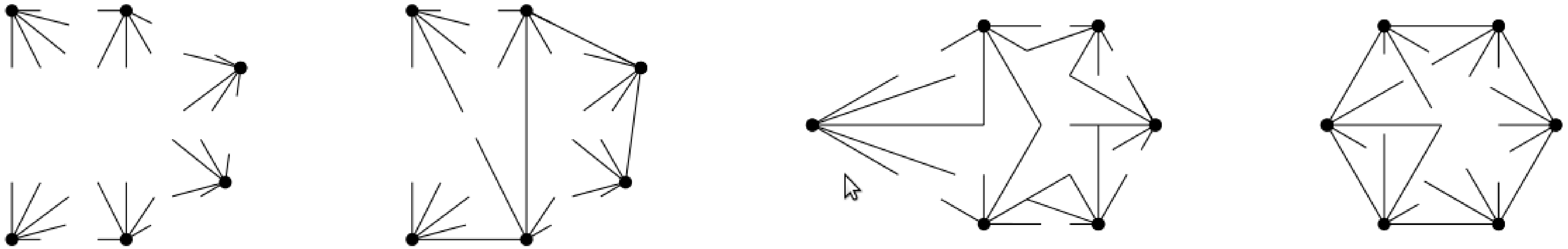
Stub = remaining edge part. Stubs may not intersect.



A Classification and Some Theory

PED = Partial Edge Drawing
can be (non-) symmetric and (non-) homogeneous

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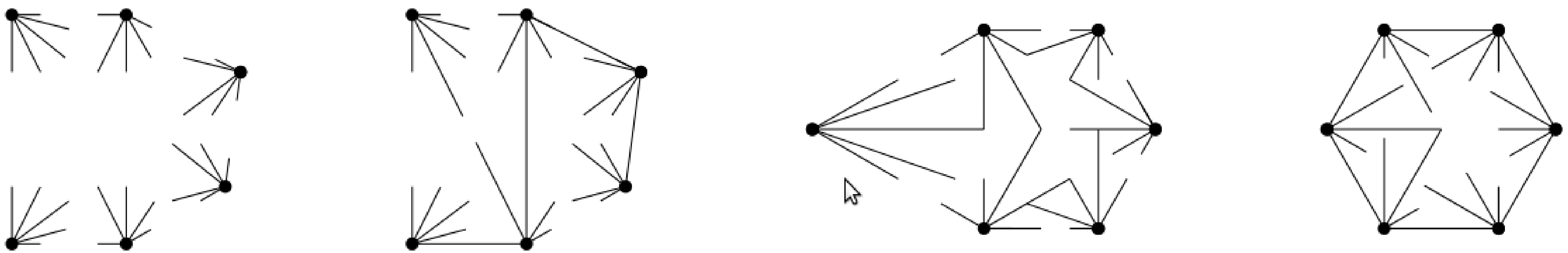

Mad at edge crossings? Break the edges!

[Bruckdorfer & Kaufmann, FUN'12]

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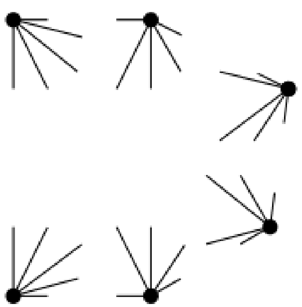
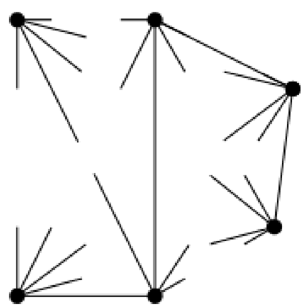
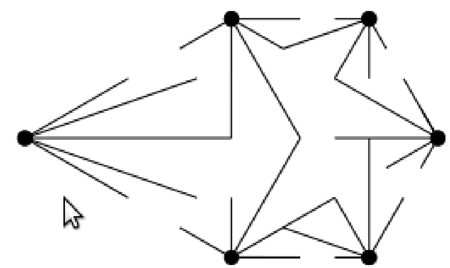


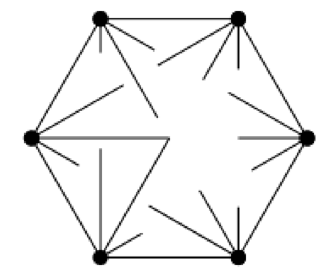
1/4-SHPED



SnHPED



nSHPED



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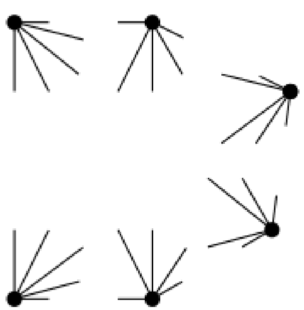

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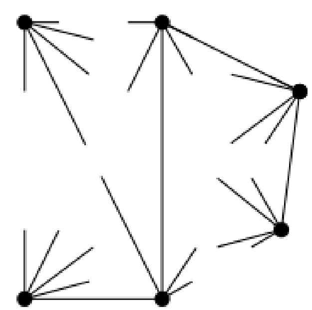

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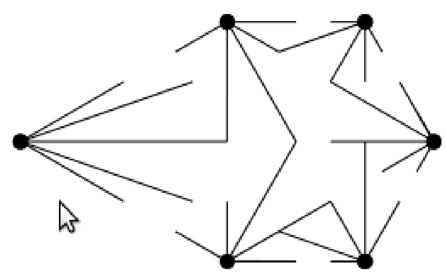

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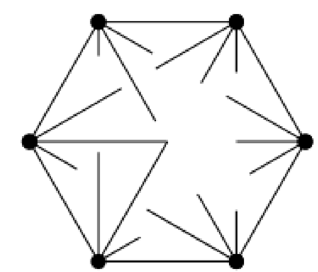

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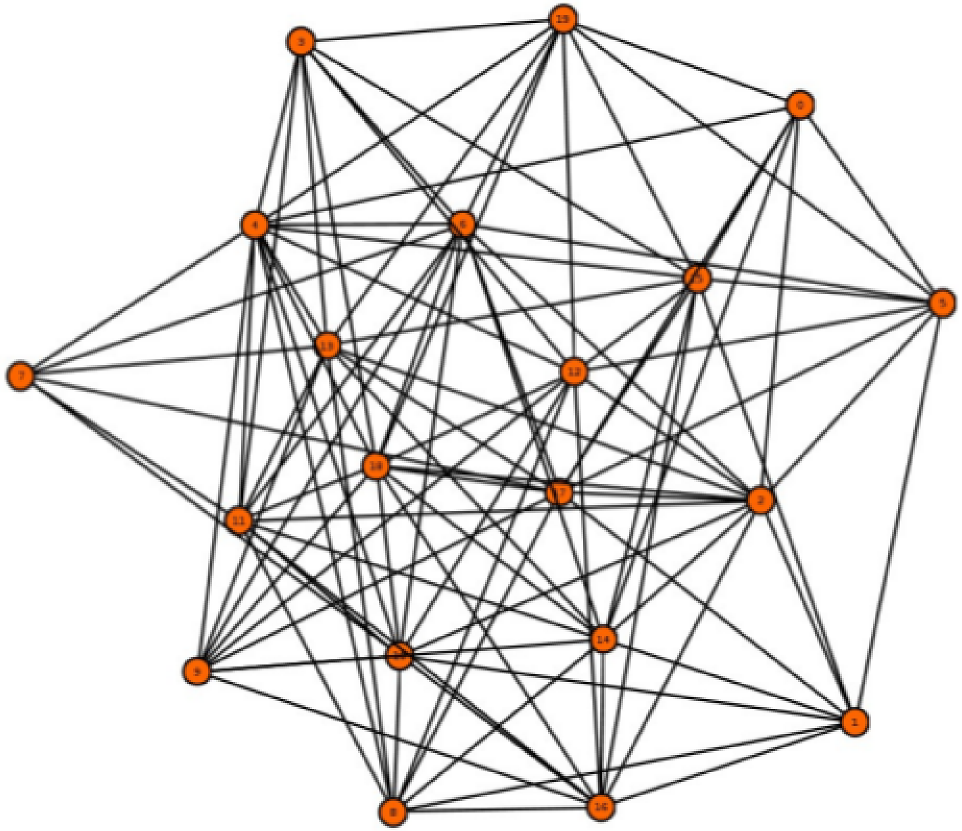
Combining Spring Embedder and SHPED

Randomly generated graph with 20 vertices and 100 edges.

Force-directed layout



413 crossings

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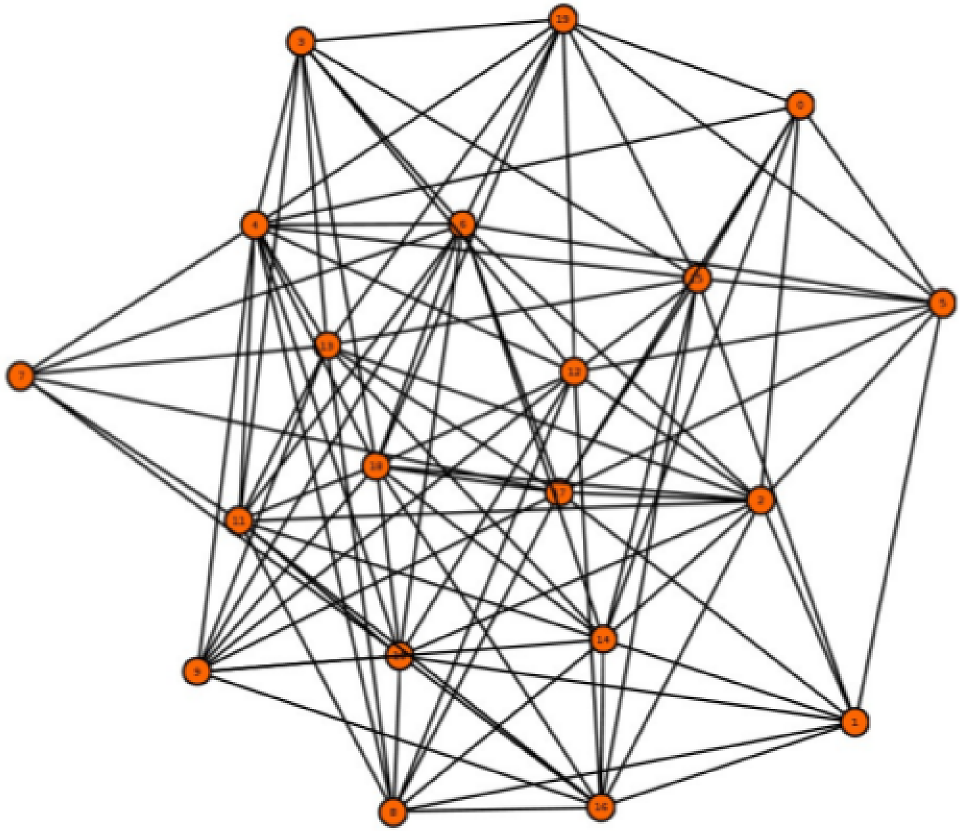

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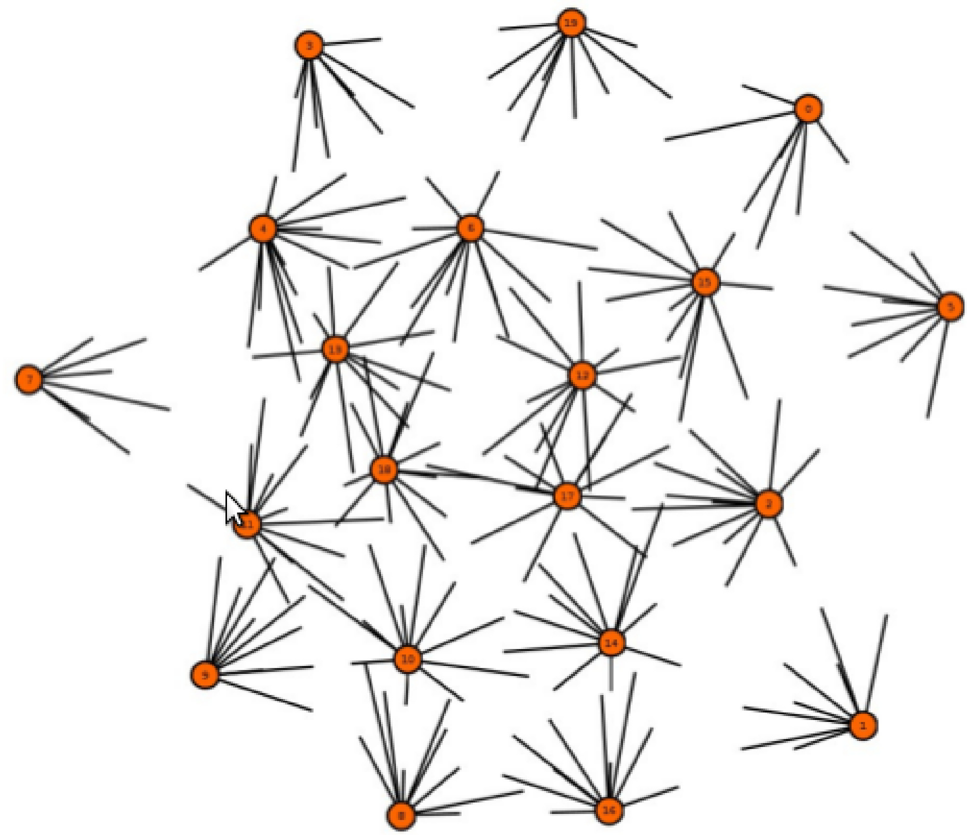
Postprocessed to 1/4-SHPED



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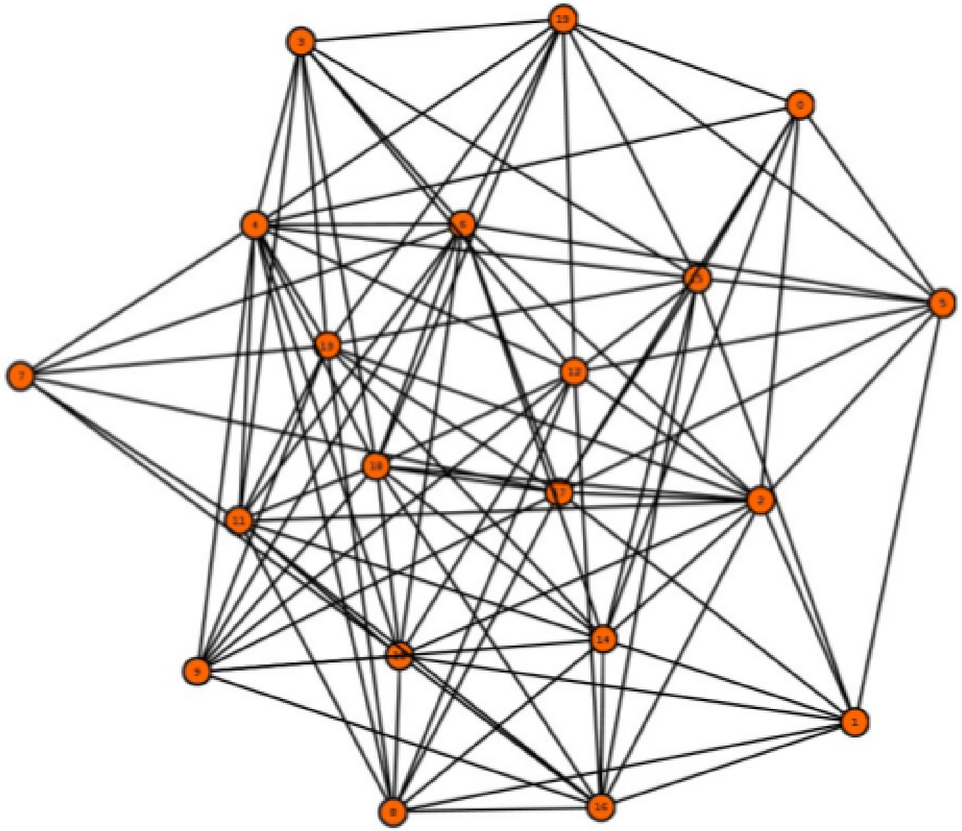

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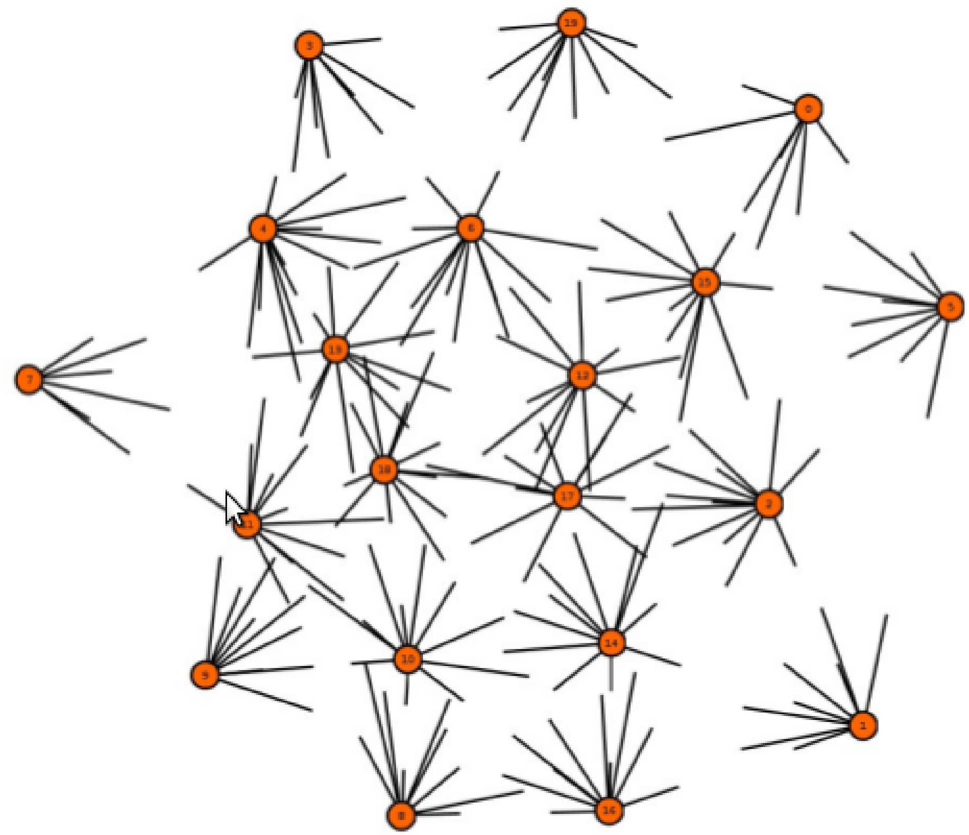

23 crossings

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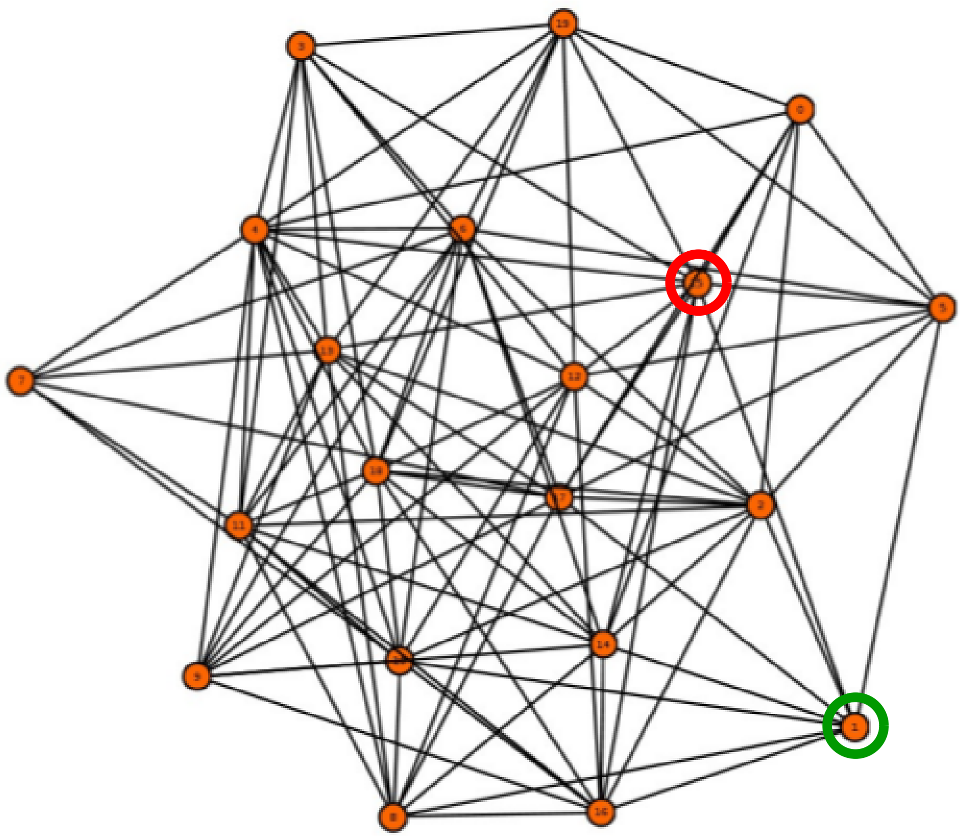

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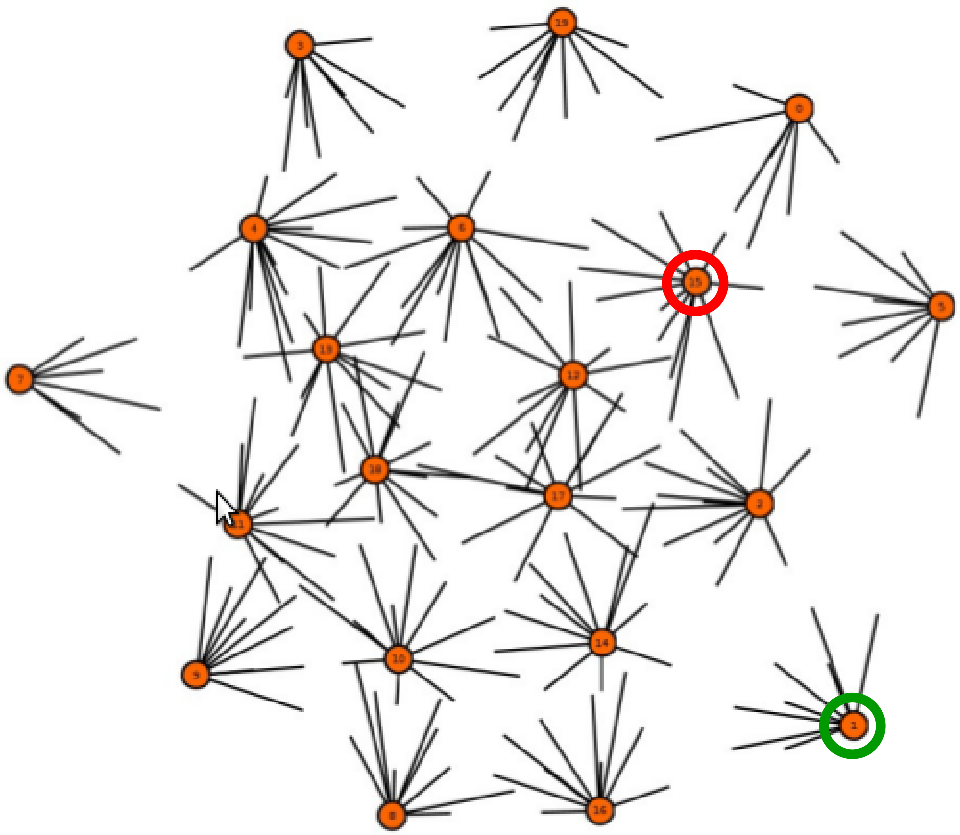

