Variants of Oriented Coloring

Dhyanagamyananda
(Swathyprabhu mj)

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BITS - K K Birla Goa Campus, Goa

Ramakrishna Mission Vivekananda University
Belur campus (1 out of 5 units pan India)
Howrah, West Bengal

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Department of Computer Science

Exordium

"It is possible to go to a graph theory conference and to ask oneself, at the end of every talk, What is the oriented analogue? What is the right definition? Does the oriented version of the theorem still hold? If so, is there an easier proof and can a stronger conclusion be obtained? If the theorem fails, can one get a proof in the oriented world assuming a stronger hypothesis? We can personally attest that this can be an entertaining pastime. If at the end of this talk you too catch the oriented bug and begin to ask these questions" *then I deem this presentation fruitful*.

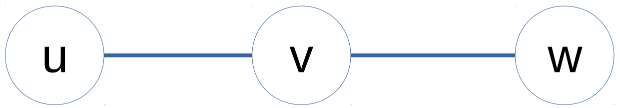
Adaptation from E.Scheinerman's introduction in Fractional Graph Theory

Outline of the Talk

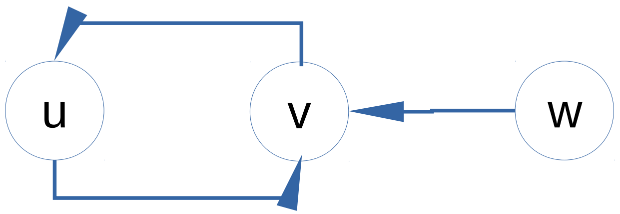
Topics	Transitions	Time(mins)
0. Graph Coloring problem	3	1
1. Oriented Vertex Coloring	18	8
2. Oriented Clique	7	1
3. Oriented Edge Coloring	12	7
4. Total Oriented Coloring	23	10
5. Oriented Fractional Coloring	19	8
6. Oriented Chromatic Polynomial		

0. Introduction

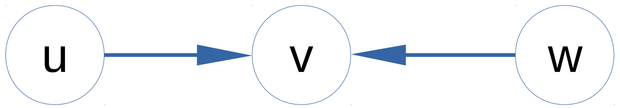
- Simple Graph



- Directed Graph



- Oriented Graph

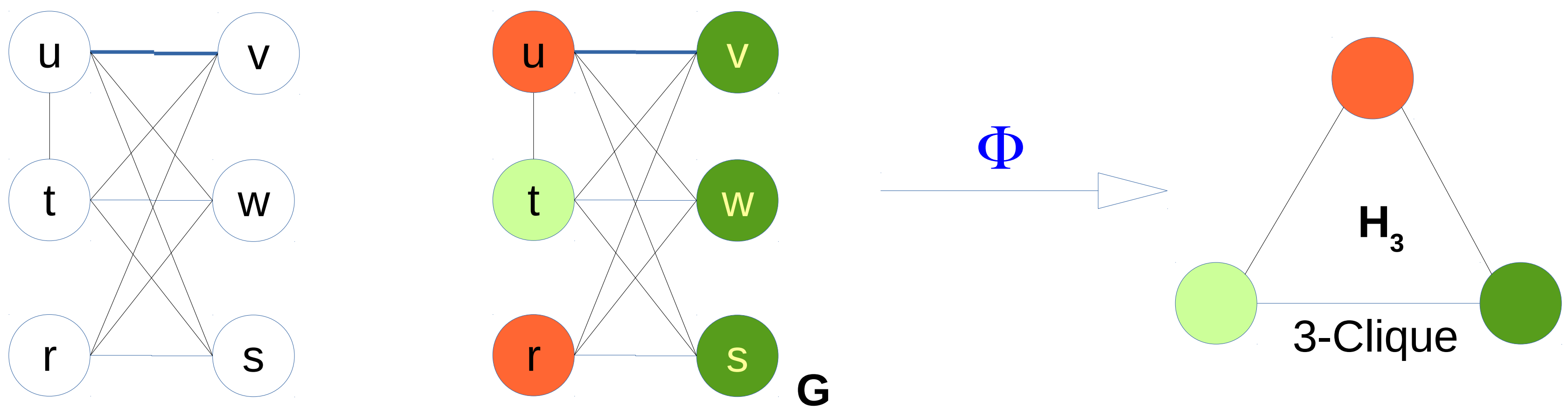


Edges in a simple graph are replaced by arcs

Meta Question: How do the properties of simple graph "get lifted" to oriented graphs?