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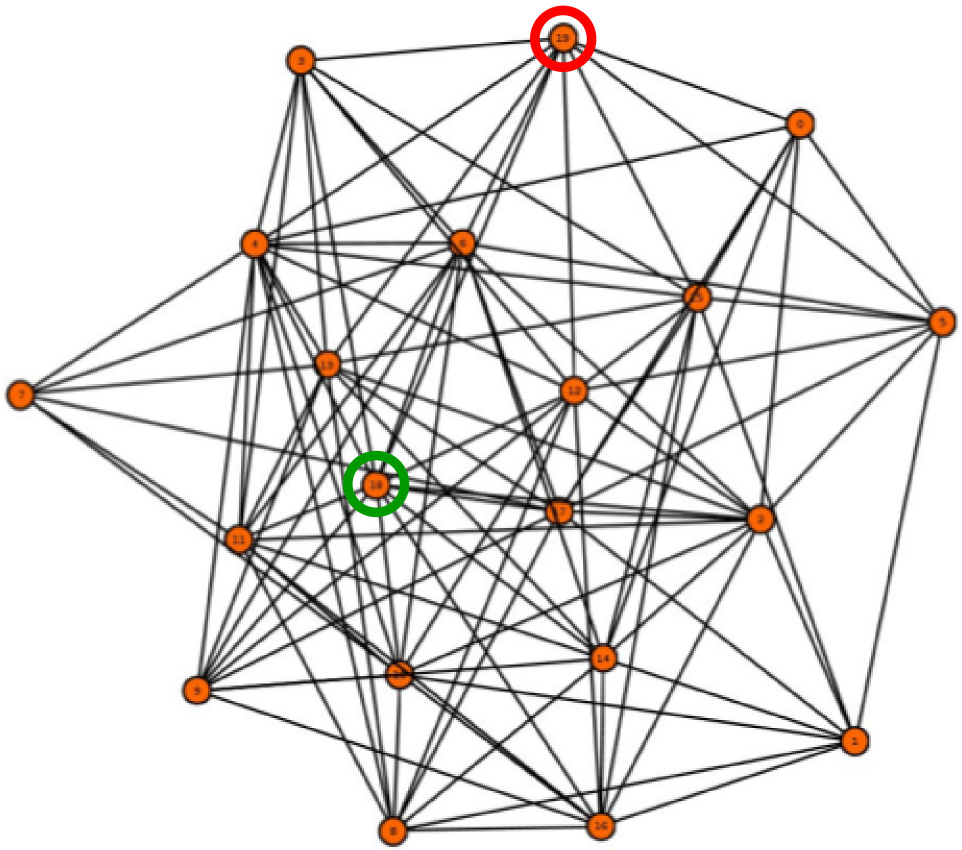

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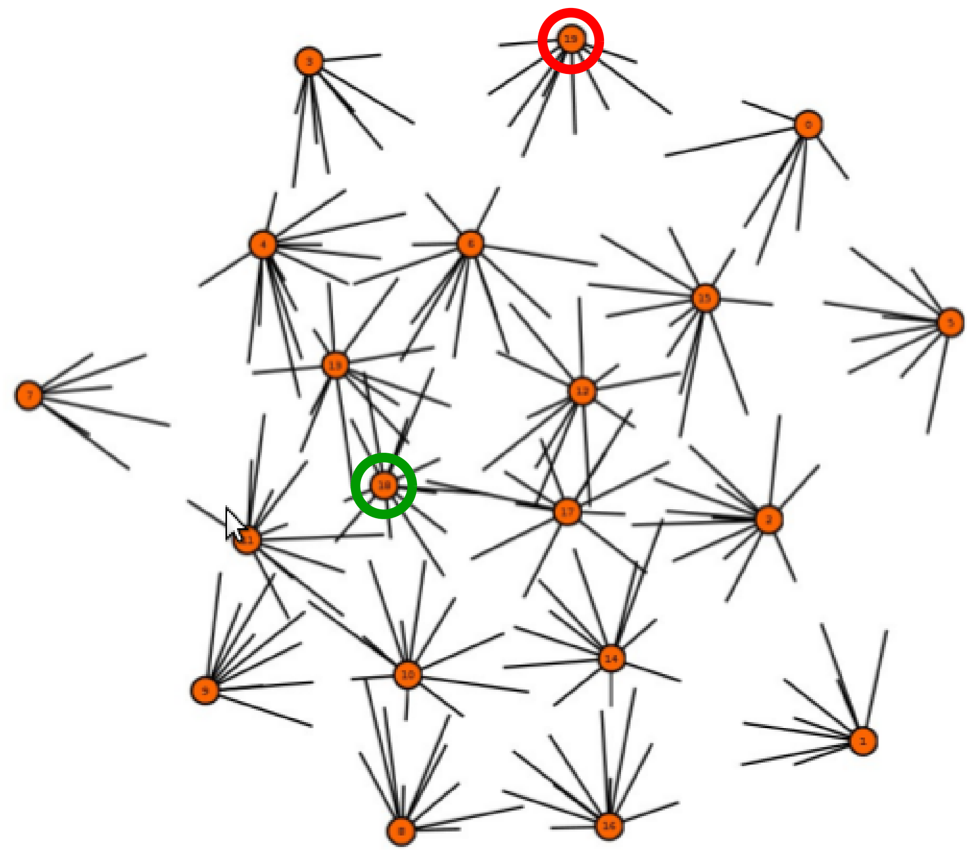
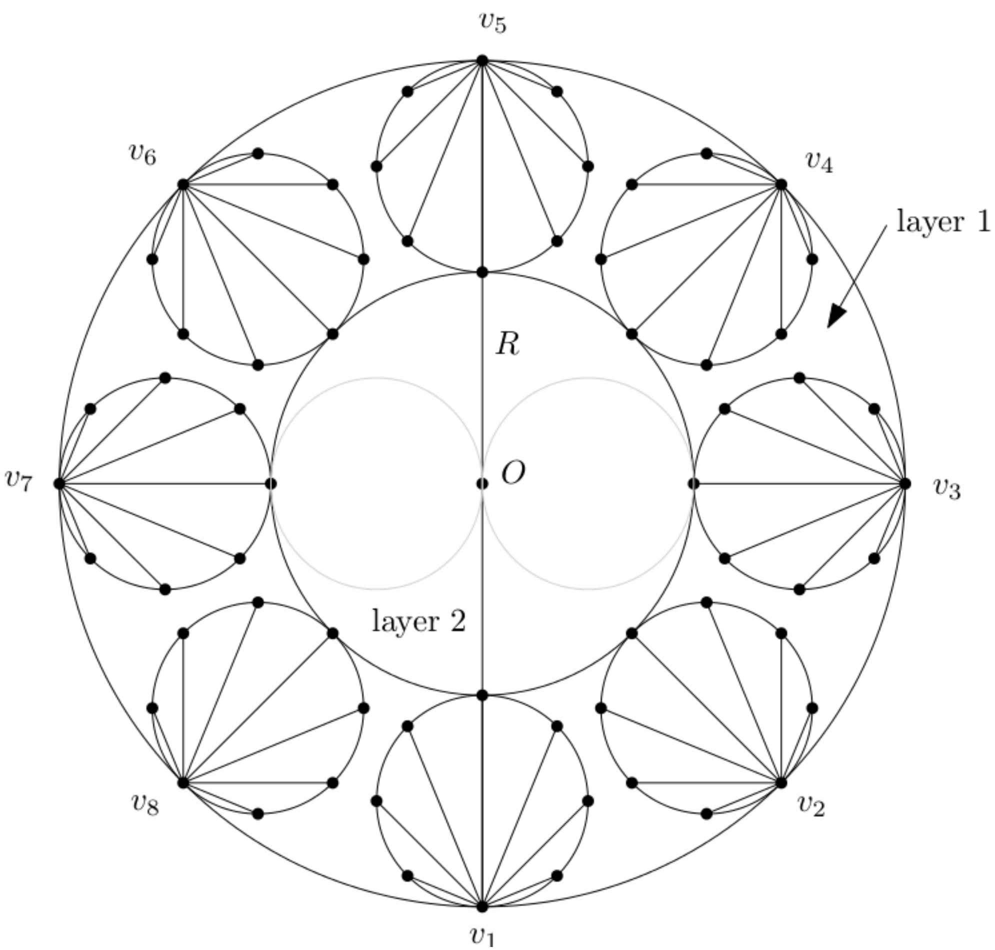


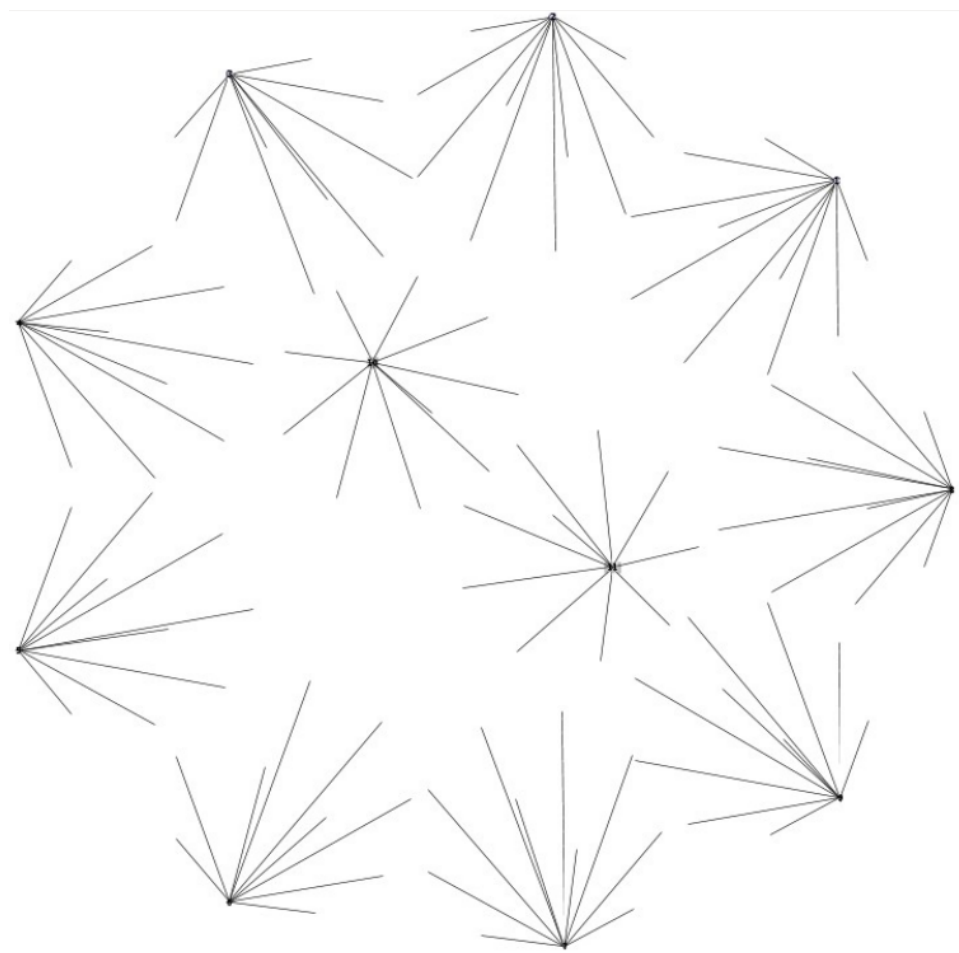
23 crossings

Thm. Let G be a subgraph of K_n.
Then G has an δ-SHPED for any $\delta \leq \frac{1}{\sqrt{4n/\pi}}$.

Construction for $\delta = 1/4$.



Idea of the construction.



Output of the algorithm.

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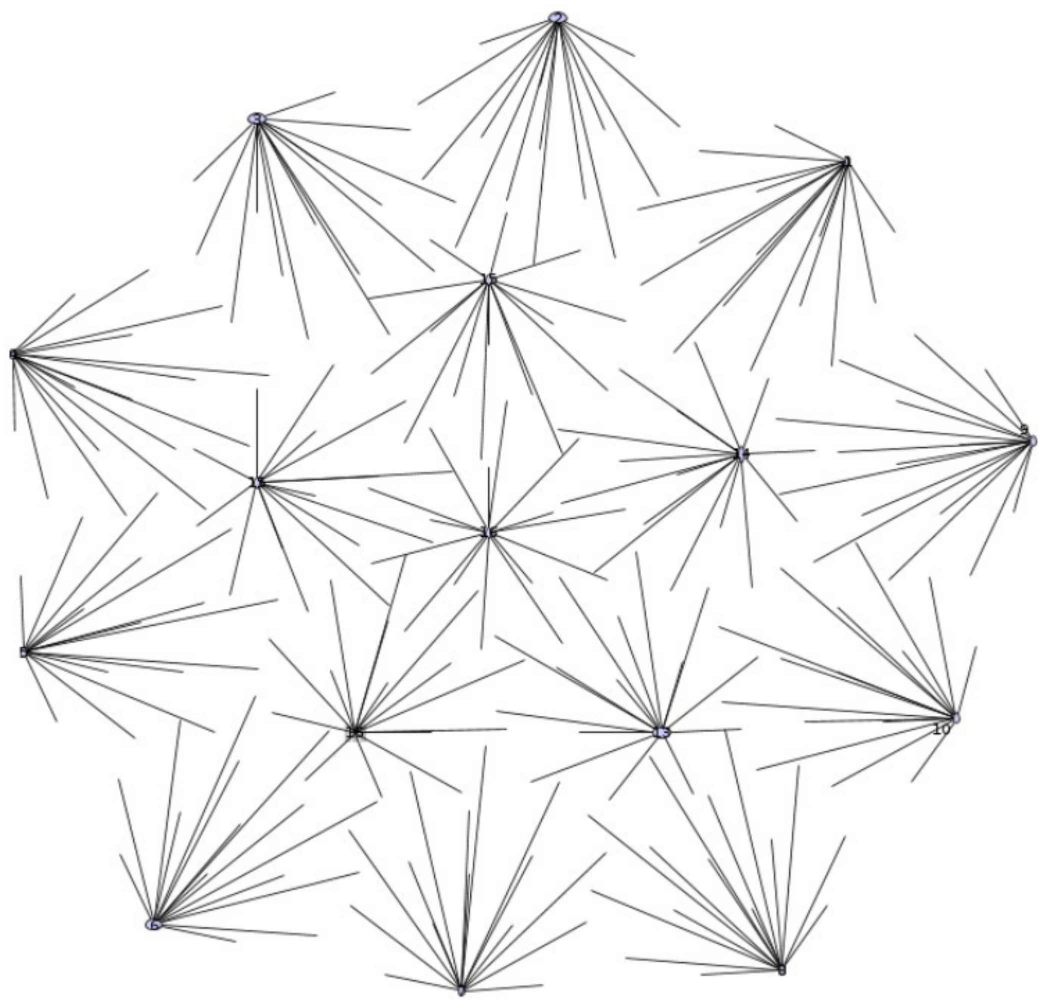
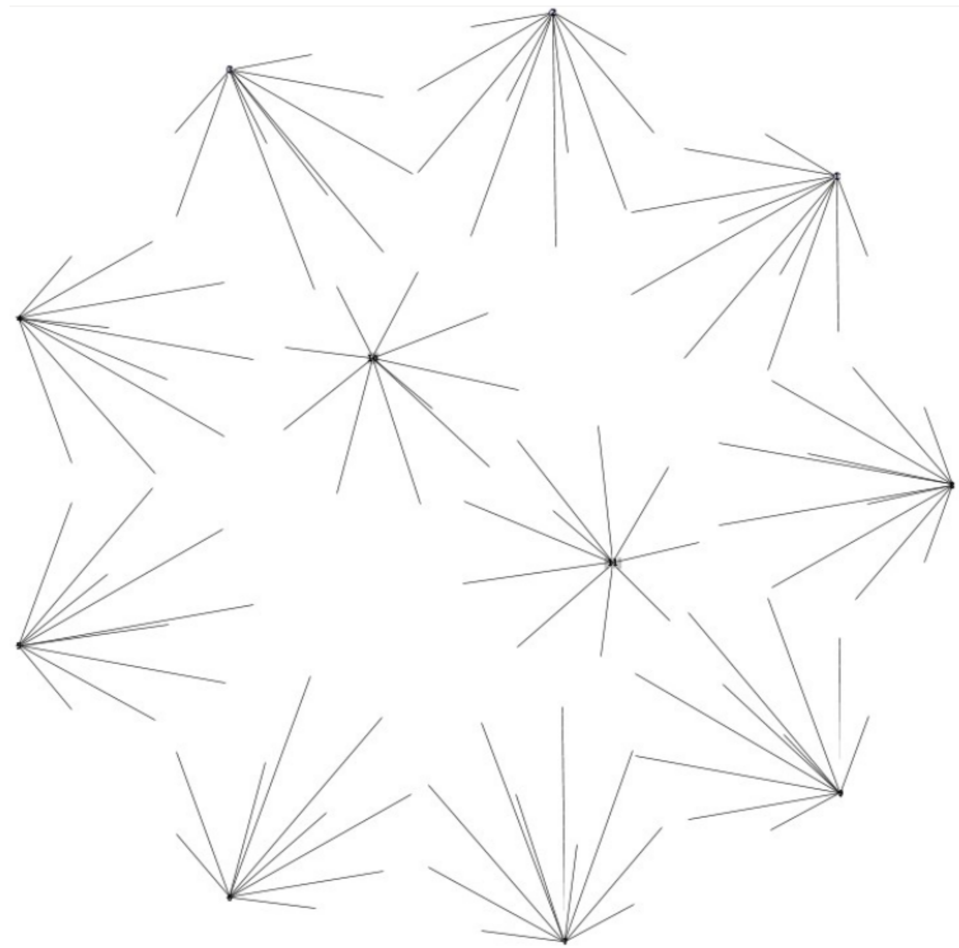


Drawn with a PED editor.



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$\delta = 1$ allows to draw K_4, but not K_5.

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Tool: Every set of

$$f(N) = \binom{2N-5}{N-2} + 1 \in O\left(\frac{4^N}{\sqrt{N}}\right)$$

points has at least N points in convex position.

[Valtr, Tóth, 2005]

In other words:

In every set of n pts, there are $\Omega(\log n)$ pts in convex position.

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Actually: K_{165}.

[Bruckdorfer, Kaufmann, Cornelsen, Gutwenger, Montecchiani, Nöllenburg, W. GD'12]

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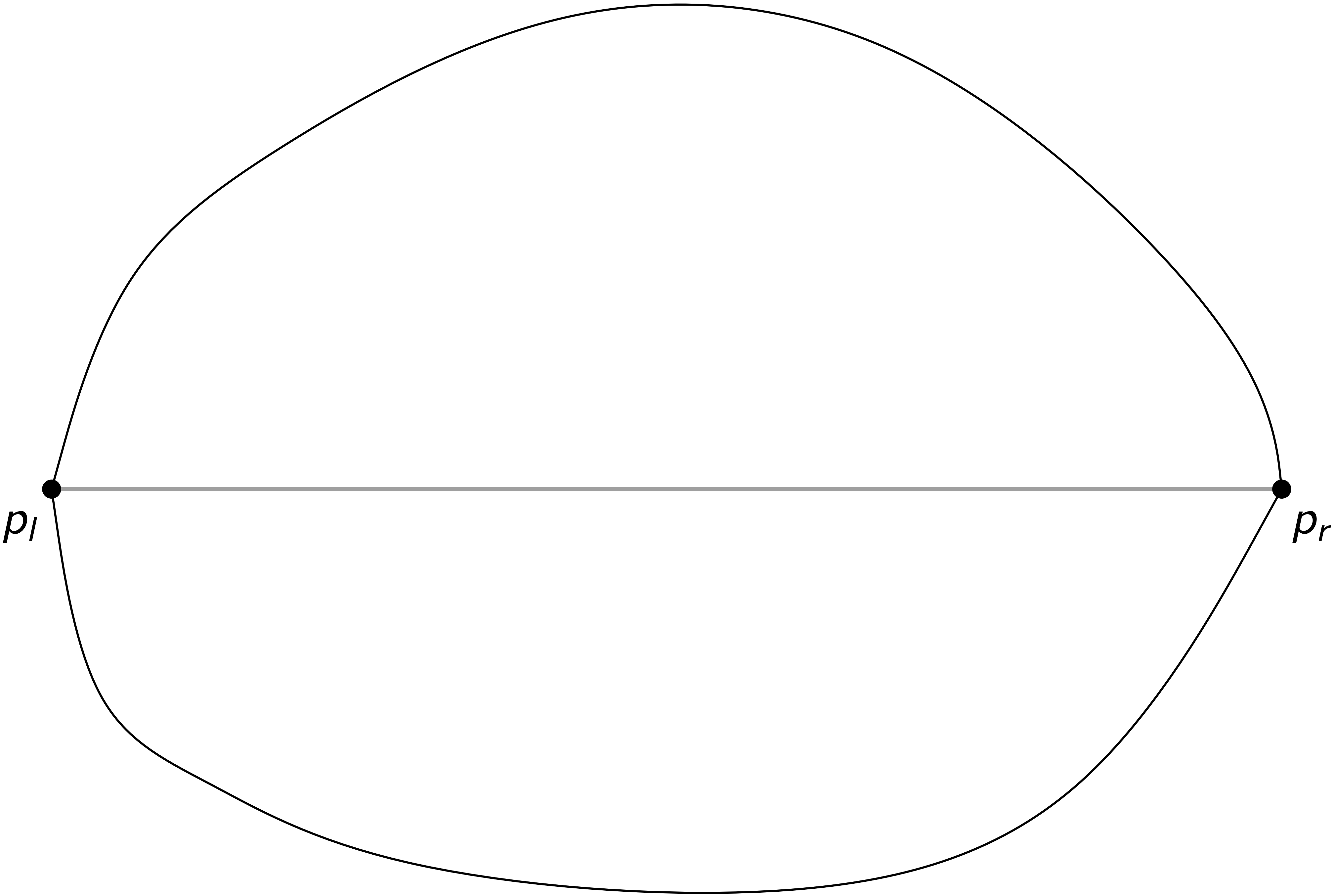
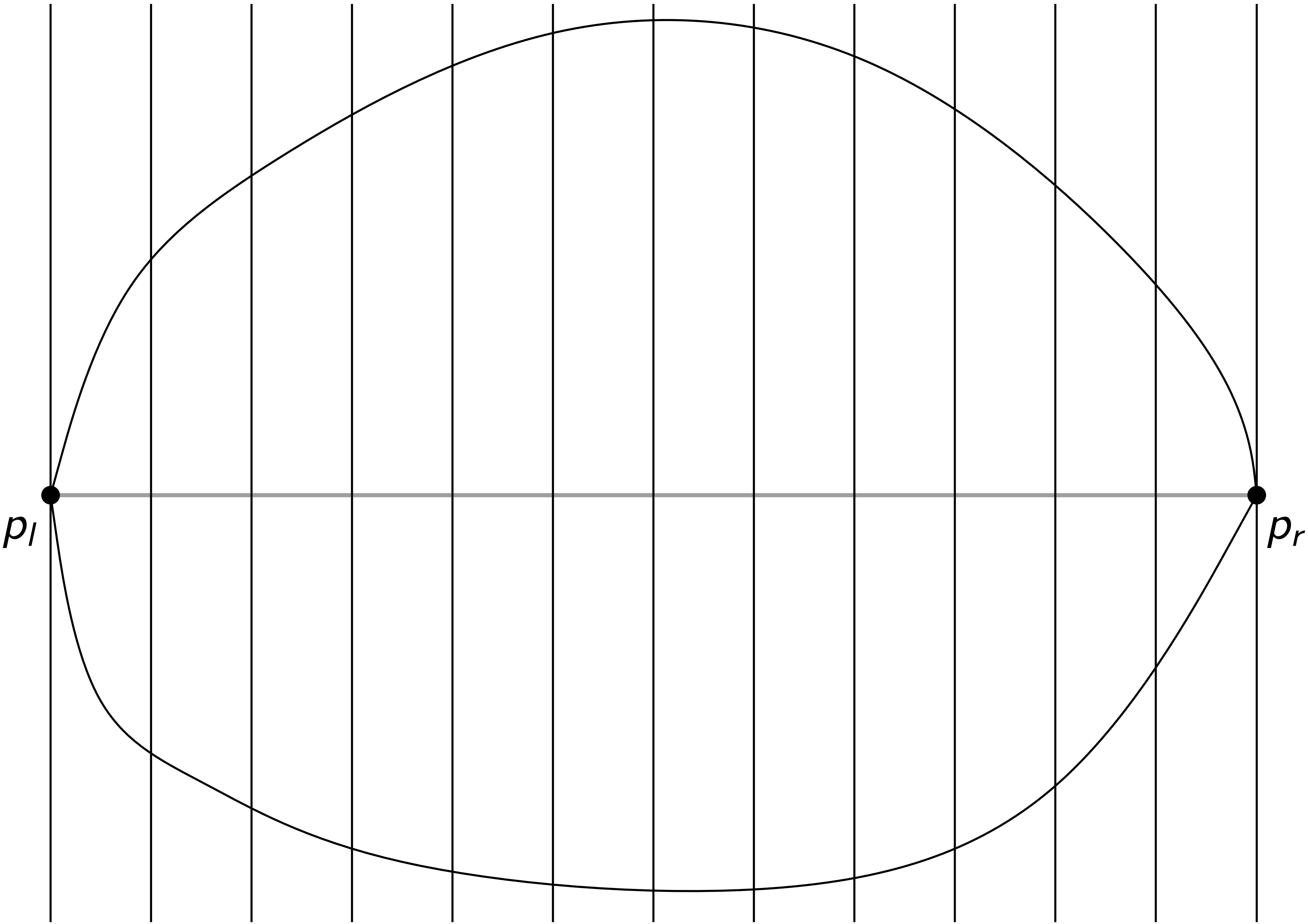

p_l
p_r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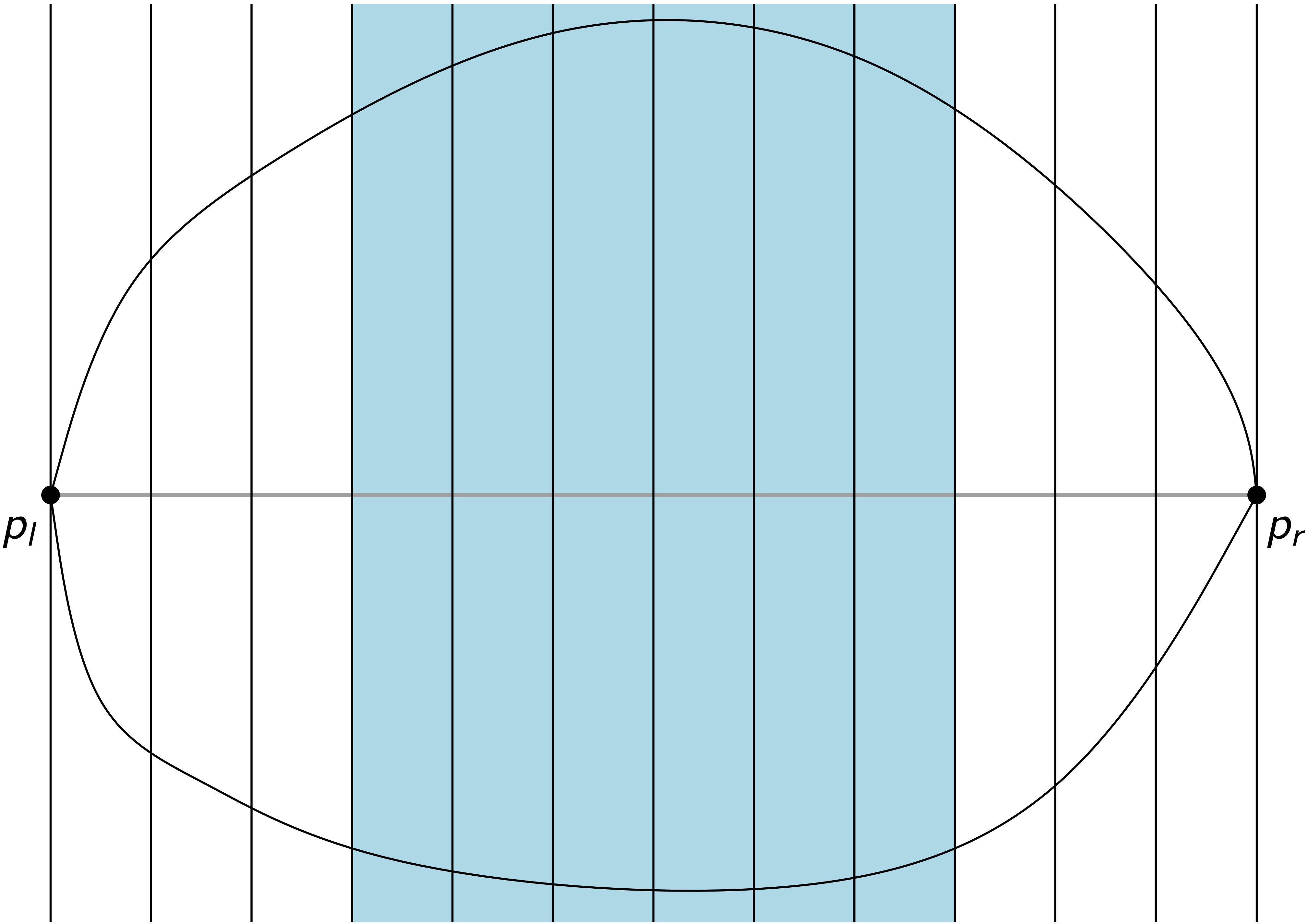
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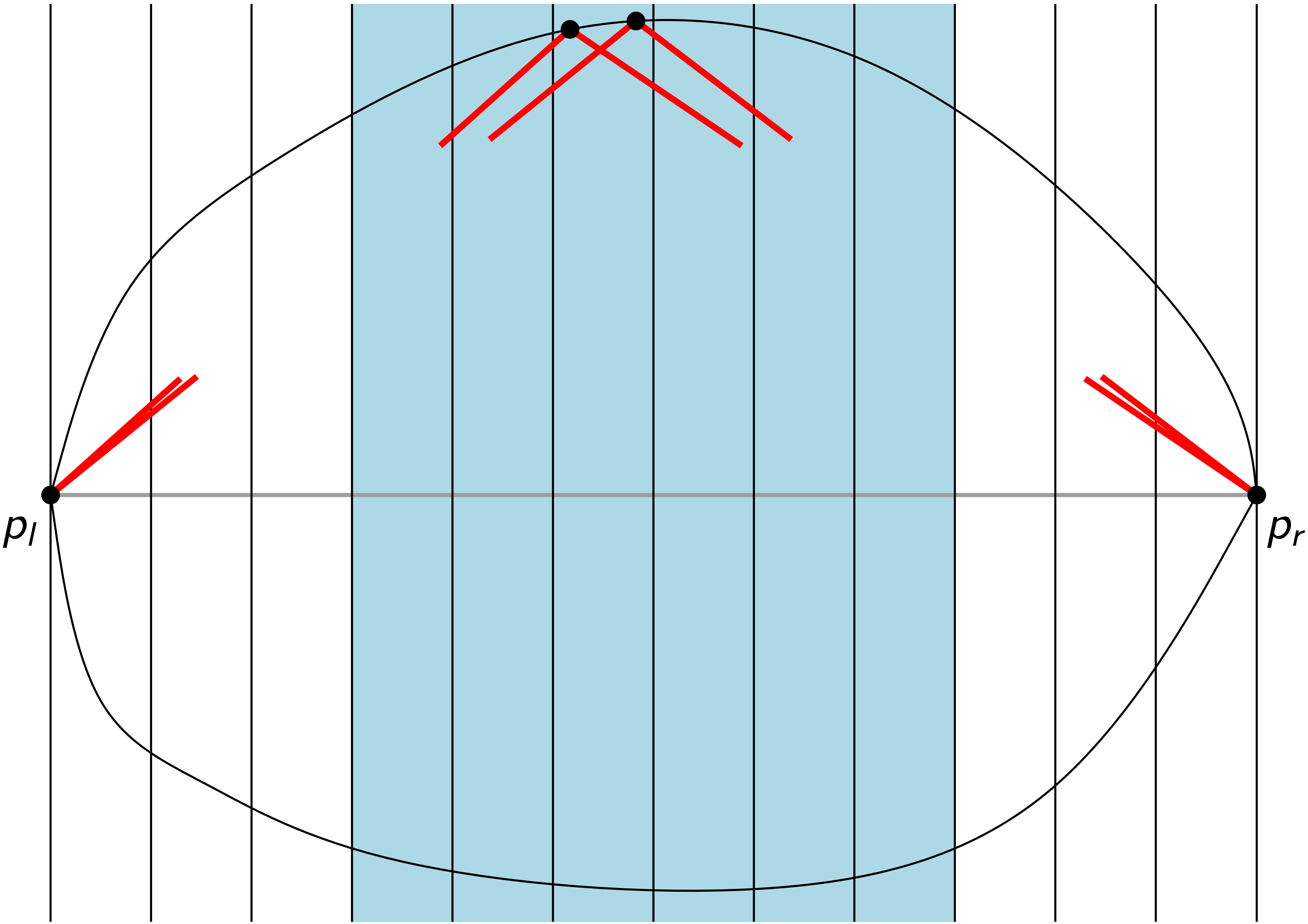
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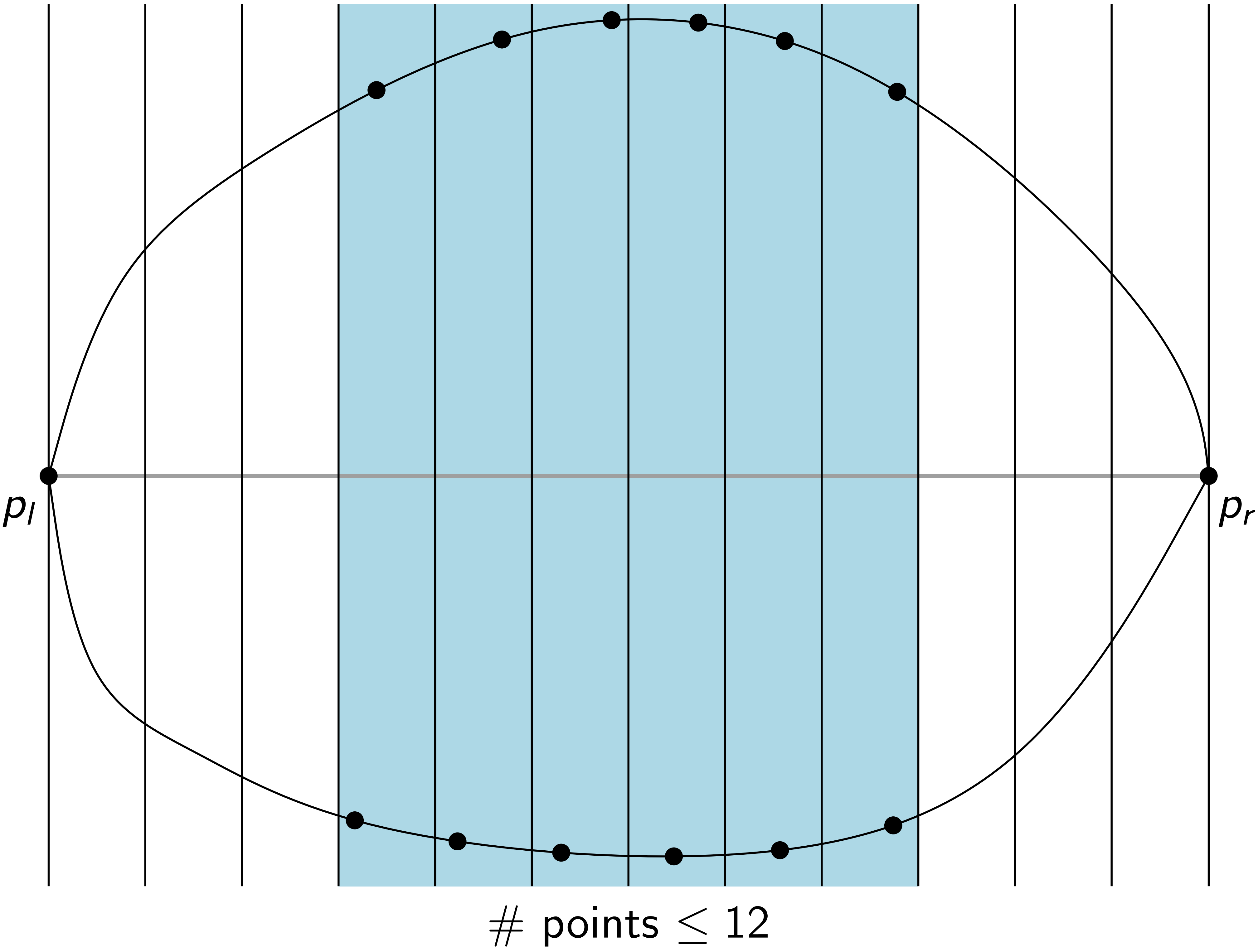
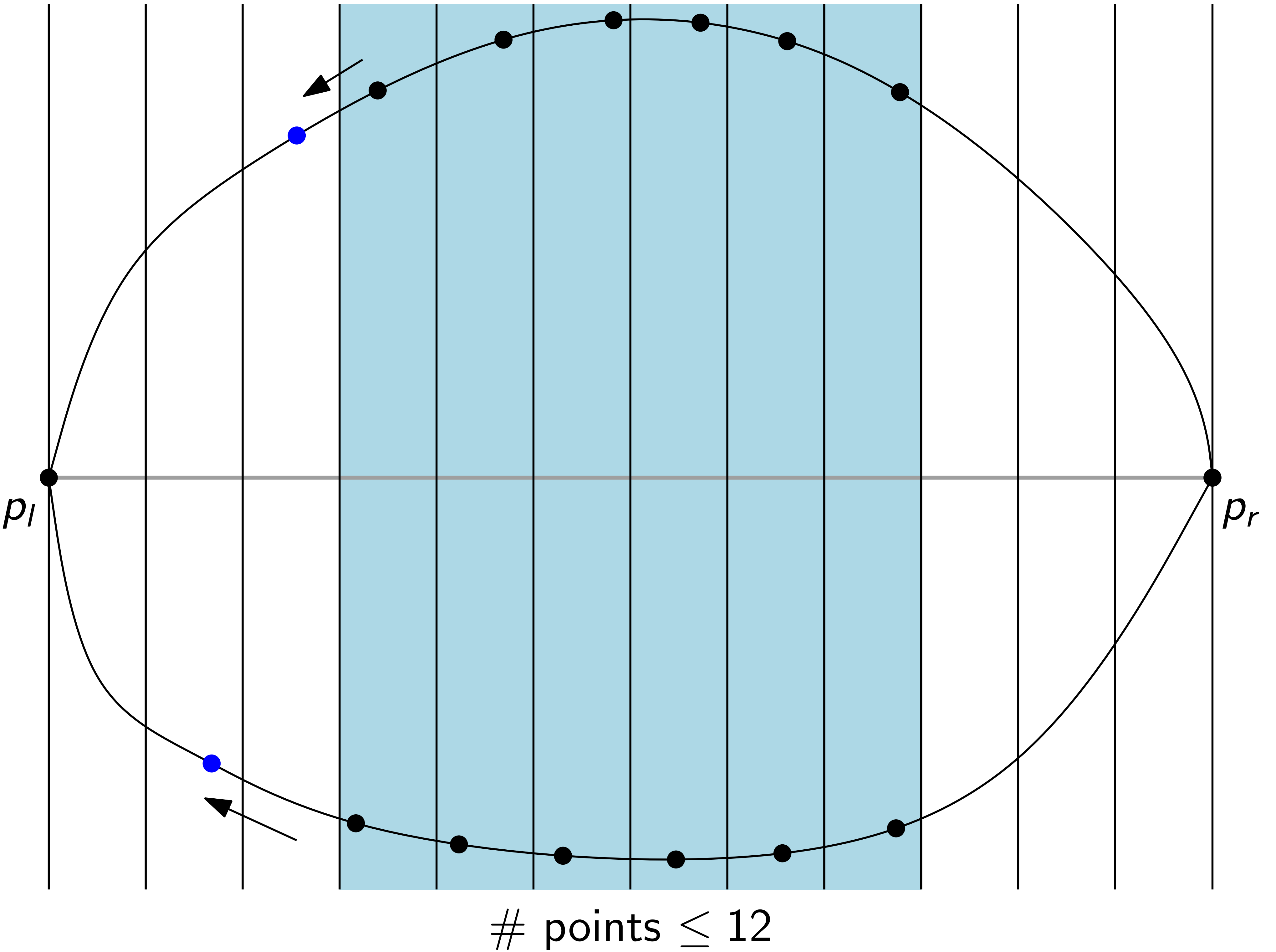


p_l
p_r
$\#$ points ≤ 12


p_l
p_r
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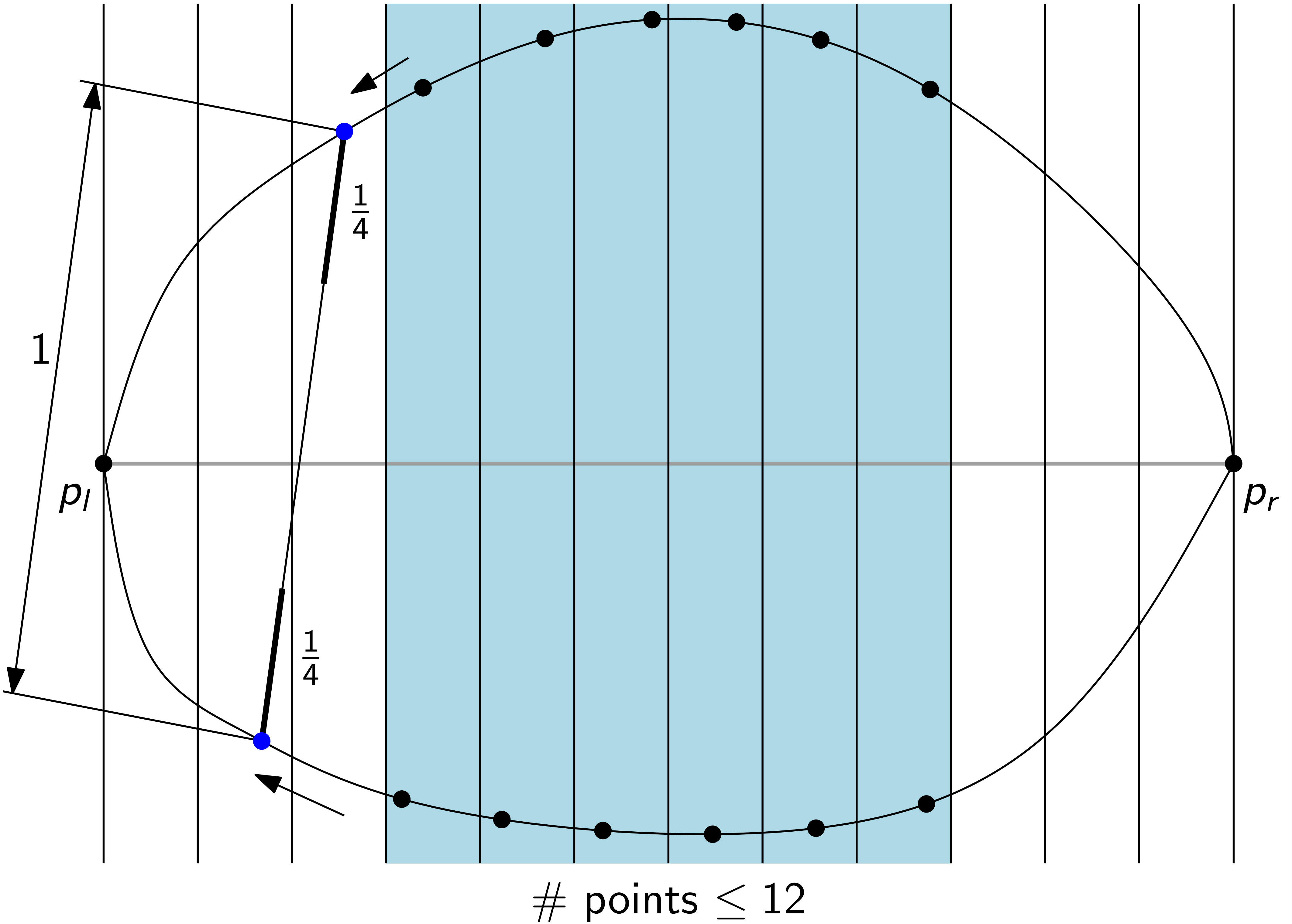
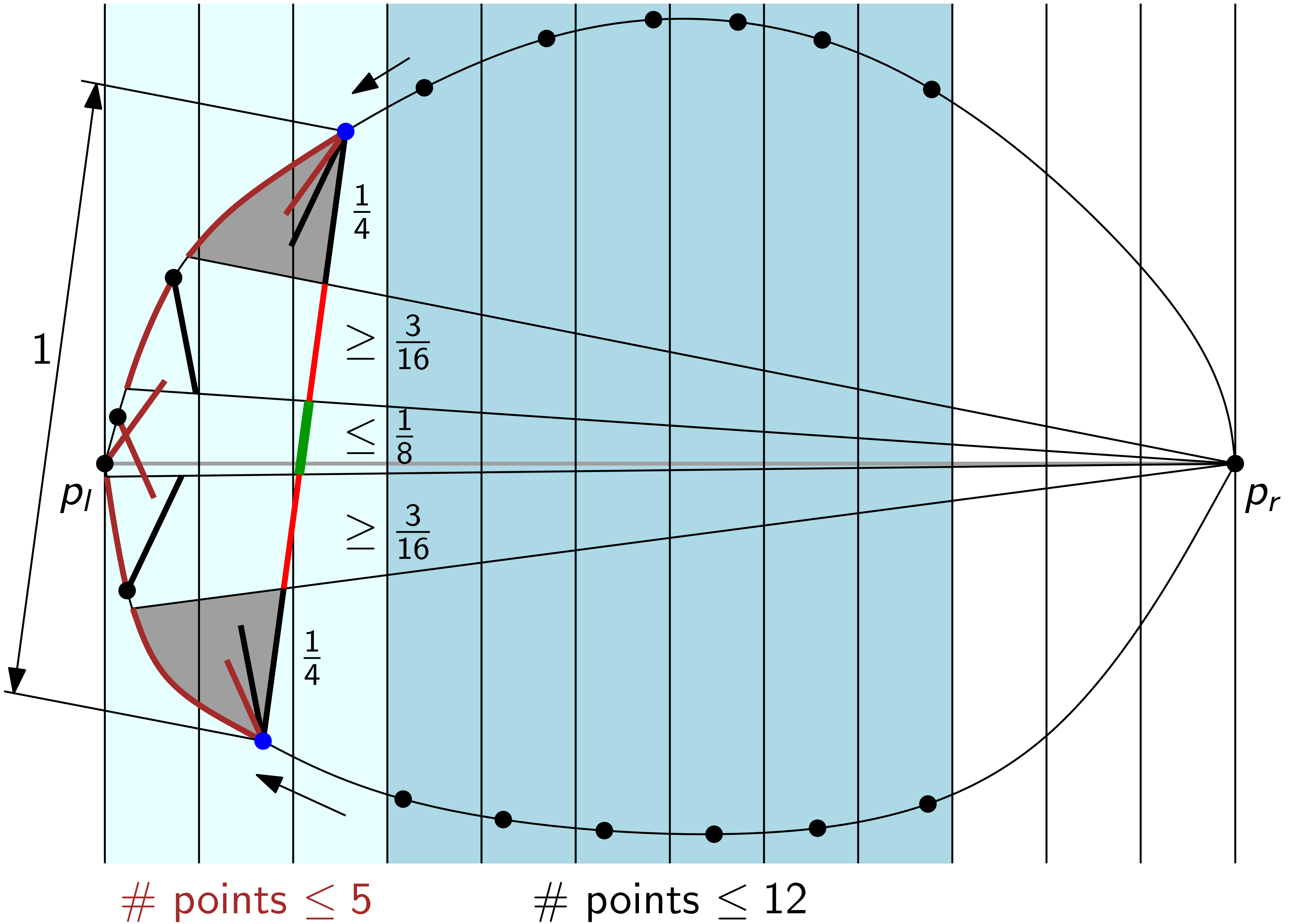
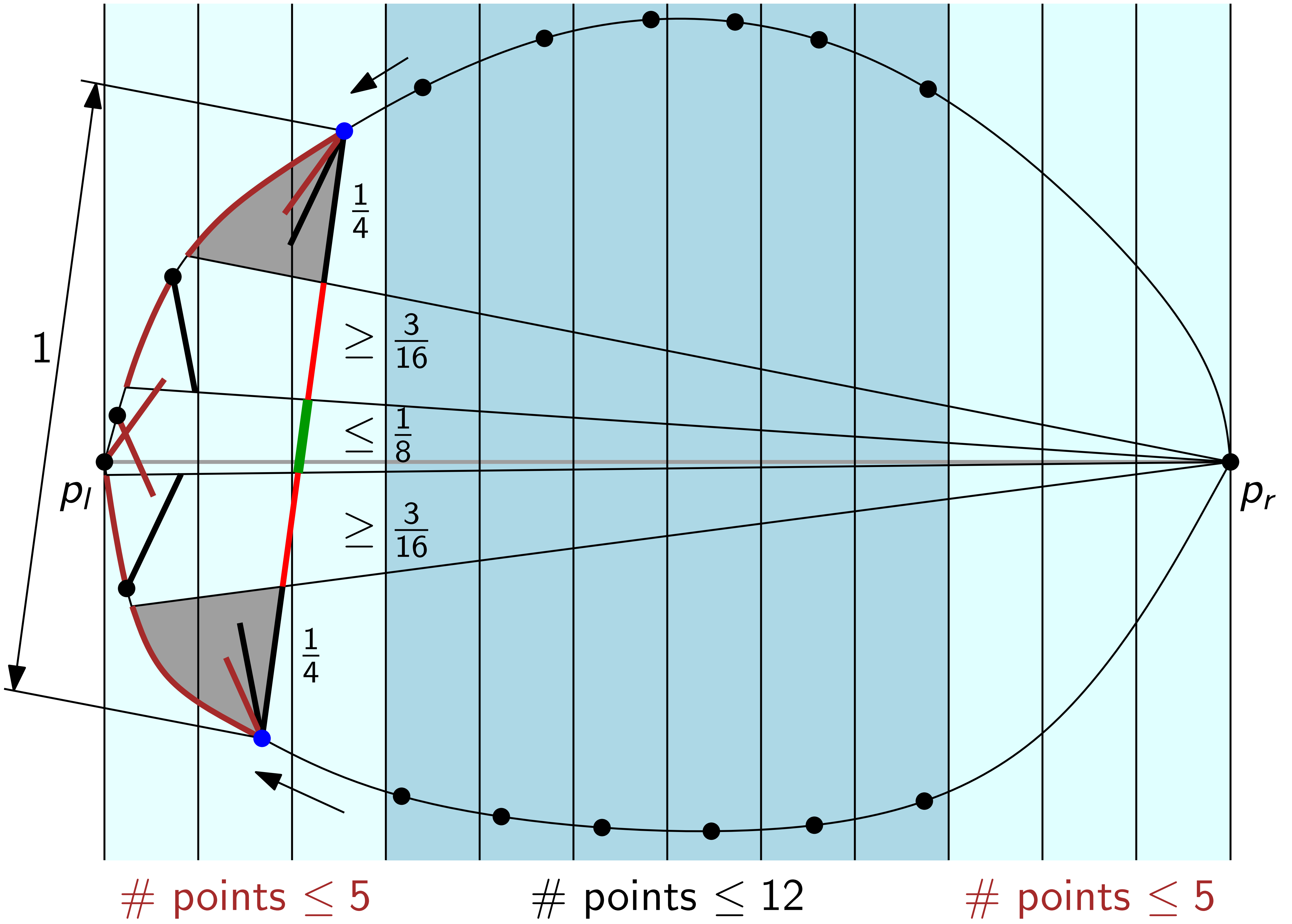


1
$\frac{1}{4}$
$\frac{1}{4}$
p_l
p_r
points ≤ 12



1
$\frac{1}{4}$
$\geq \frac{3}{16}$
$\leq \frac{1}{8}$
$\geq \frac{3}{16}$
$\frac{1}{4}$
p_l
p_r
points ≤ 5
points ≤ 12



1
$\frac{1}{4}$
$\geq \frac{3}{16}$
$\leq \frac{1}{8}$
p_l
p_r
$\geq \frac{3}{16}$
$\frac{1}{4}$
points ≤ 5
points ≤ 12
points ≤ 5

Drawing graphs with low visual complexity

- slope number
- segment / arc number
- line / plane cover number

Geometric restrictions

- point-set embeddability
- track number

Making crossings nicer

- RAC / LAC drawings
- (smooth / slanted) orthogonal drawings
- edge casings

Eliminating crossings

- partial edge drawings
- confluent drawings

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Questions?