For Instance,

- Graph Coloring



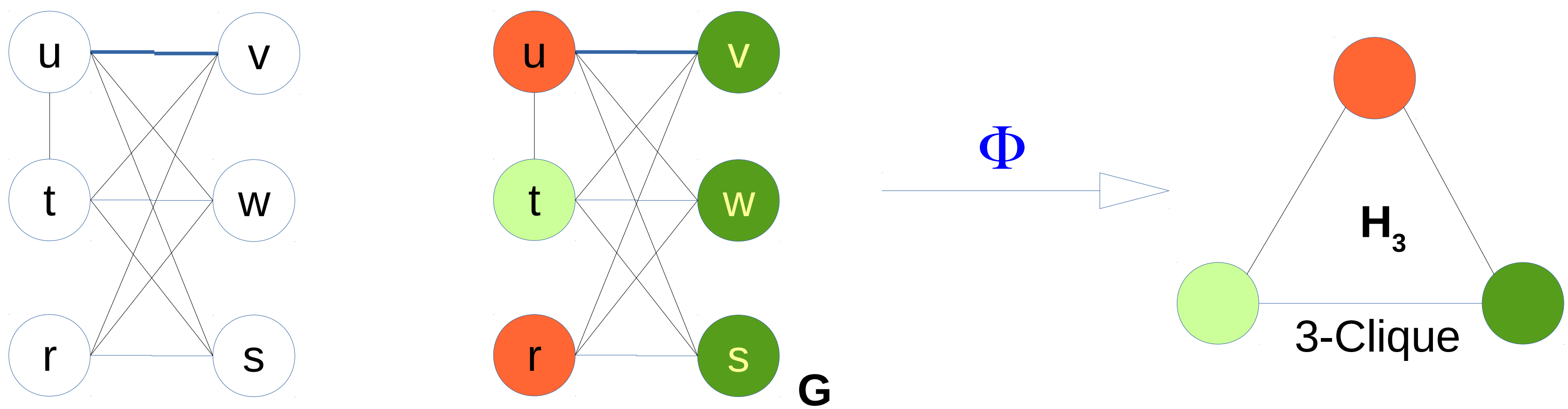
Φ(u) != Φ(v) whenever u and v are adjacent

Optimization parameter: Chromatic Number $\chi(G)$

Pavol Hell, Jaroslav Nešetřil: Graphs and Homomorphisms

For Instance,

- Graph Coloring



$\Phi(u) \mathrel{!=} \Phi(v)$ whenever u and v are adjacent

Φ is an adjacency preserving mapping,
also called a homomorphism from G to H_k

Pavol Hell, Jaroslav Nešetřil:
Graphs and Homomorphisms

1: Oriented Coloring

- An oriented coloring of oriented graph G is a homomorphic mapping

$$\phi : G \to H_k$$

i) $\phi(u) \neq \phi(v)$ when u and v are adjacent

ii) for two arcs $u \to v, x \to y$ of G,

$$\phi(u) = \phi(y) \implies \phi(v) \neq \phi(x)$$

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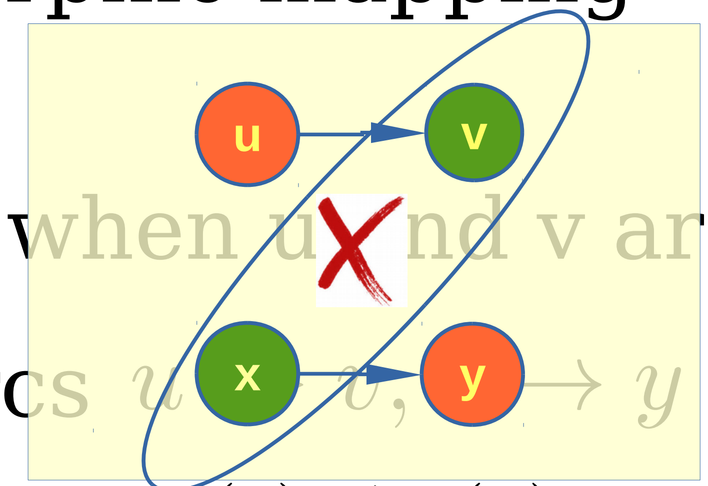

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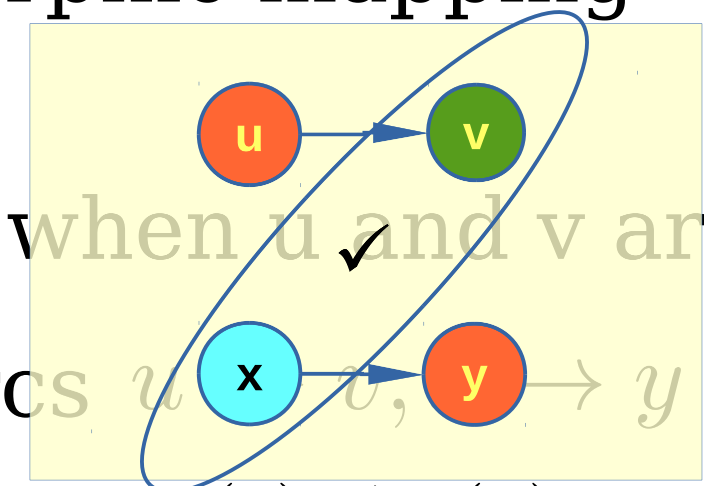