Drawing Graphs on Few Lines and Few Planes

Steven Chaplick Krzysztof Fleszar
Fabian Lipp Alexander Wolff
Julius-Maximilians-Universität Würzburg, Germany

Alexander Ravsky
National Academy of Science of Ukraine, Lviv, Ukraine

Oleg Verbitsky
Humboldt-Universität zu Berlin, Germany

Kuratowski's Theorem

Thm. [Kuratowski 1930: *Sur le problème des courbes gauches en topologie*]
Let G be a simple graph. Then:
G planar $\Leftrightarrow$ neither K_5 nor $K_{3,3}$ is minor of G.

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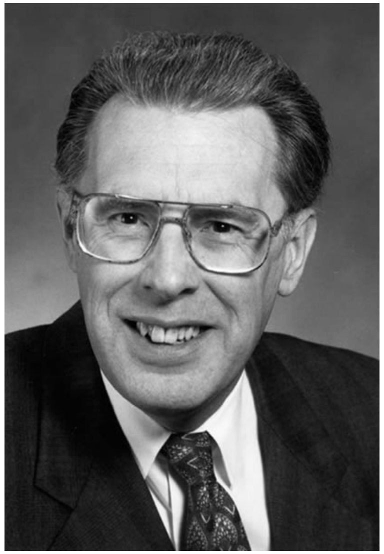
Thm. [Robertson & Seymour 1983–94: *Graph minors I–XIV*]
Every minor closed graph class has a *finite* obstruction set.

Planar Graphs: Testing and Drawing

Thm. [Hopcroft & Tarjan, J. ACM 1974]
Let G be a simple graph with n nodes.
It takes $O(n)$ time to test whether G is planar.

John Edward Hopcroft
*1939, Seattle, WA, U.S.A.



Robert Endre Tarjan
*1948 Pomona, CA, USA

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Every planar graph can be drawn *straightline*.

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Cologne 1910–2000

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Gyula, Hungary 1922
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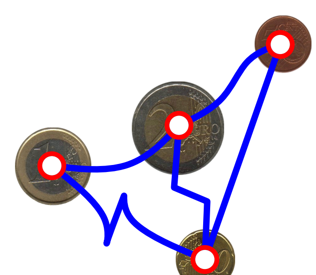
Thm. [Koebe 1936: *Kontaktprobleme der konformen Abbildung*]
Every planar graph can be represented as the *contact graph* of disks (*coin graph*).



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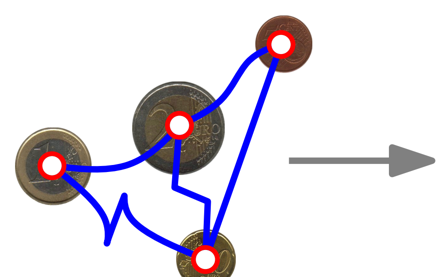

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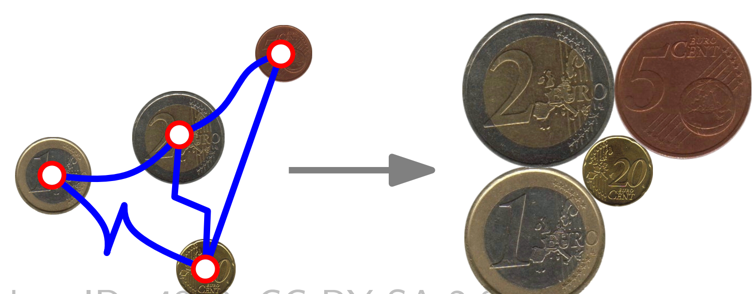

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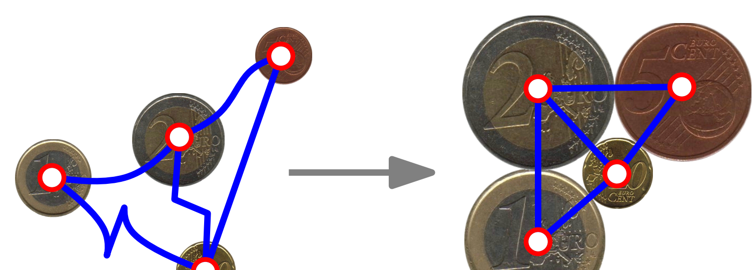
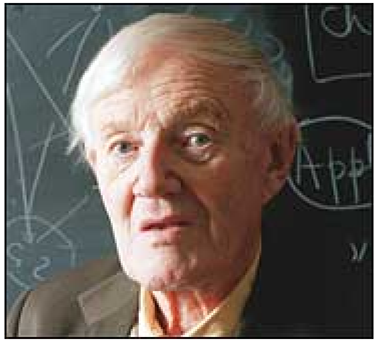


Drawing Planar Graphs

Thm. [Tutte 1963: *How to draw a graph*]
A (3-connected) planar graph can be drawn straightline (and convex); in linear time.



William Thomas Tutte
Newmarket, GB 1917–2002 Kitchener, Kanada

Drawing Planar Graphs

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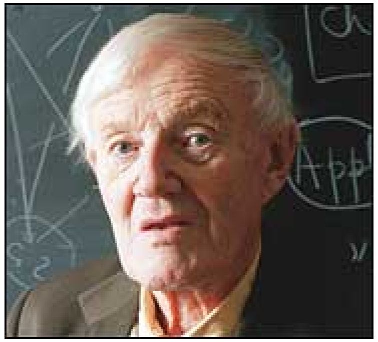

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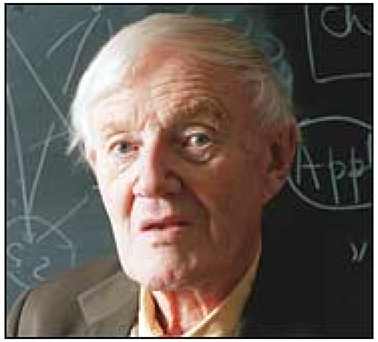

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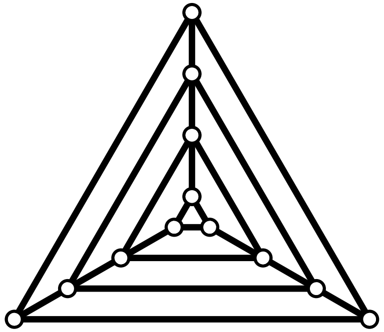

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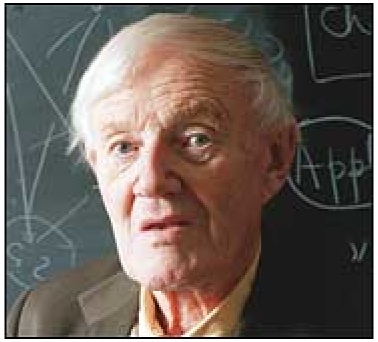

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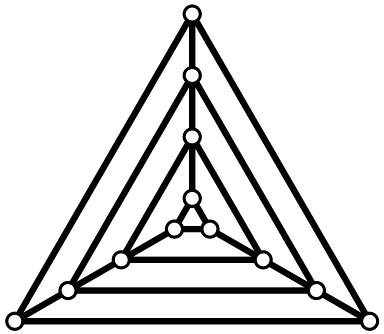

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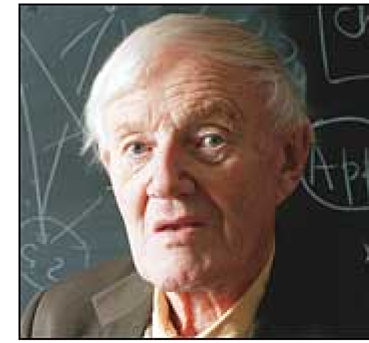

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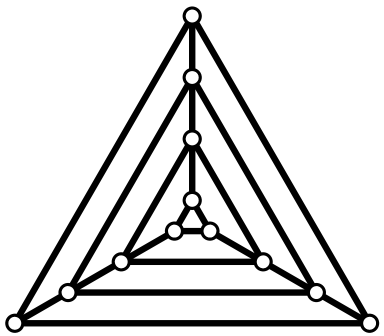

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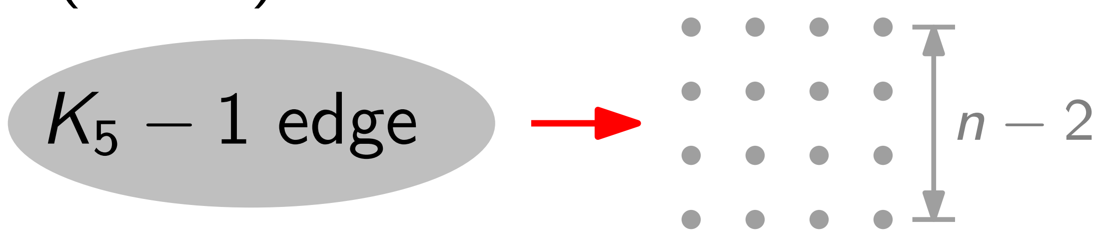

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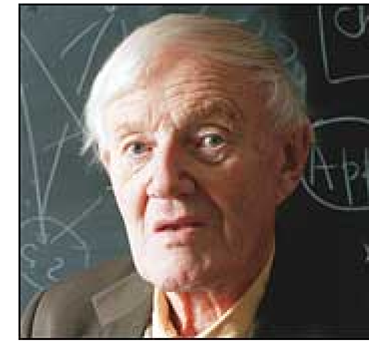

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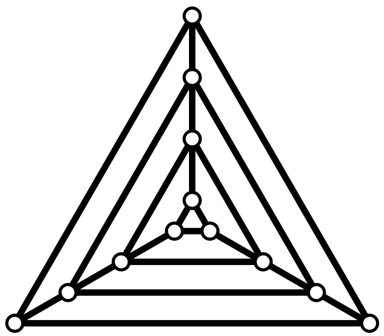

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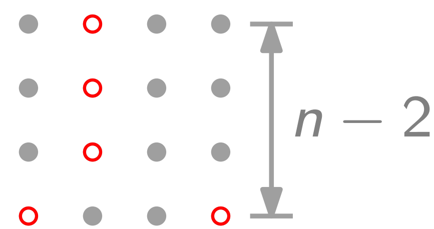

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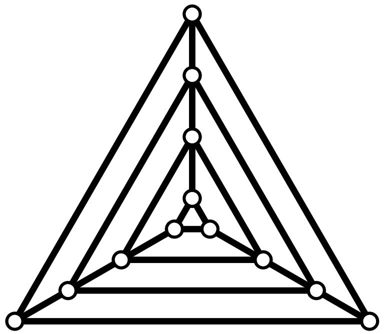

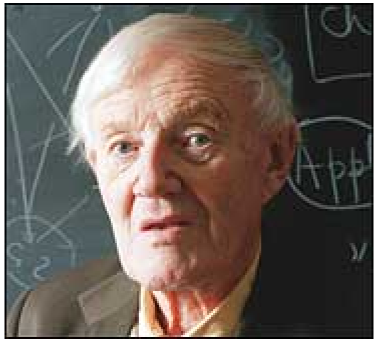

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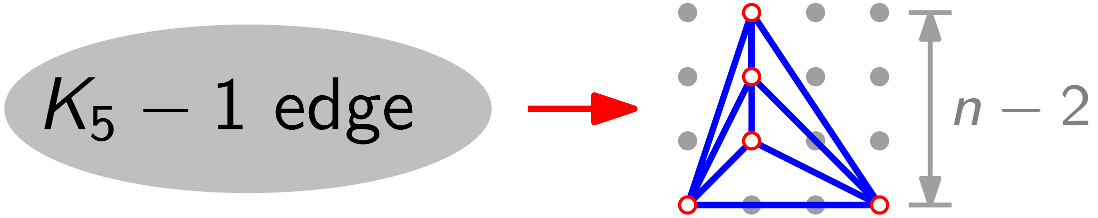

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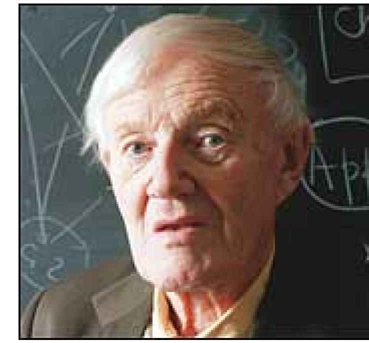

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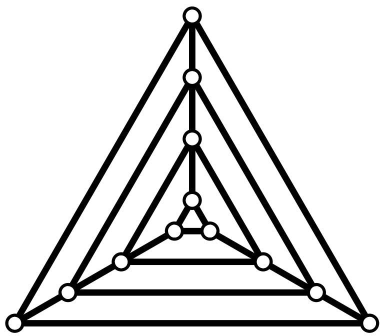

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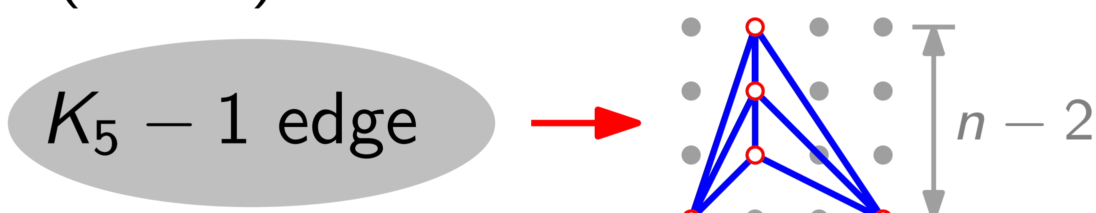

Thm. [De Fraysseix, Pach, Pollack: Combinatorica 1990]

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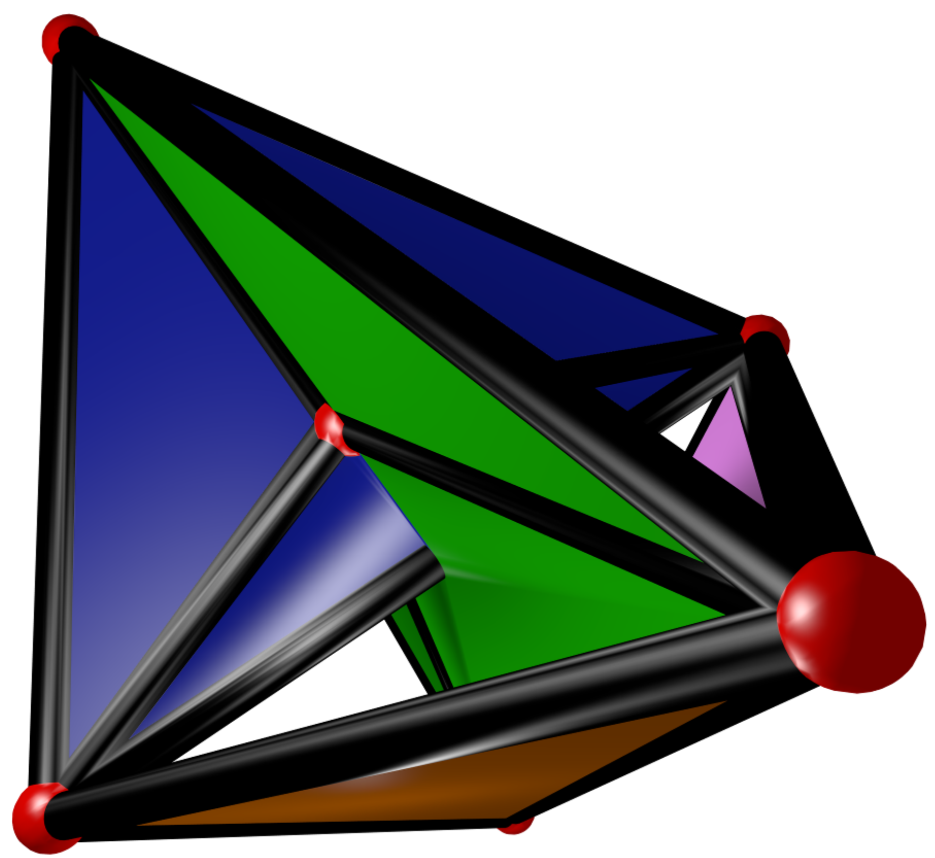

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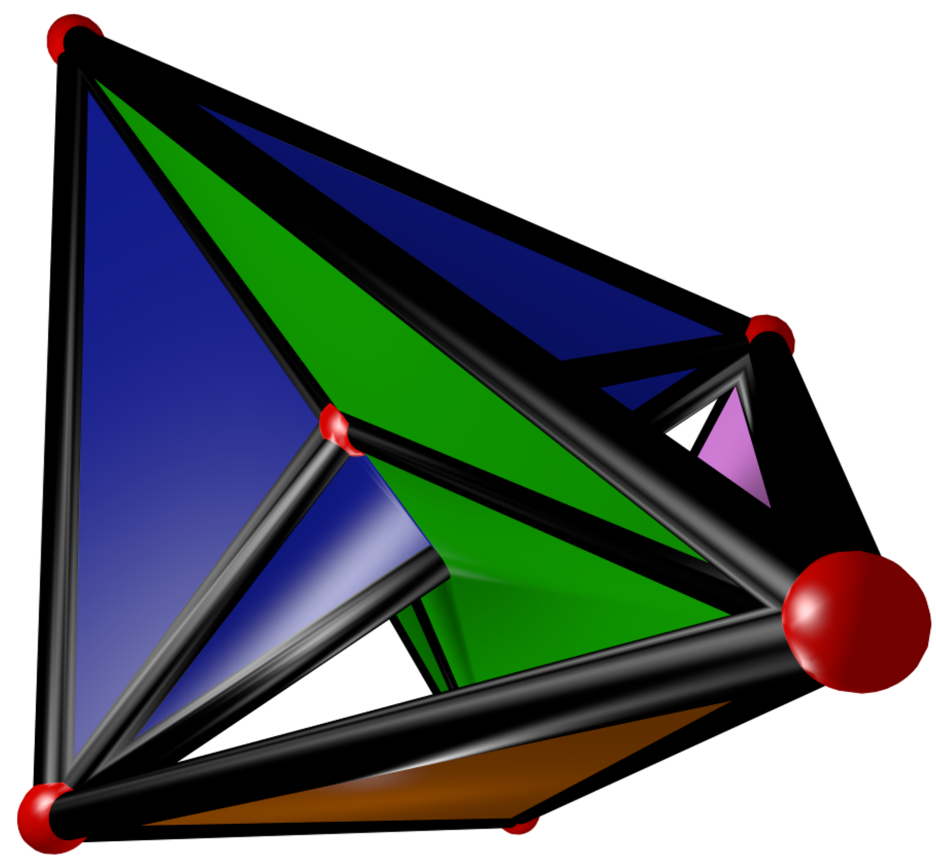

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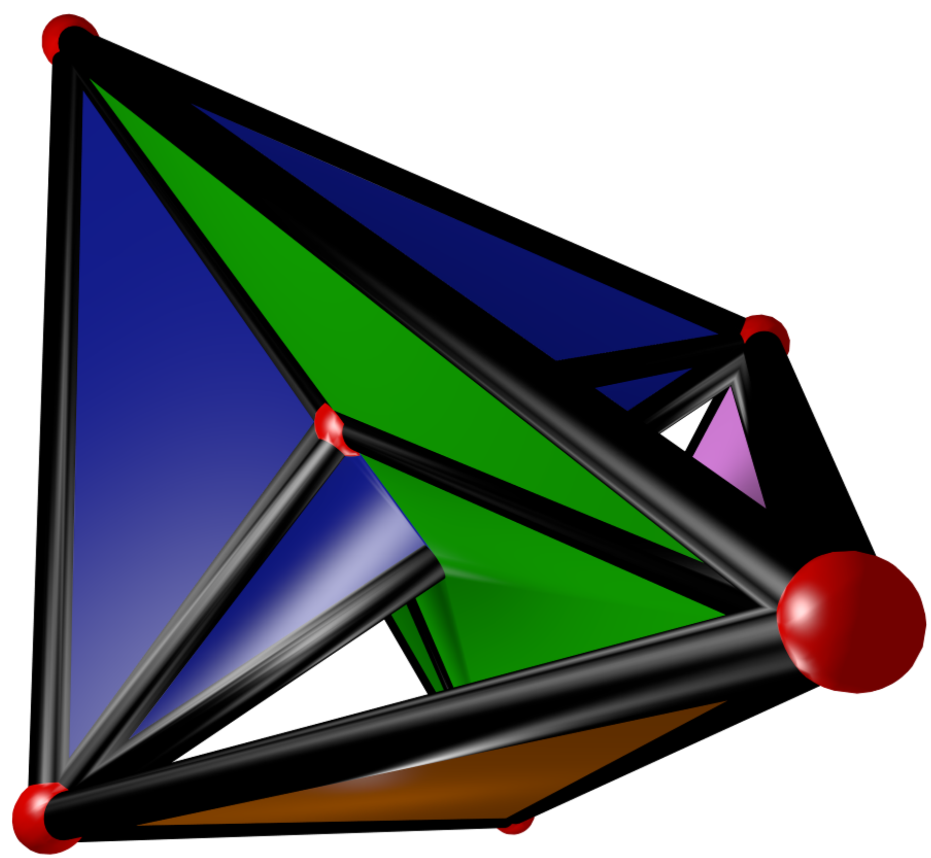

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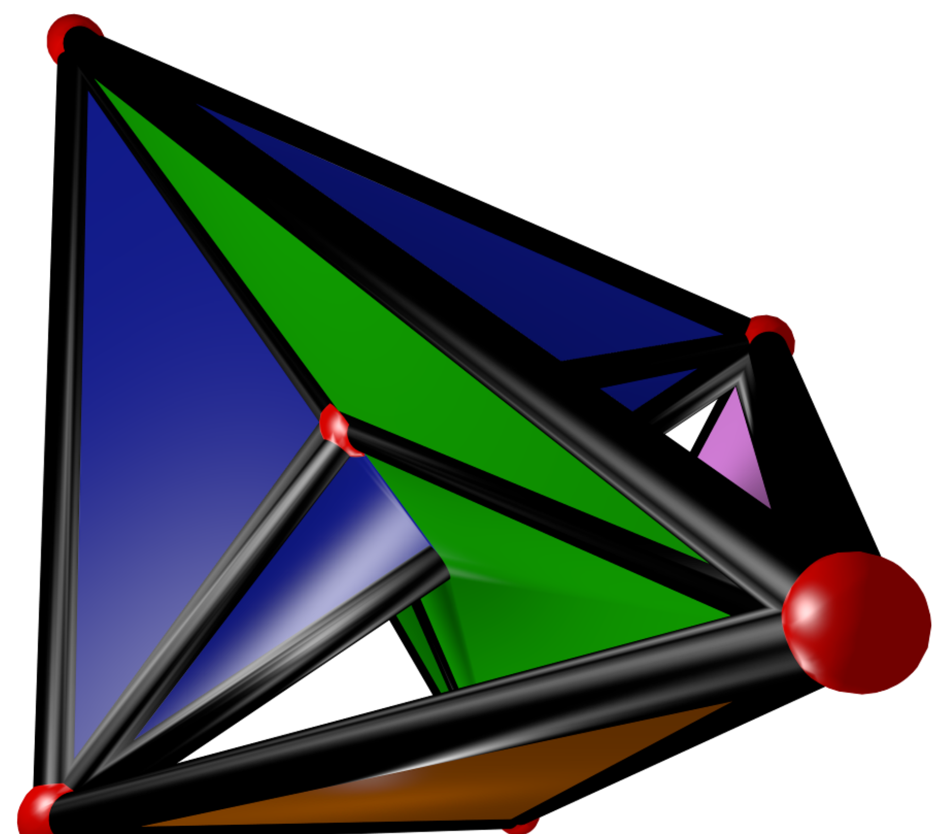

For any planar graph G, clearly $\rho_3^2(G) = 1$.

We can consider ρ_3^2 as a parameter for classifying non-planar graphs.

Formal Definition

Let G be a graph and $1 \leq m < d$.

Affine cover number $\rho^m_d(G)$:

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Complexity: Computing ρ_2^1, ρ_3^1, ρ_3^2, π_3^1, and π_3^2 is NP-hard.

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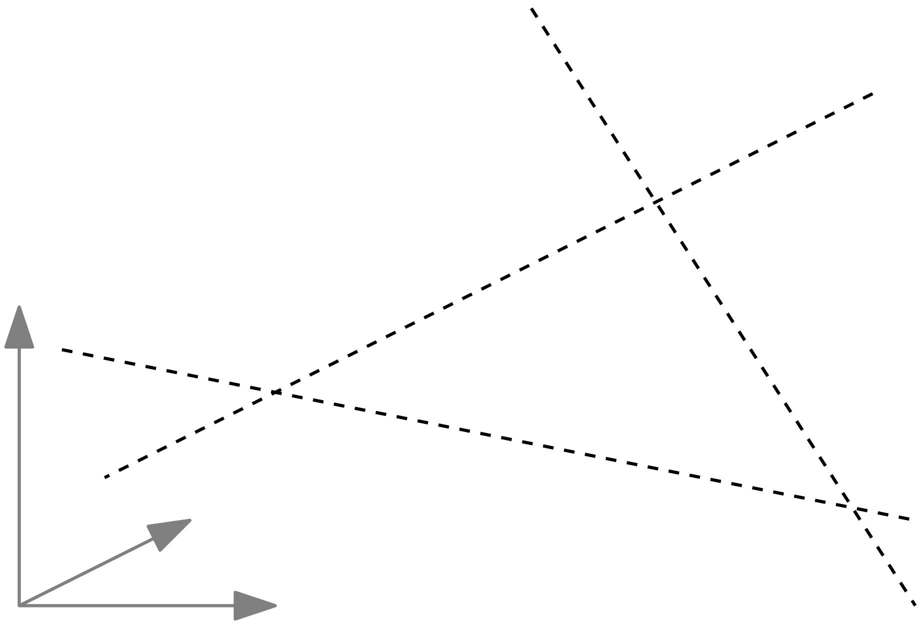

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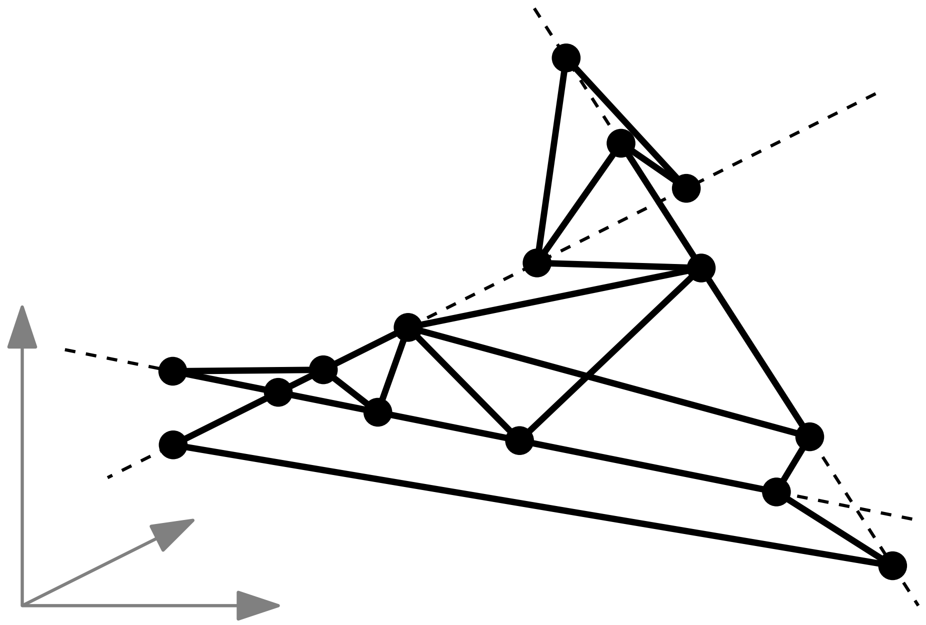

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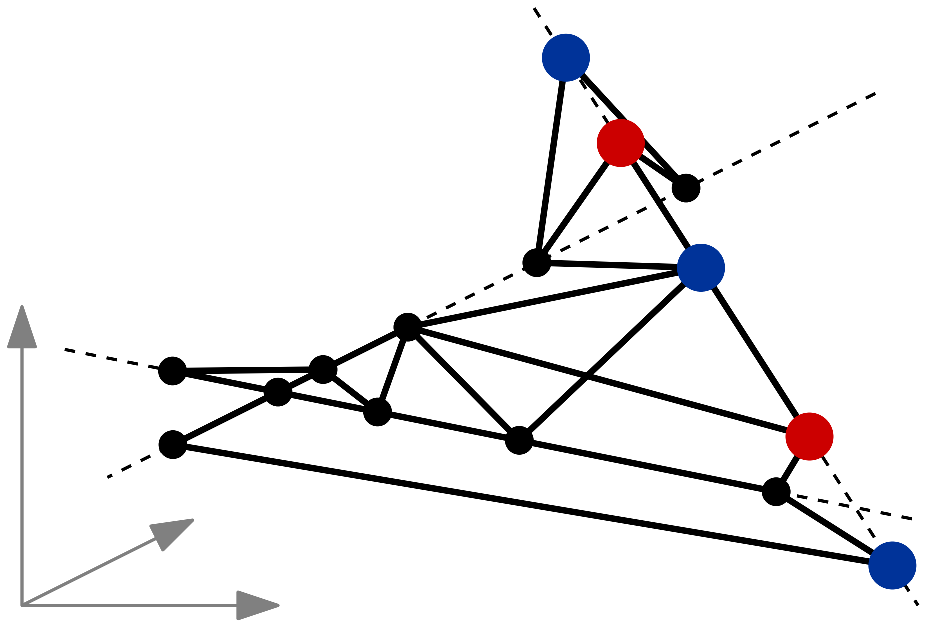

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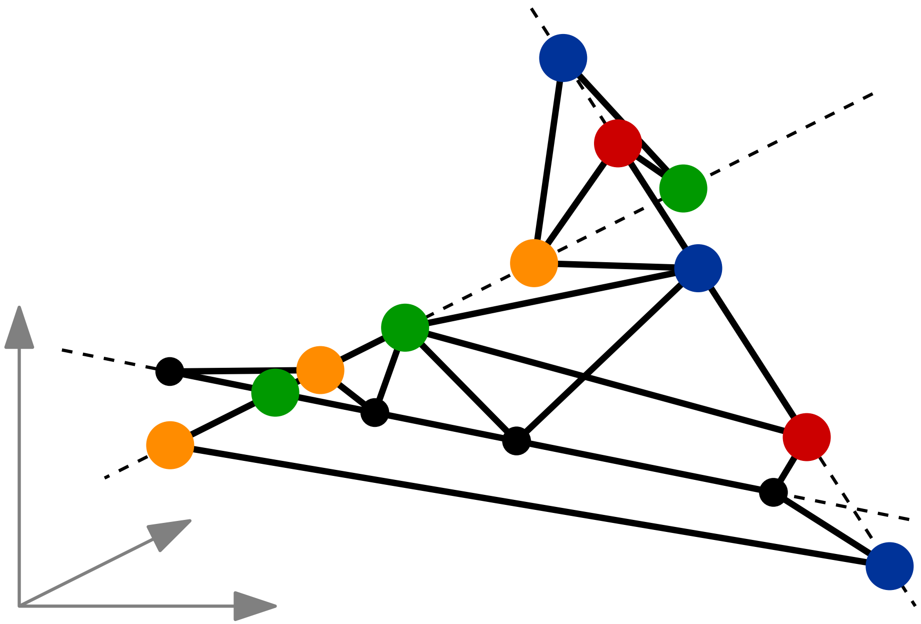

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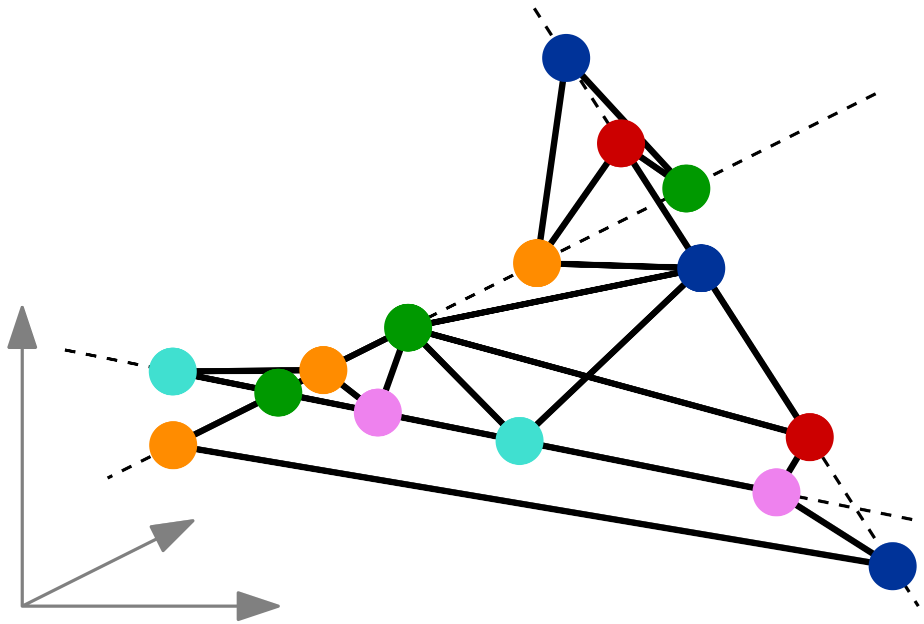

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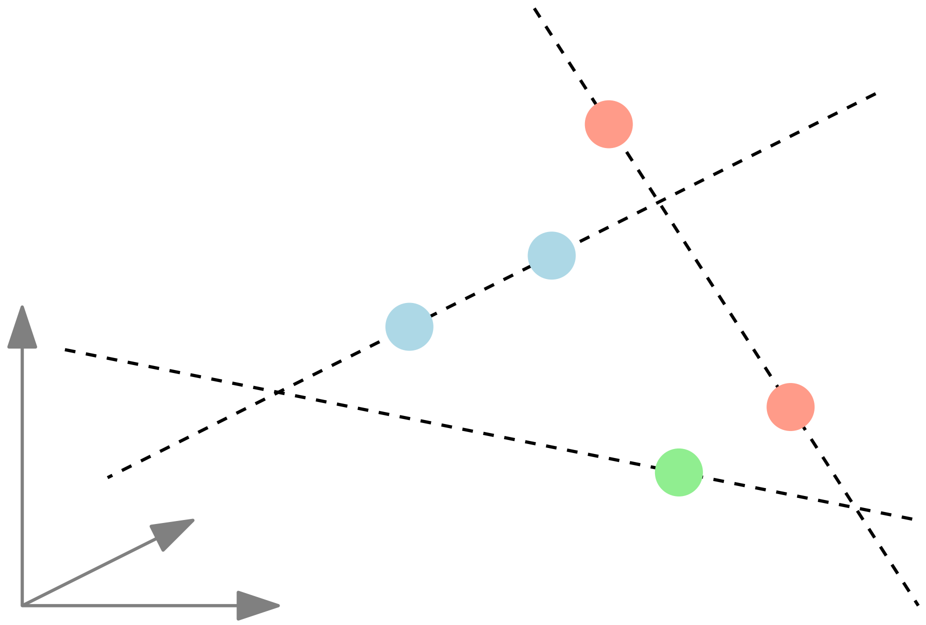

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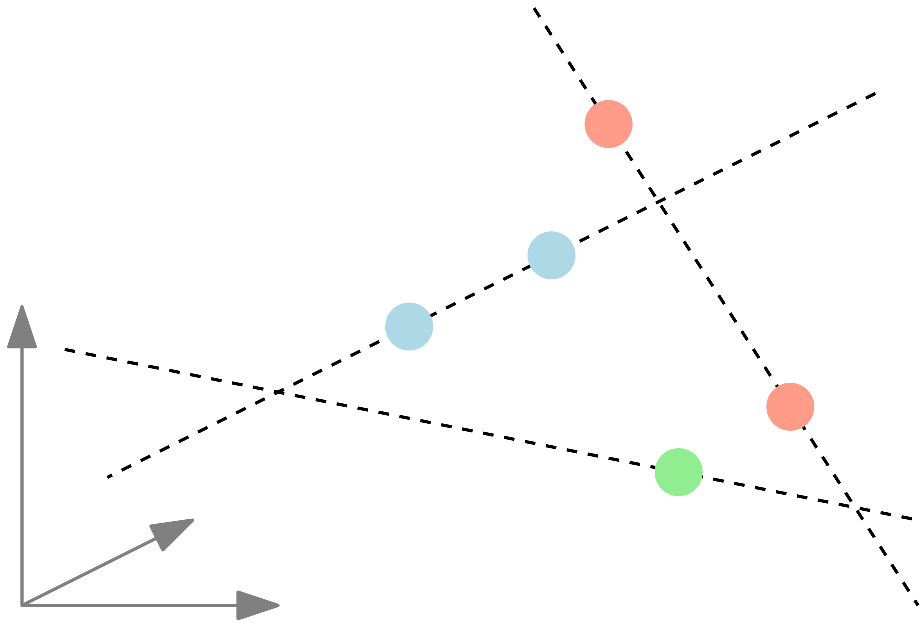

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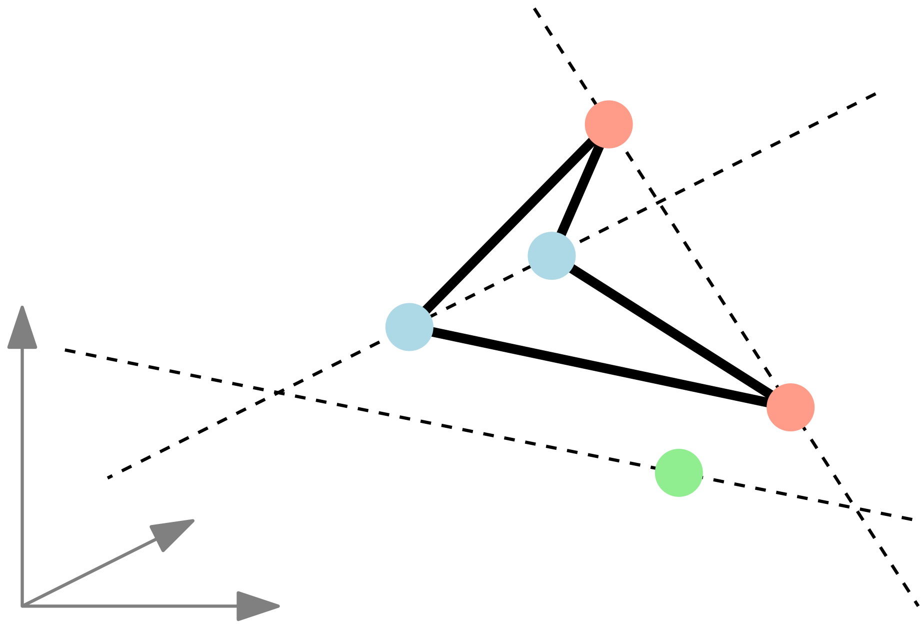

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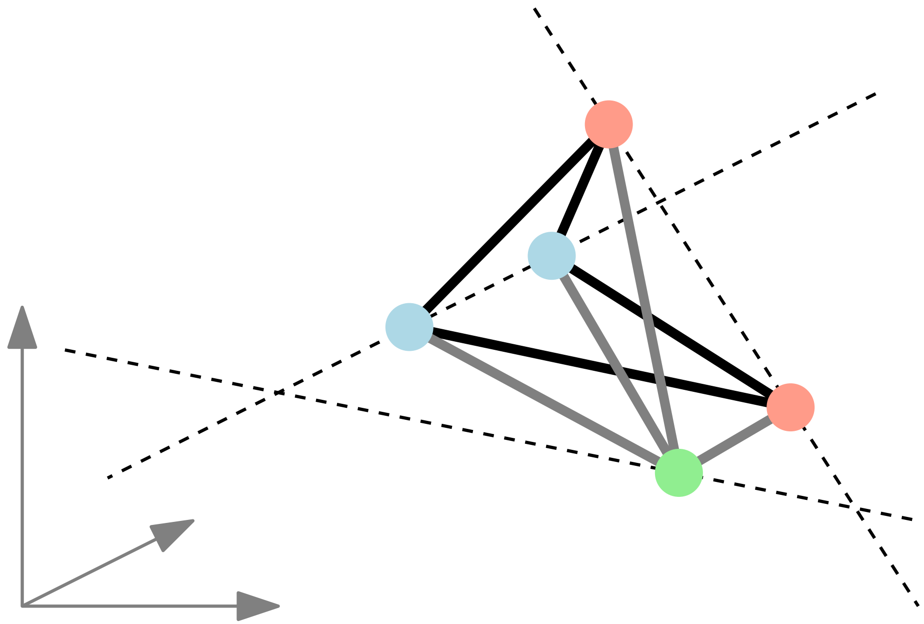

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- $\pi_3^2(G) = \mathrm{vt}(G)$
  vertex thickness: smallest partition of V such that each set induces a planar graph

Relations to Other Parameters

For planar, connected graphs G:

$$\boxed{\mathrm{slope}(G) \leq \rho_2^1(G) \leq \mathrm{segm}(G)}$$

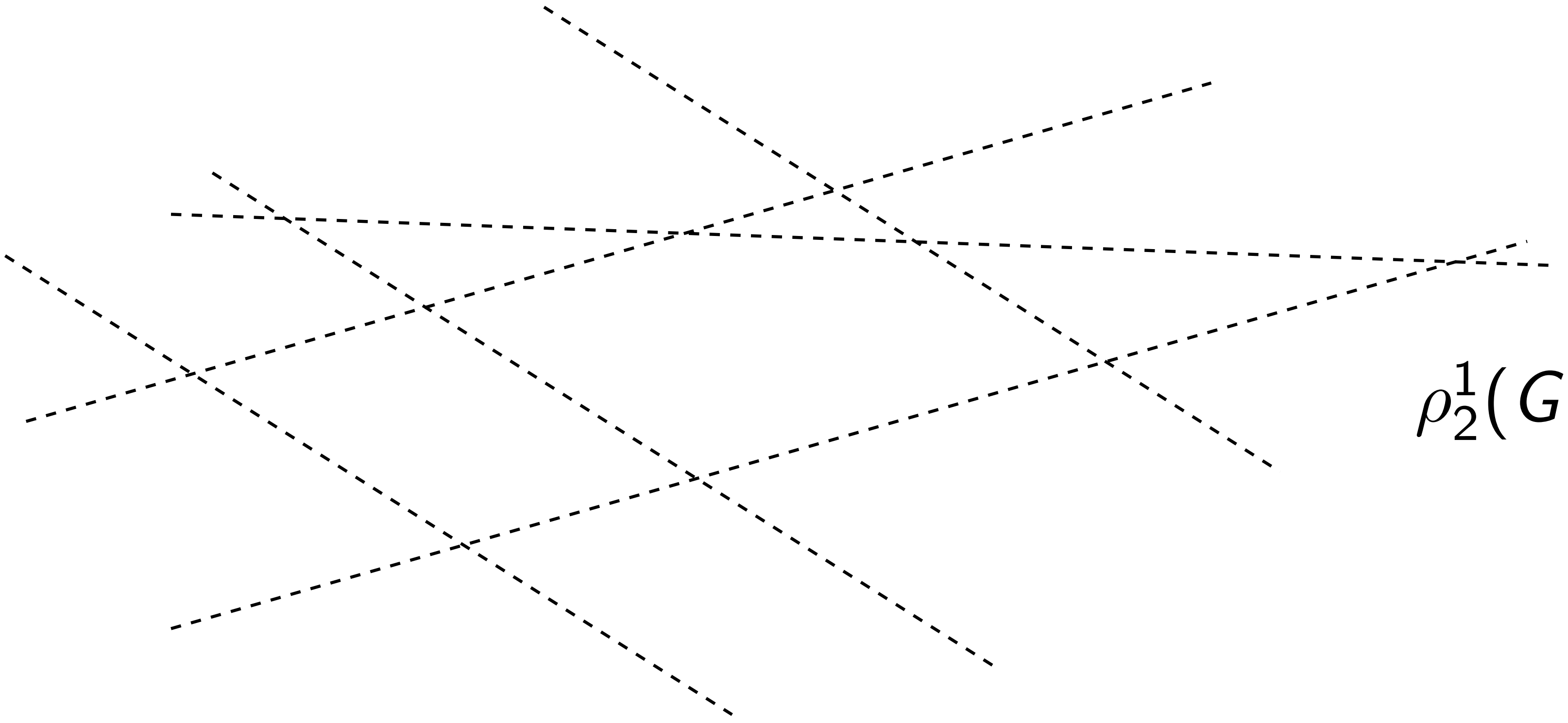
Relations to Other Parameters

For planar, connected graphs G:

minimum number of different slopes needed to draw G

[Dujmović et al. 2007]

$\boxed{\text{slope}(G) \leq \rho_2^1(G) \leq \text{segm}(G)}$



$\rho_2^1(G) \leq 6$

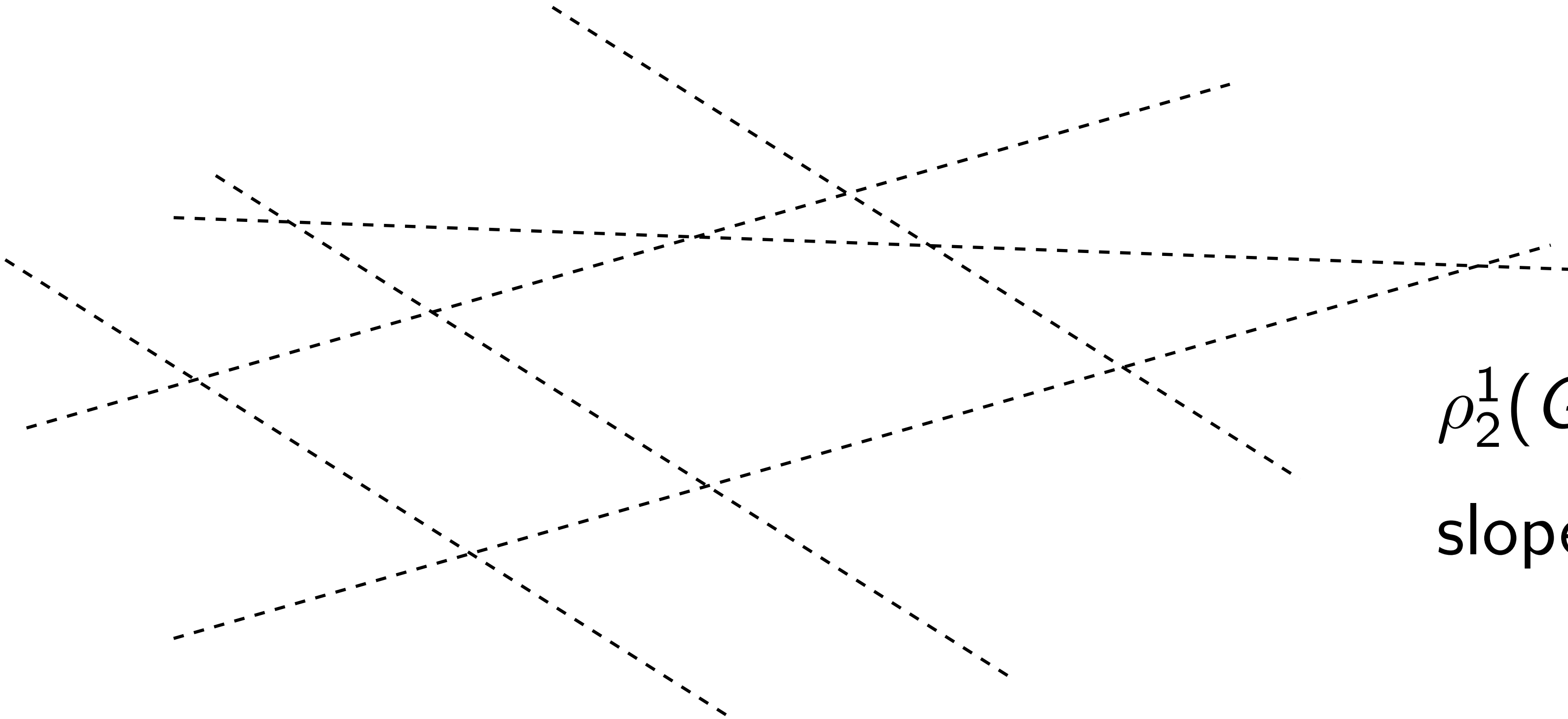
Relations to Other Parameters

For planar, connected graphs G:

minimum number of different slopes needed to draw G

[Dujmović et al. 2007]

$$\boxed{\mathrm{slope}(G) \leq \rho^1_2(G) \leq \mathrm{segm}(G)}$$



$\rho^1_2(G) \leq 6$

$\mathrm{slope}(G) \leq 3$

Relations to Other Parameters

For planar, connected graphs G:

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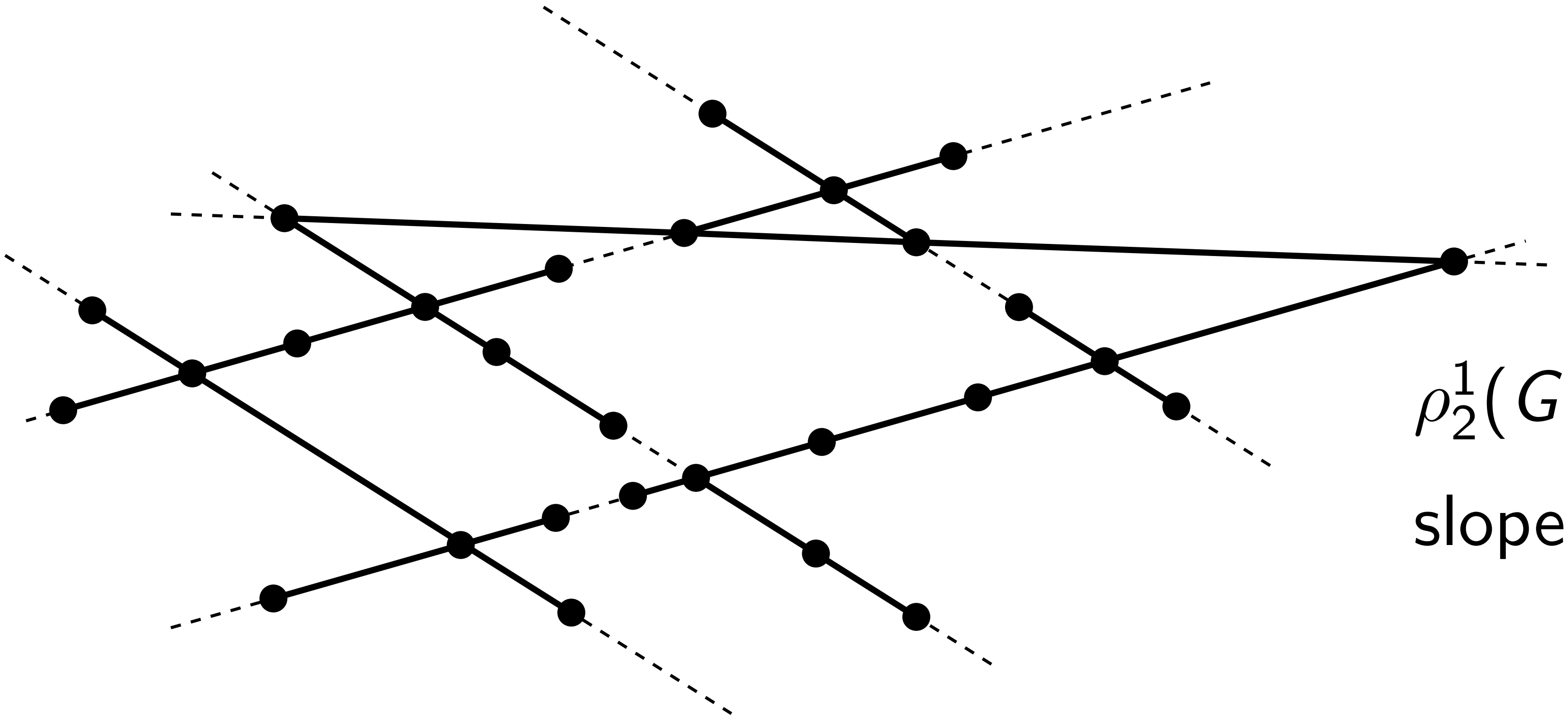

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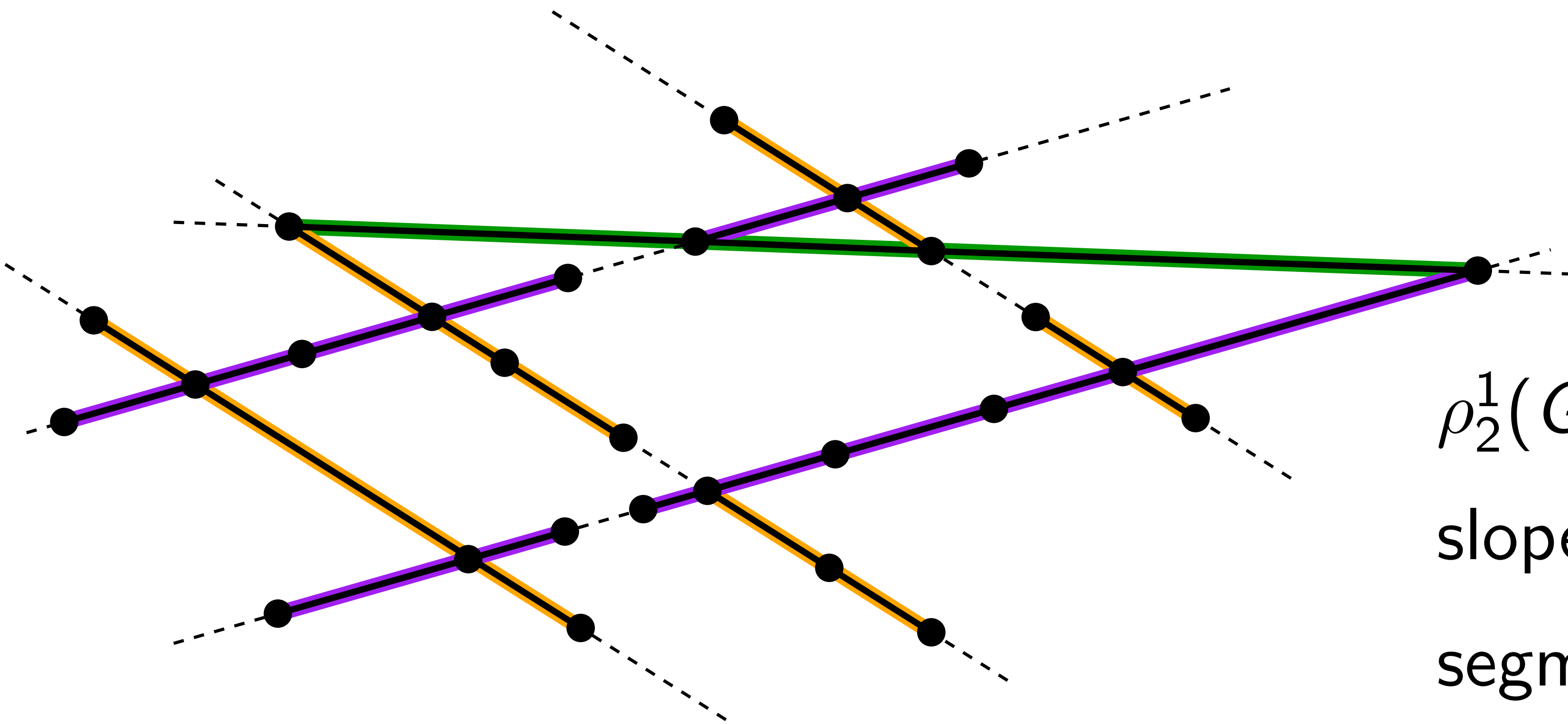
$$\boxed{\mathrm{slope}(G) \leq \rho_2^1(G) \leq \mathrm{segm}(G)}$$

minimum number of line segments needed to draw G



$\rho_2^1(G) \leq 6$

$\mathrm{slope}(G) \leq 3$

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Relations to Other Parameters

For planar, connected graphs G:

minimum number of different slopes needed to draw G

[Dujmović et al. 2007]

$$\boxed{\mathrm{slope}(G) \leq \rho_2^1(G) \leq \mathrm{segm}(G) = O(\rho_2^1(G)^2)}$$

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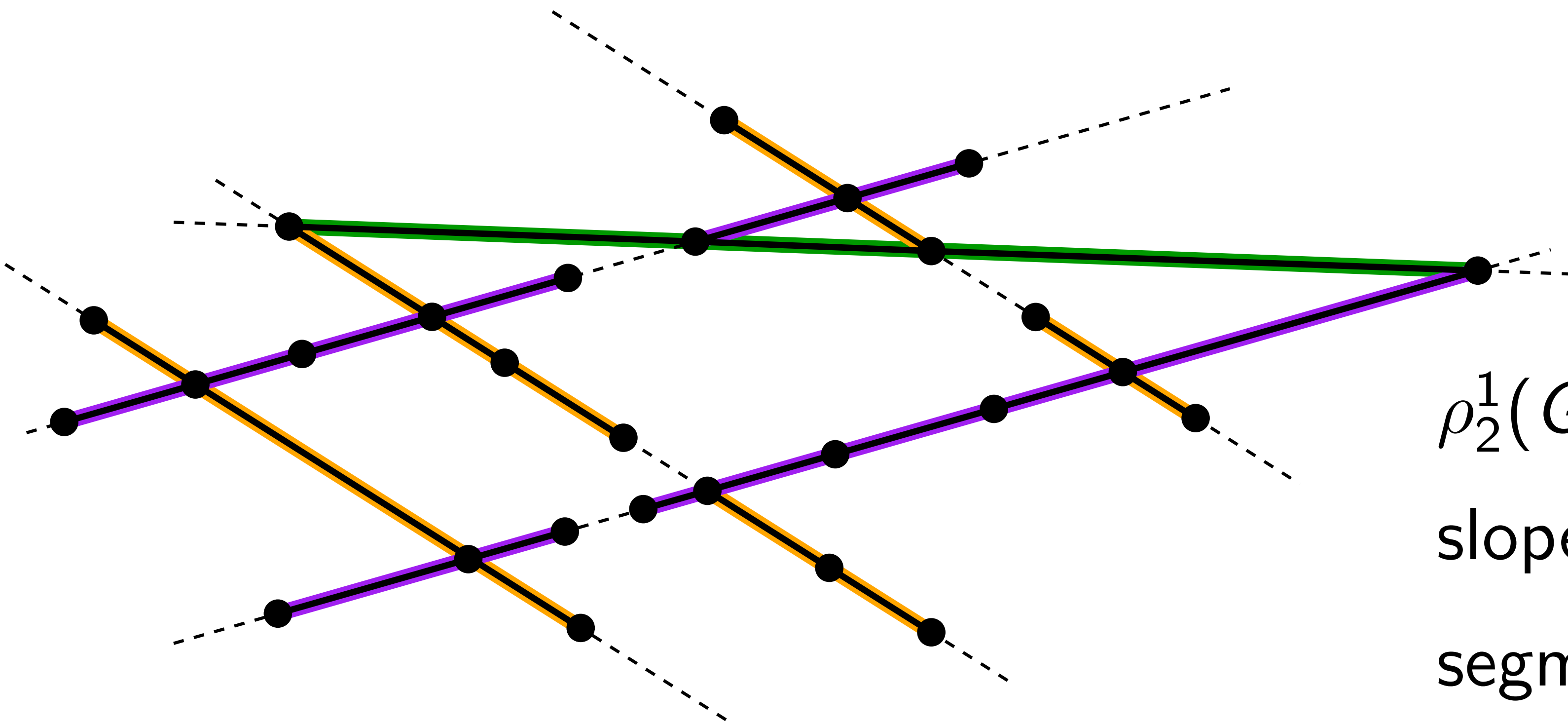

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Our Results

Complexity: Computing ρ_2^1, ρ_3^1, ρ_3^2, π_3^1, and π_3^2 is NP-hard.

Upper bound for outerplanar graphs π_2^1

ρ_2^1 Relations to other graph parameters

Relations to other graph parameters $\begin{cases} \pi_3^1 \\ \pi_3^2 \end{cases}$

ρ_3^1 Lower bound

ρ_3^2 Bounds for K_n

Complete Graphs K_n

- $\pi_3^1(K_n) =$

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- $\pi^1_3(K_n) =$ *Idea:* Draw K_n in general position. Place lines s.t. each covers two vertices.

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at most one K_4 on any plane

distribute $n(n-1)/2$ edges s.t.
each plane contains ≤ 6 edges

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use one plane for each K_3

Results on *Steiner triple systems* imply that the edge set of K_n can be covered *exactly* by copies of K_3 iff $n \equiv 1 \pmod 6$ or $n \equiv 3 \pmod 6$

[Kirkman, 1847]

Our Results

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Upper bound for outerplanar graphs π_2^1

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Relations to other graph parameters $\left\{ \begin{array}{l} \pi_3^1 \\ \\ \pi_3^2 \end{array} \right.$

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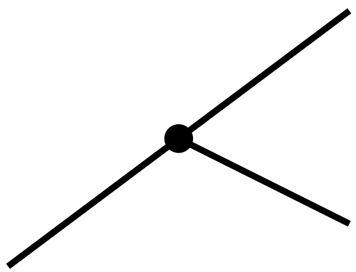
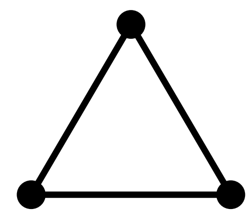
Lower Bound for $\rho_3^1(G)$

Essential vertex: degree ≥ 3 or belongs to a K_3





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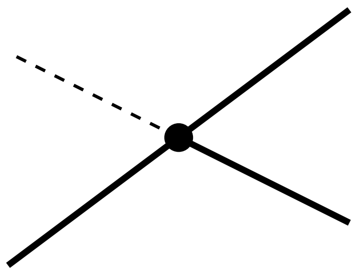

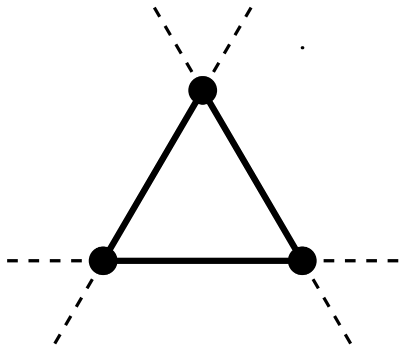

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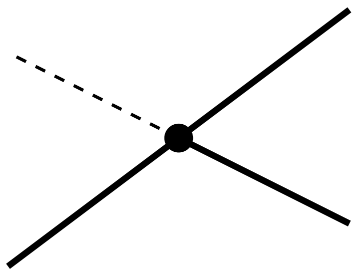

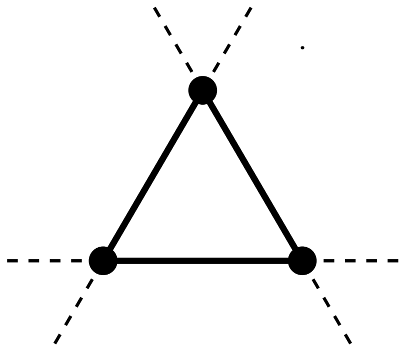

- Essential vertices on intersections of at least two lines
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- $\mathrm{es}(G) \leq \binom{\rho_3^1(G)}{2}$

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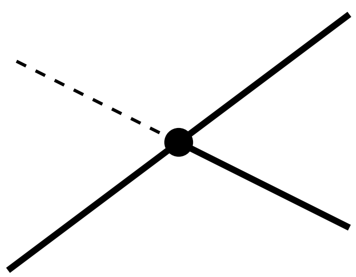

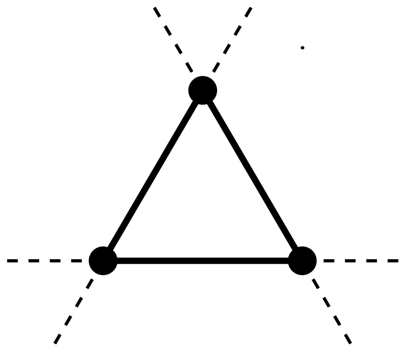

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Additionally we can show: $\boxed{\rho^1_3(G) \geq \mathrm{tw}(G)/3}$

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Upper bound for outerplanar graphs	π_2^1	ρ_2^1	Relations to other graph parameters
Relations to other graph parameters	π_3^1	ρ_3^1	Lower bound
	π_3^2	ρ_3^2	Bounds for K_n

π_2^1 for Outerplanar Graphs

Outerplanar graphs are track drawable.

[Felsner, Liotta, Wismath, JGAA 2003]

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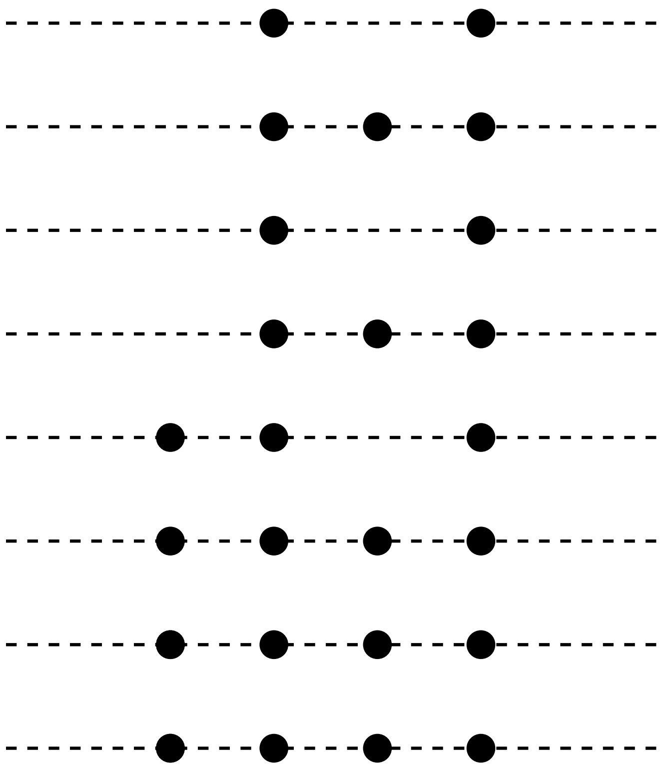

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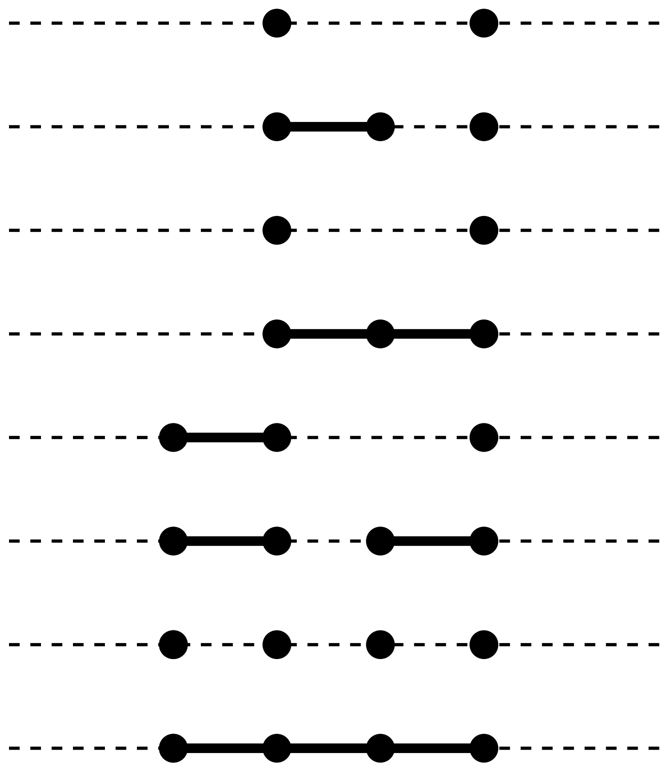

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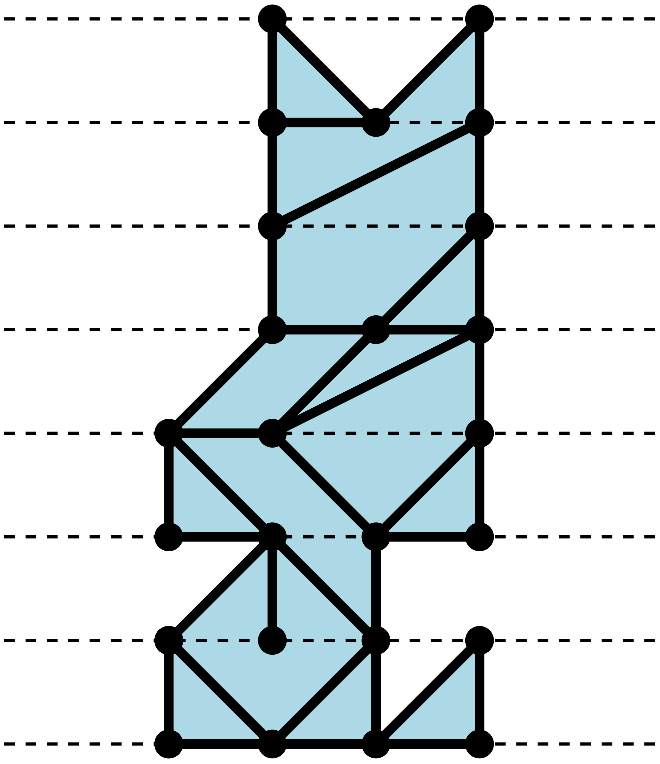

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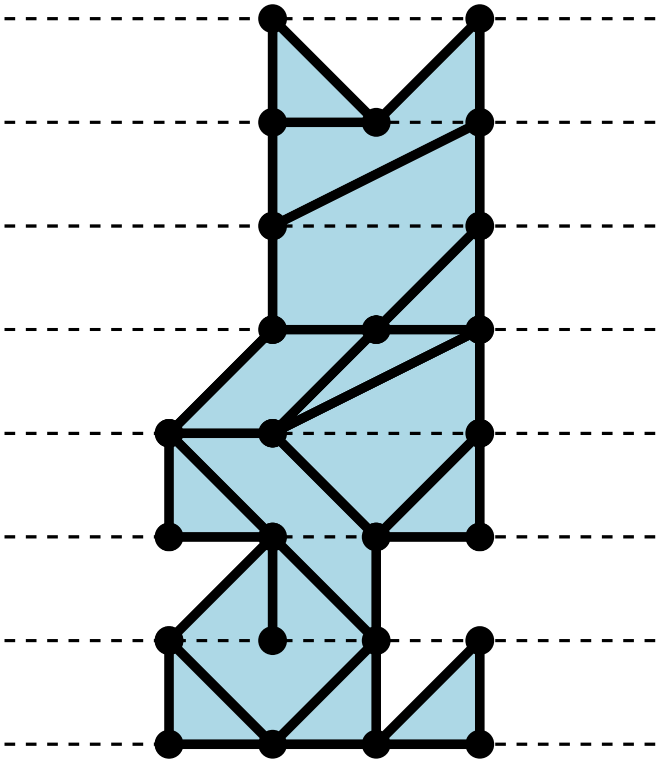