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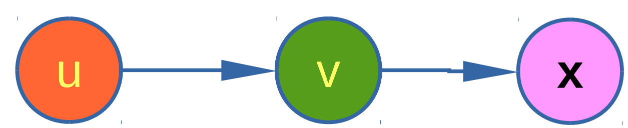
- Immediate consequence: 2-dipath needs three colors



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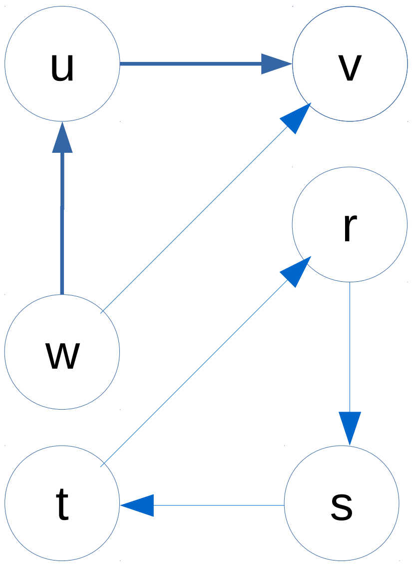
(Clique vs) 2: Oclique

- Oriented Graph Coloring



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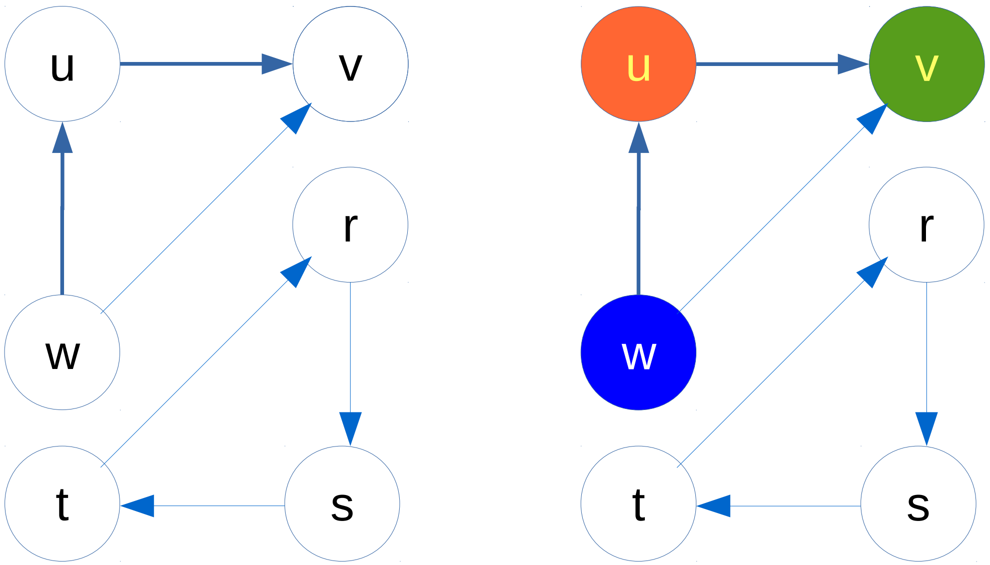

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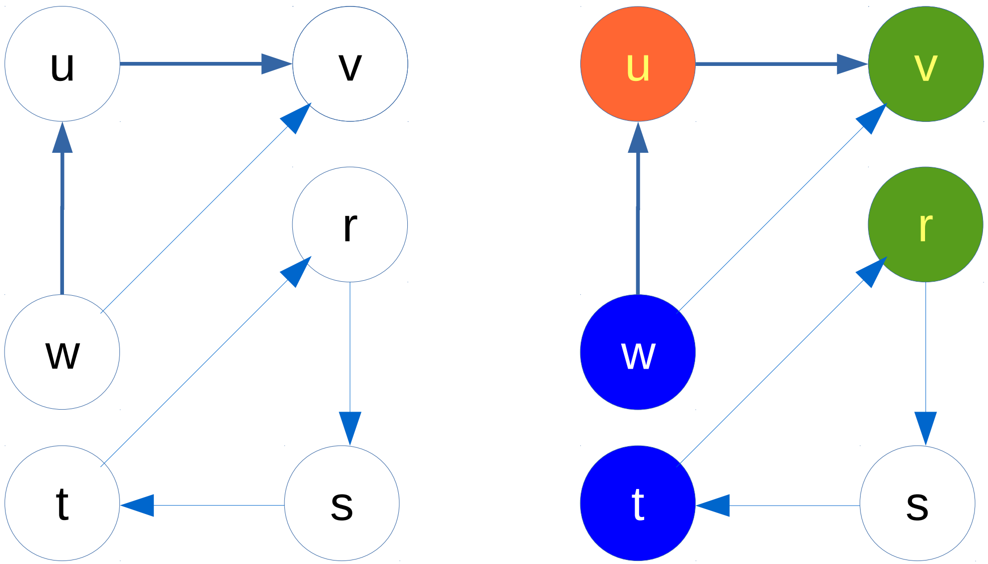