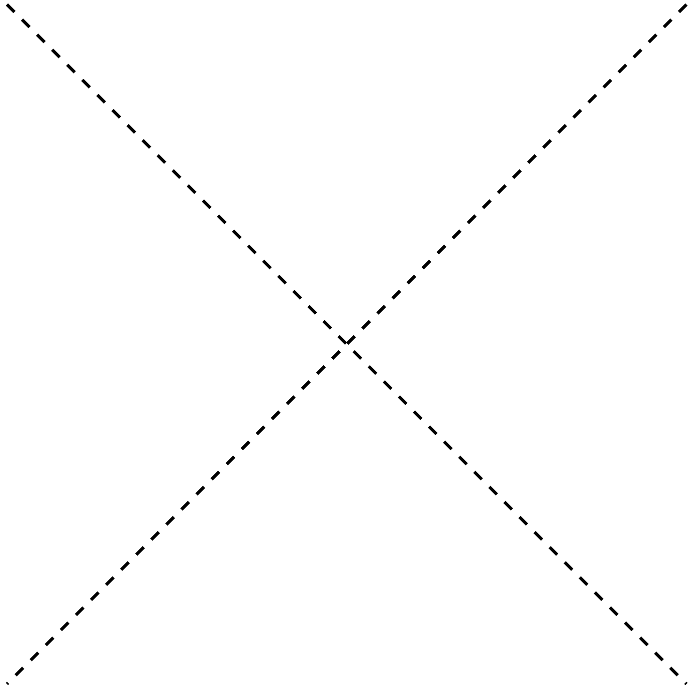

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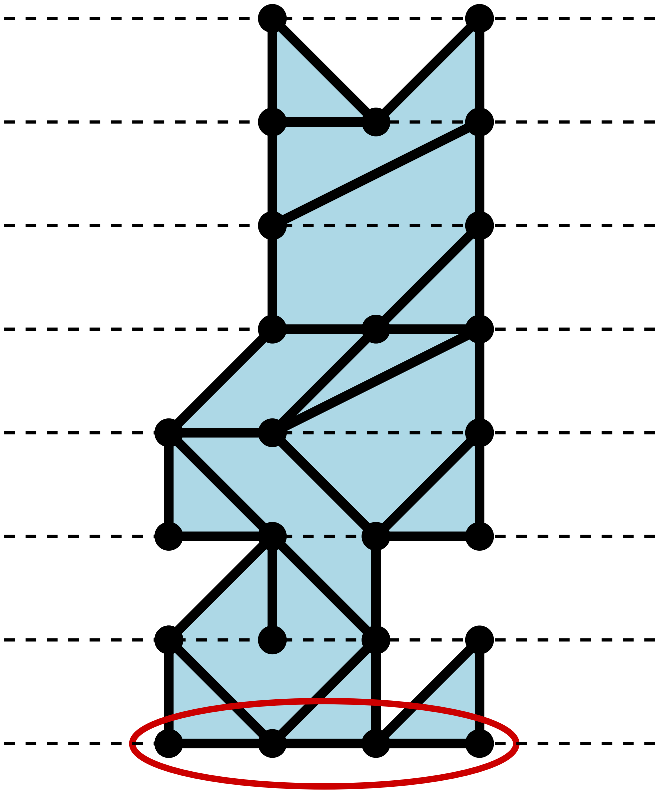

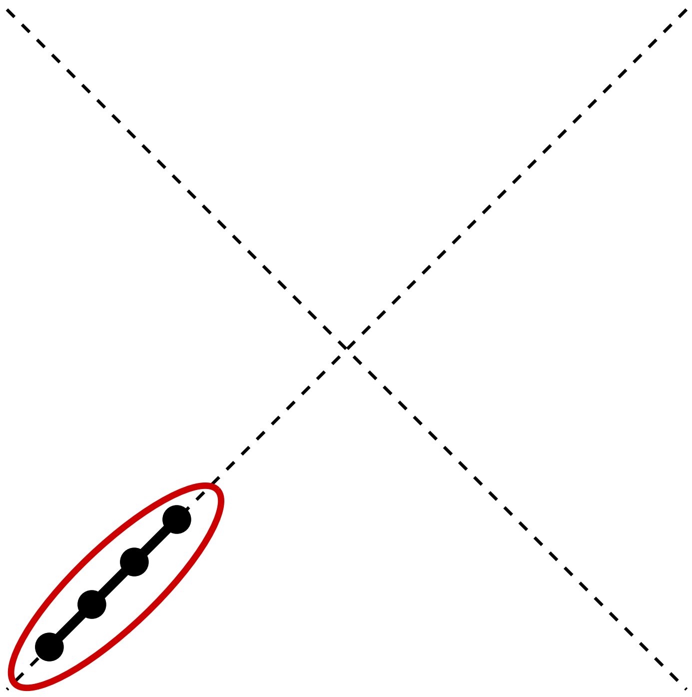

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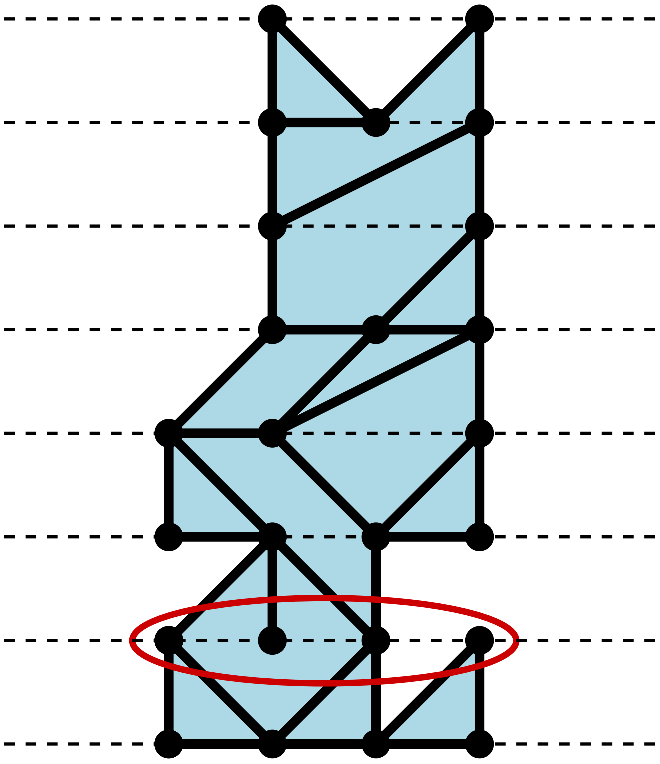

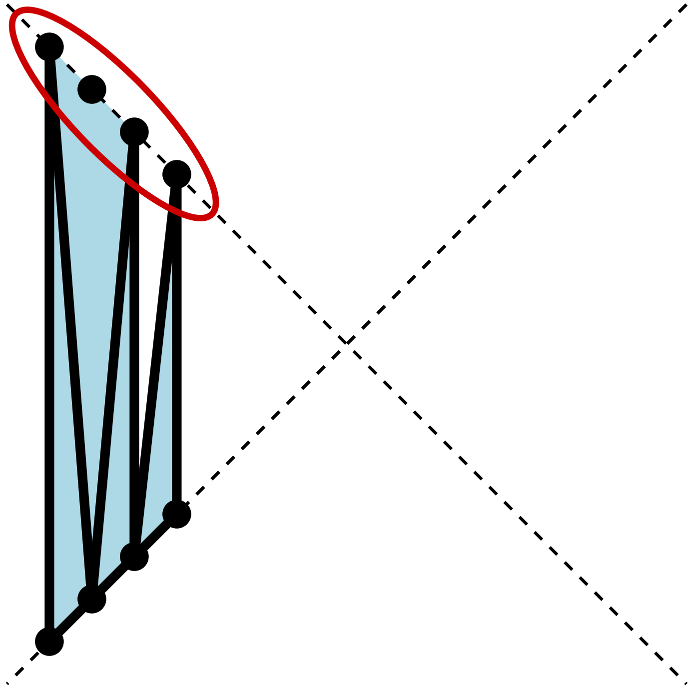

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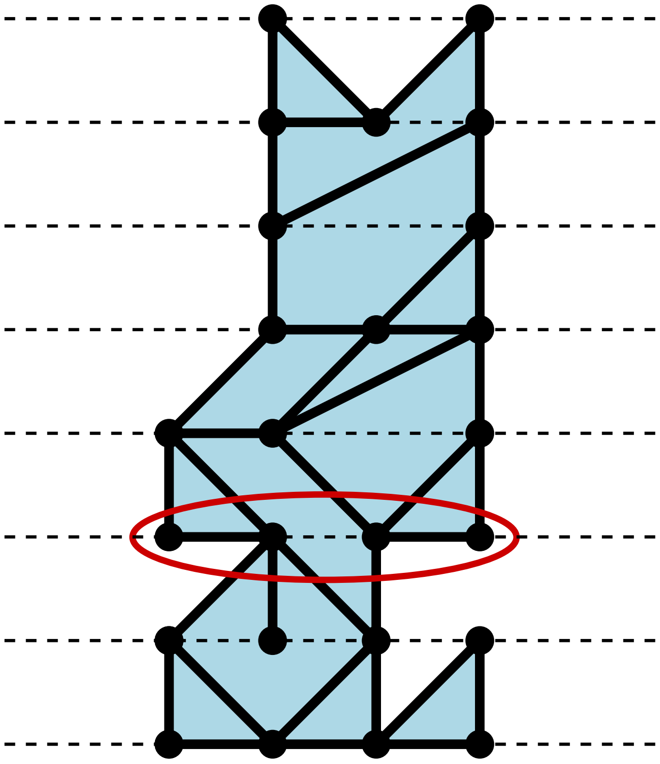

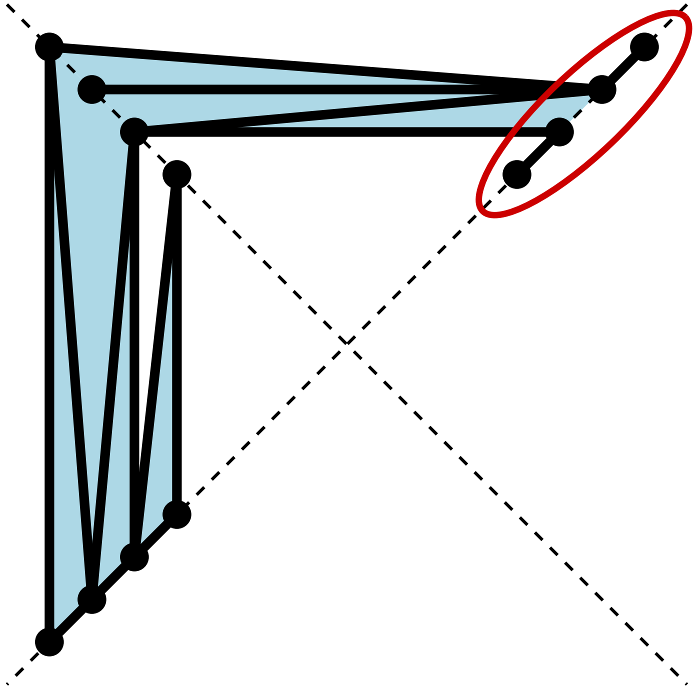

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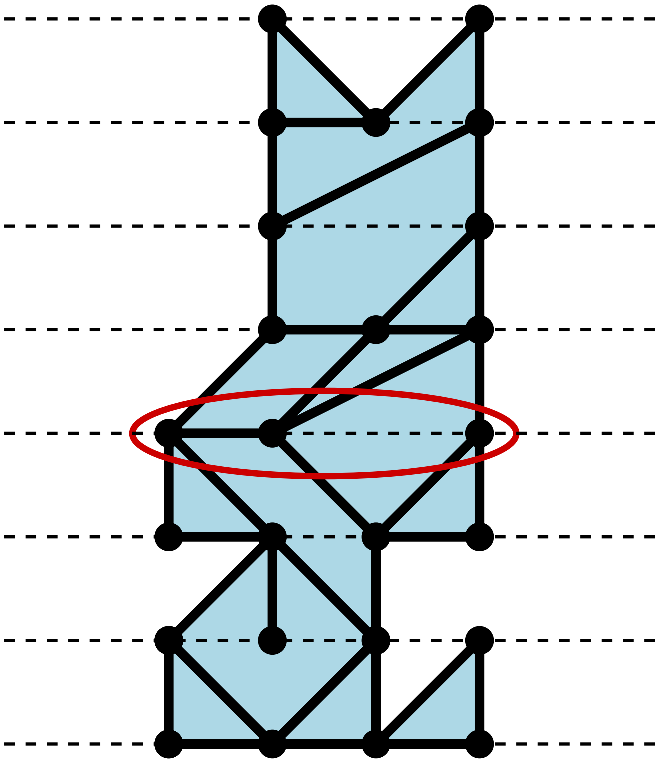

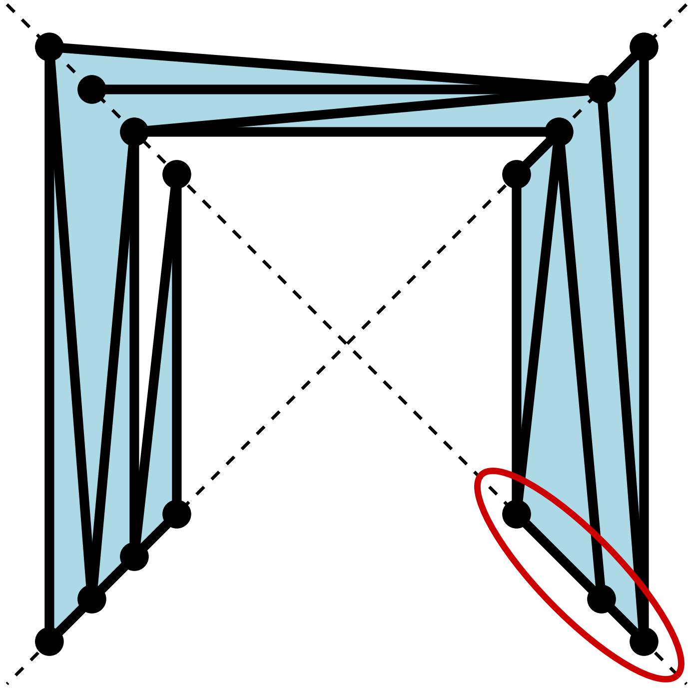

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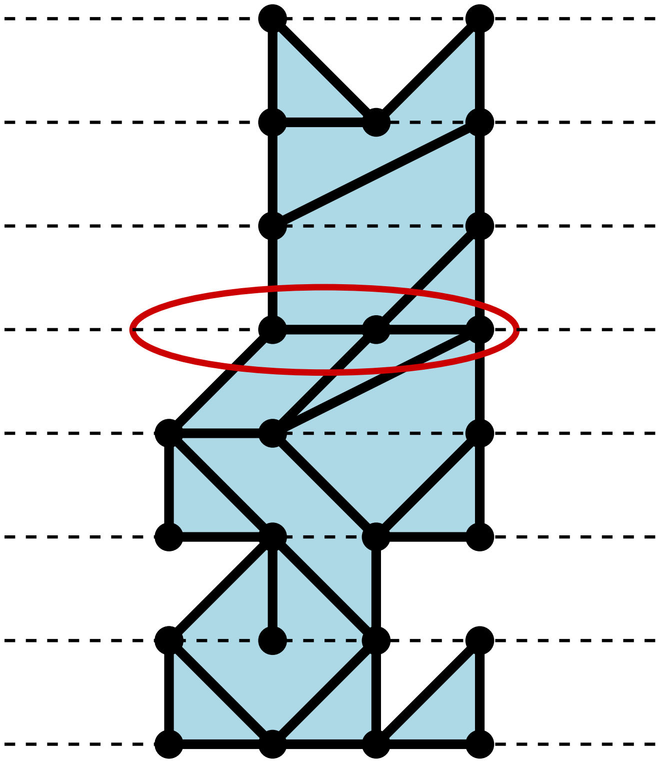

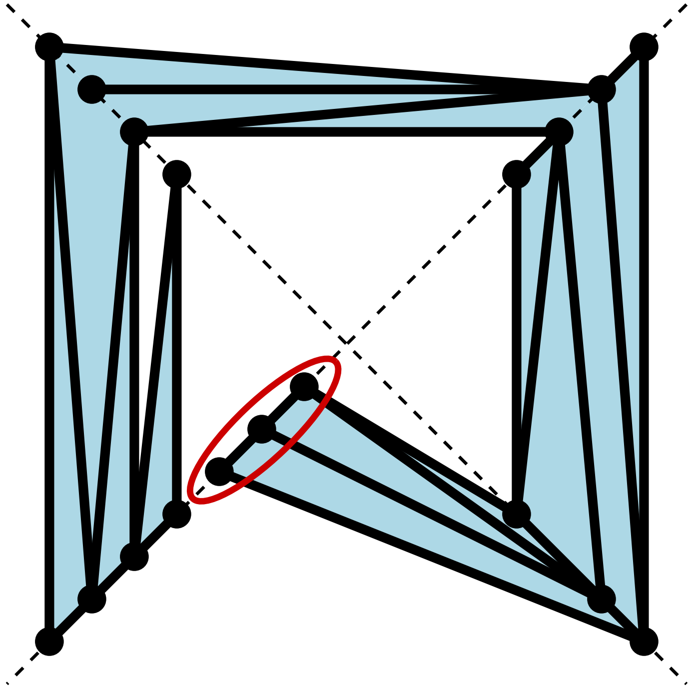

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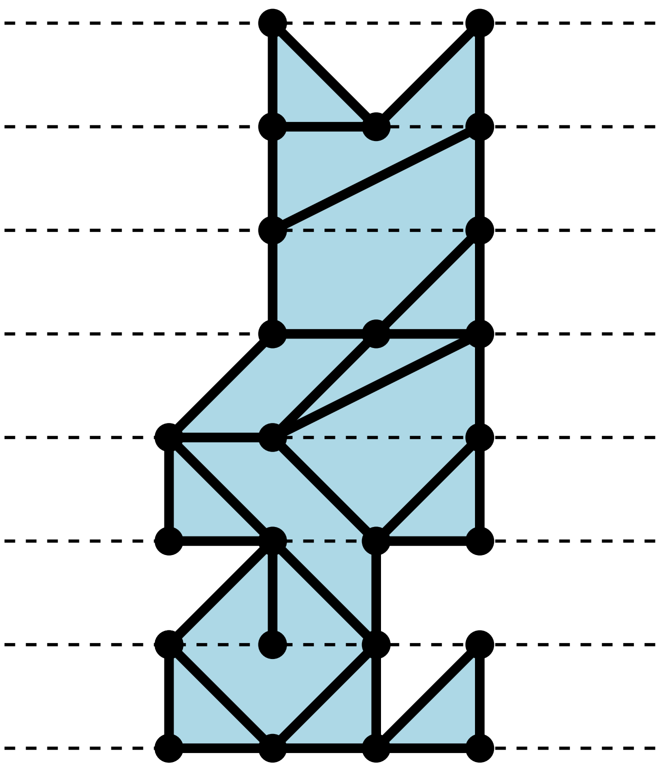

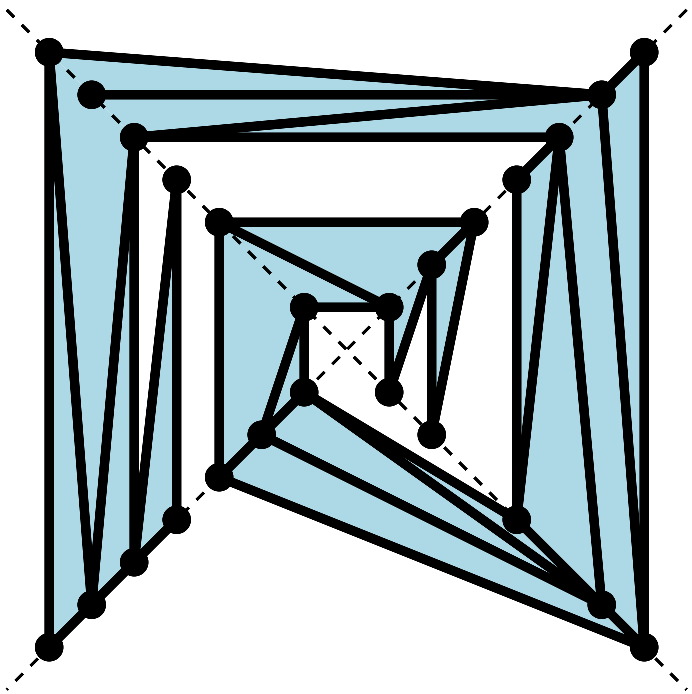

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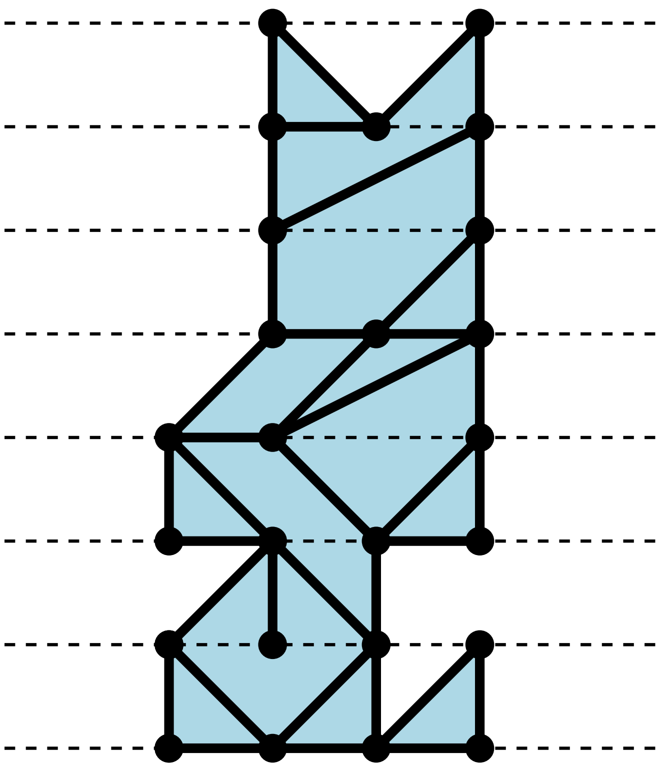

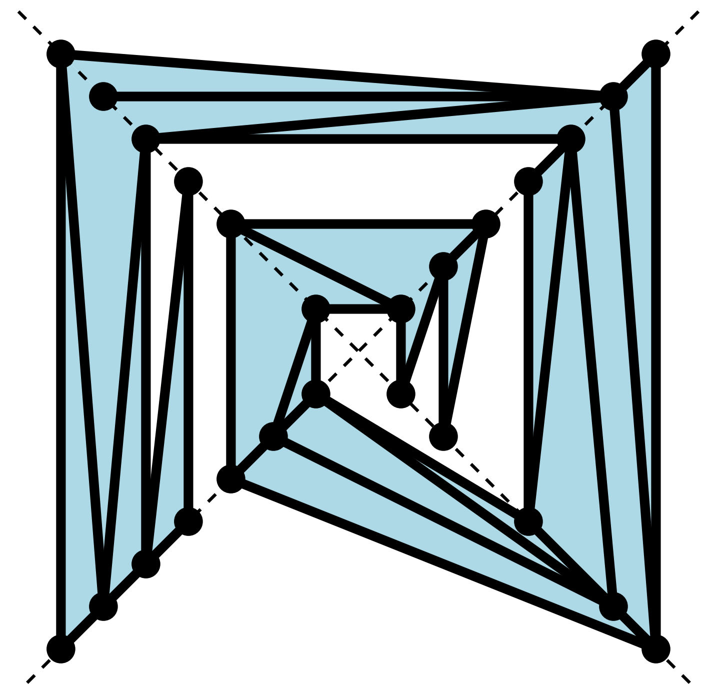

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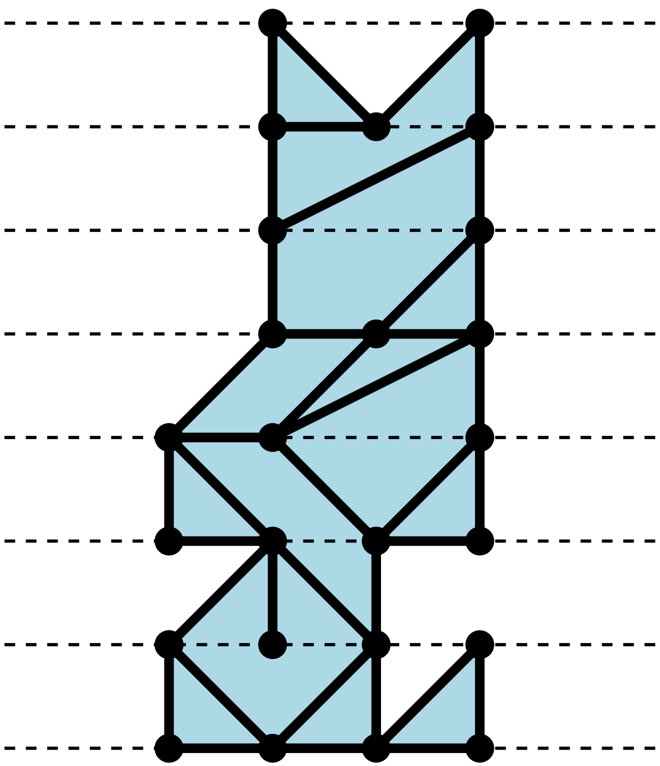

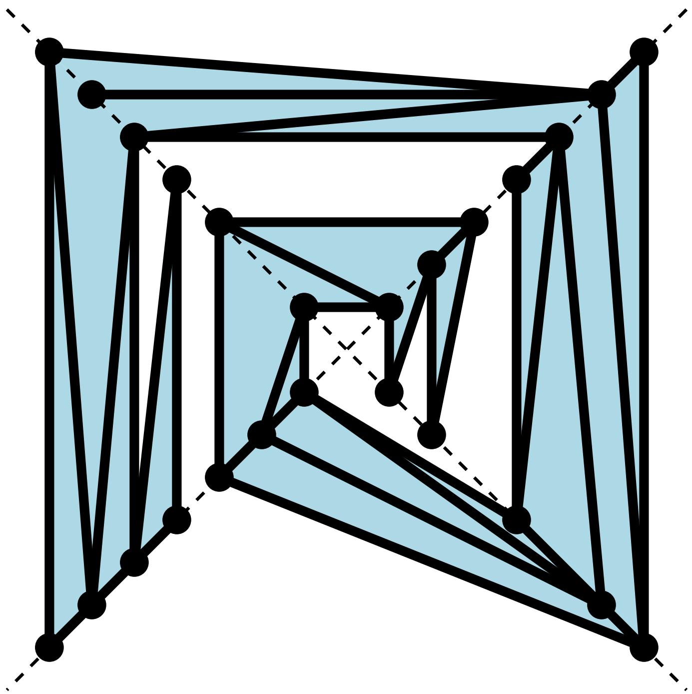

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On the other hand: There are infinitely many triangulations G with $\Delta(G) \leq 12$ and $\pi_2^1(G) \geq n^{0.01}$

Further Problems that We Considered

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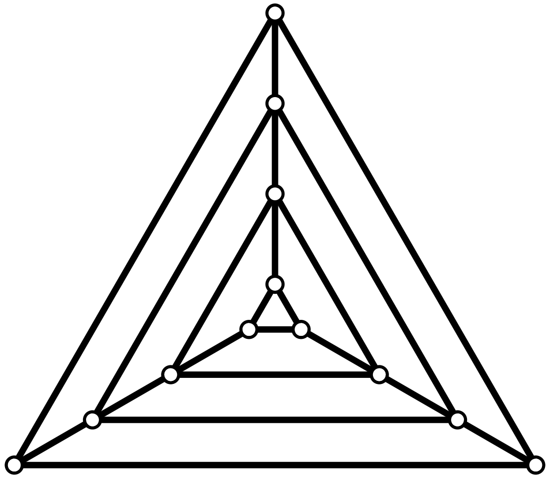
  with $\rho_2^1(G) = \Omega(n)$
  and $\rho_3^1(G) = O(n^{2/3})$



Further Problems that We Considered

- Variants with parallel lines: $\bar{\pi}_2^1$ and $\bar{\pi}_3^1$
- Topological variant—don't insist on straight lines:
  $\tau_3^2(G) = \text{thickness}(G)$ $= \text{min. \# planar graphs into which } E(G) \text{ can be partitioned.}$
- Separation between ρ_2^1 and ρ_3^1:
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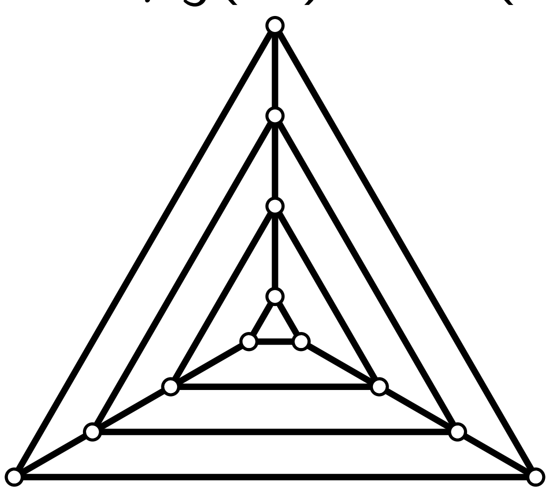

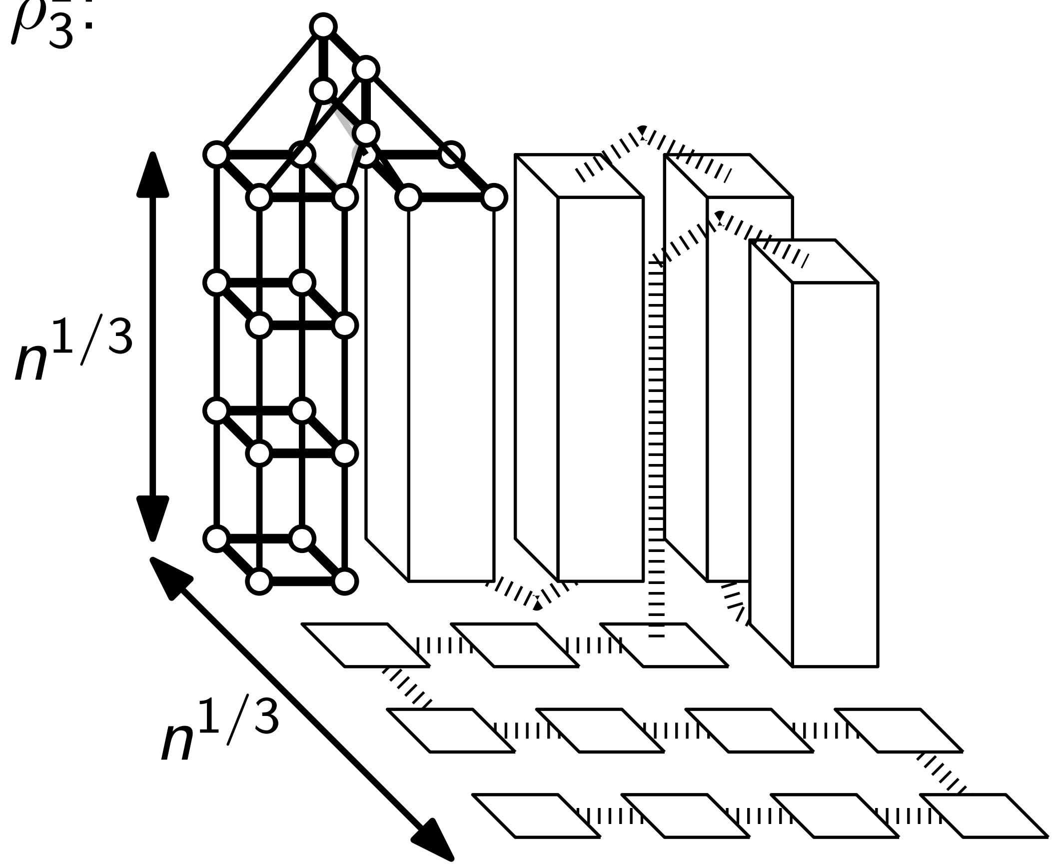

A Great Tool for Making Presentations

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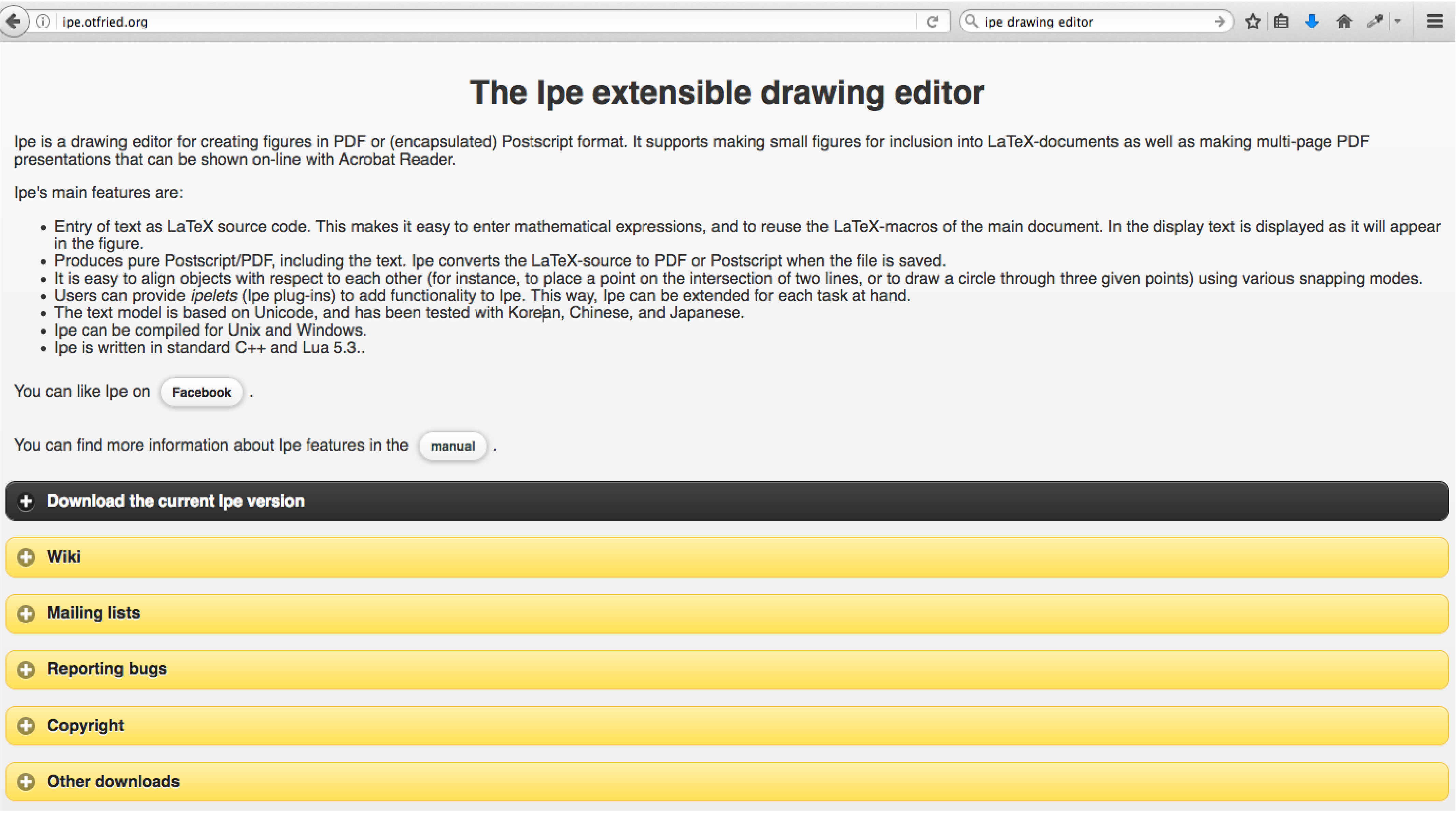

The Ipe extensible drawing editor

Ipe is a drawing editor for creating figures in PDF or (encapsulated) Postscript format. It supports making small figures for inclusion into LaTeX-documents as well as making multi-page PDF presentations that can be shown on-line with Acrobat Reader.

Ipe's main features are:

- Entry of text as LaTeX source code. This makes it easy to enter mathematical expressions, and to reuse the LaTeX-macros of the main document. In the display text is displayed as it will appear in the figure.
- Produces pure Postscript/PDF, including the text. Ipe converts the LaTeX-source to PDF or Postscript when the file is saved.
- It is easy to align objects with respect to each other (for instance, to place a point on the intersection of two lines, or to draw a circle through three given points) using various snapping modes.
- Users can provide *ipelets* (Ipe plug-ins) to add functionality to Ipe. This way, Ipe can be extended for each task at hand.
- The text model is based on Unicode, and has been tested with Korean, Chinese, and Japanese.
- Ipe can be compiled for Unix and Windows.
- Ipe is written in standard C++ and Lua 5.3..

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Open problems

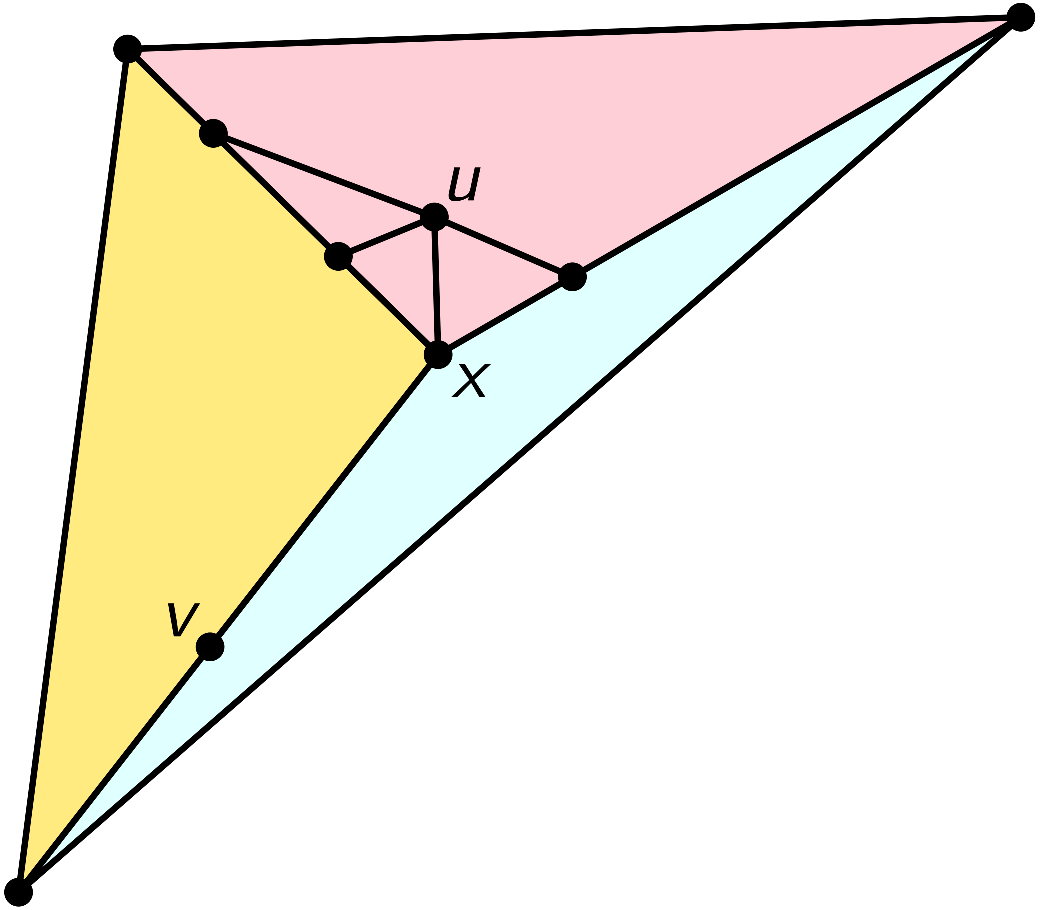
Near-planar graphs

Task: add one edge uv to a planar graph G and create a ρ_3^2-optimal drawing.

Near-planar graphs

Task: add one edge uv to a planar graph G and create a ρ_3^2-optimal drawing.

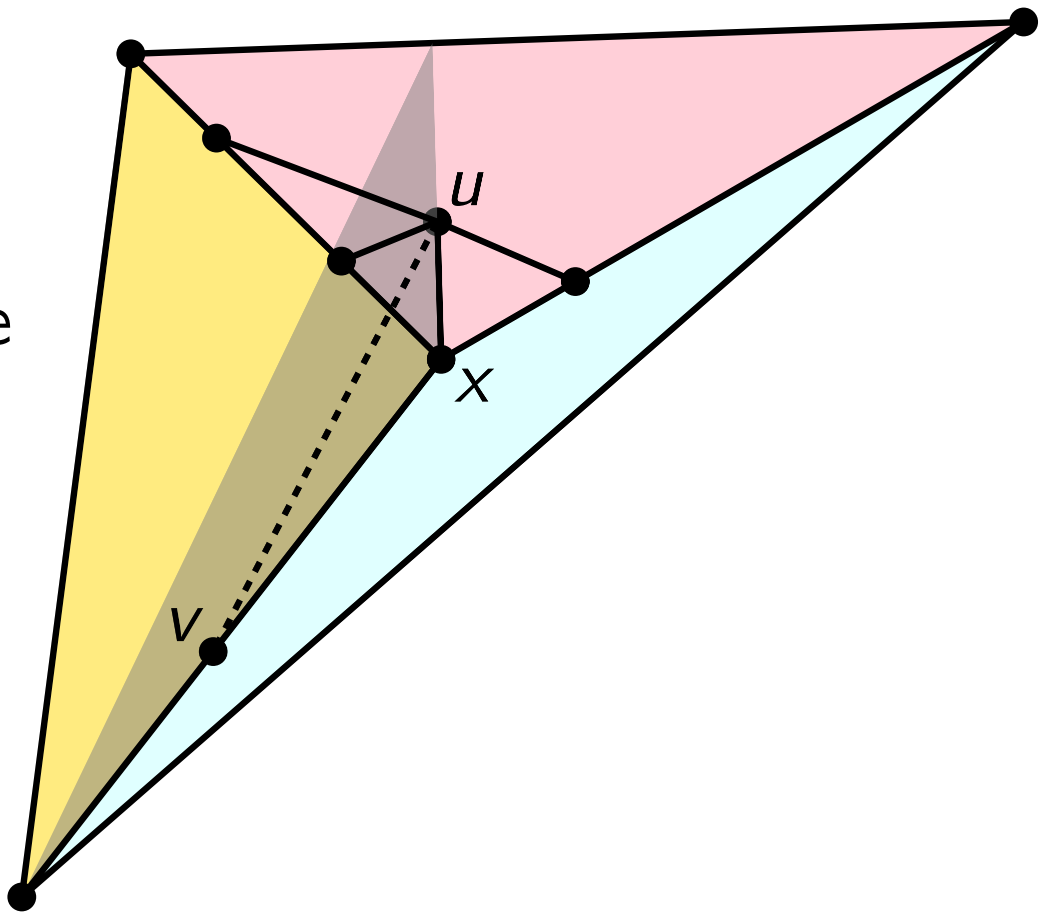
- Split the planar graph onto three planes



Near-planar graphs

Task: add one edge uv to a planar graph G and create a ρ_3^2-optimal drawing.

- Split the planar graph onto three planes
- Add uv on a fourth plane



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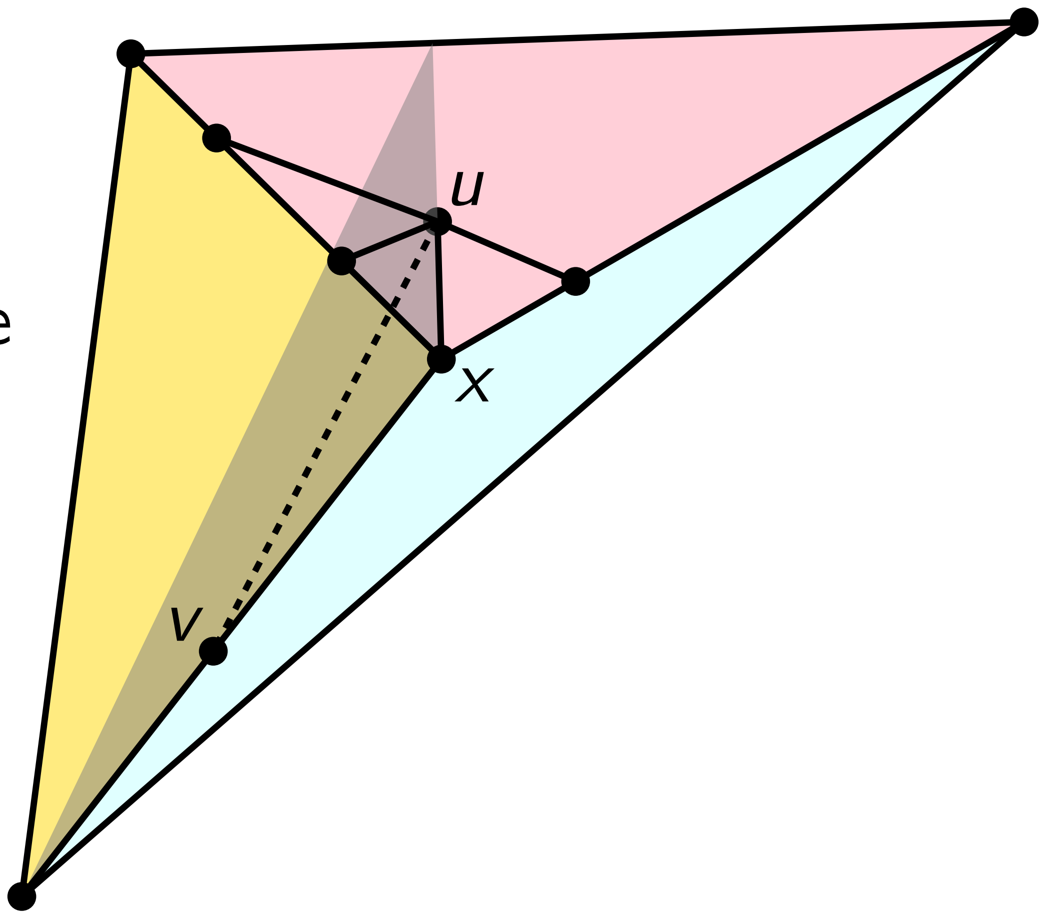
$\Rightarrow \rho_3^2(G + uv) \leq 4$



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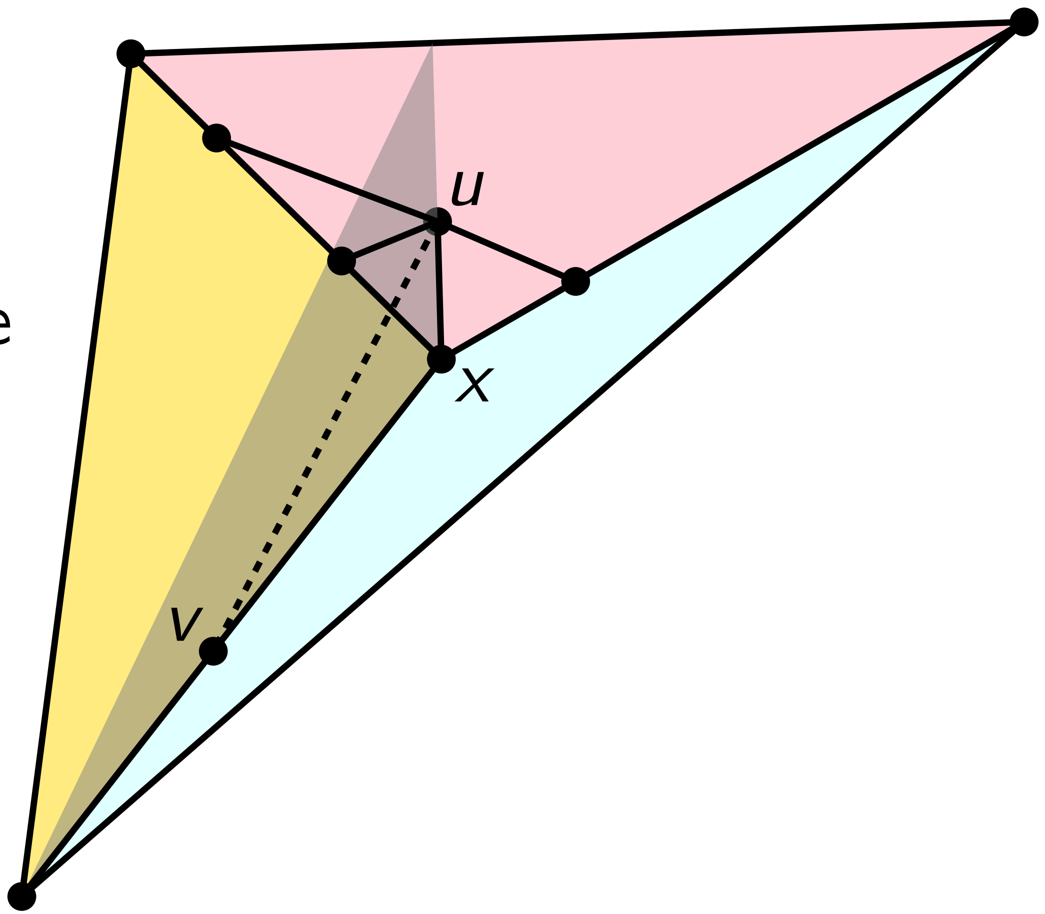

Is this bound worst-case optimal?

Open Problems

- We know $n^2/12 \lessapprox \rho_3^2(K_n) \lessapprox n^2/6.$

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  Determine exact values!

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$\rho_3^2(K_n) \geq$	1	3	4	6	6	7	**?**
$\rho_3^2(K_n) \leq$	1	3	4	6	**7**	**?**	**?**

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Open Problems

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$\rho^2_3(K_n) \geq$	1	3	4	6	6	7	**?**
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- Is $\rho^1_3(G) = \rho^1_2(G)$ for trees?

- Bound ρ^2_3 for 1-planar graphs or RAC graphs.