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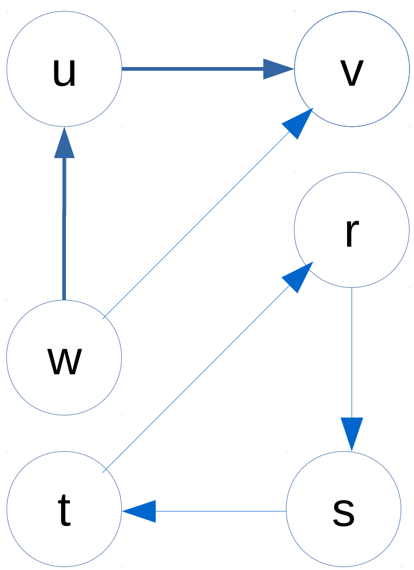

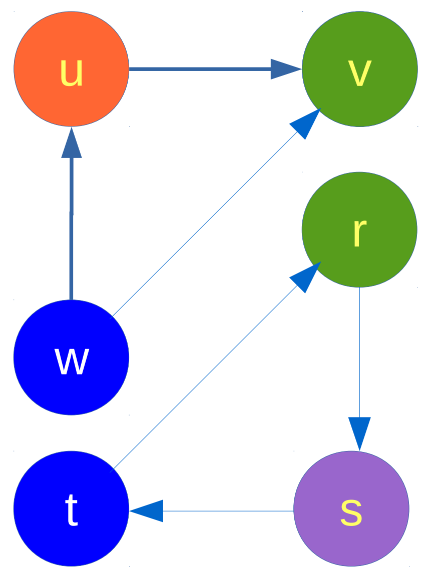

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(Clique vs) 2: Oclique

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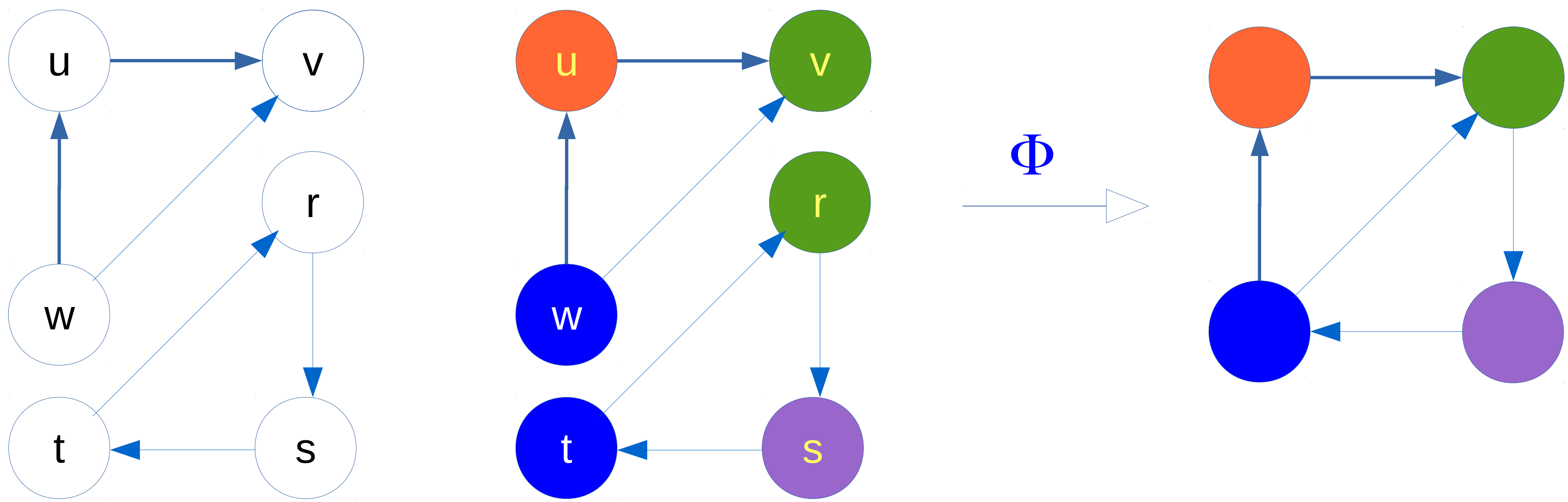
Oriented Absolute Clique
or Oclique of order 4



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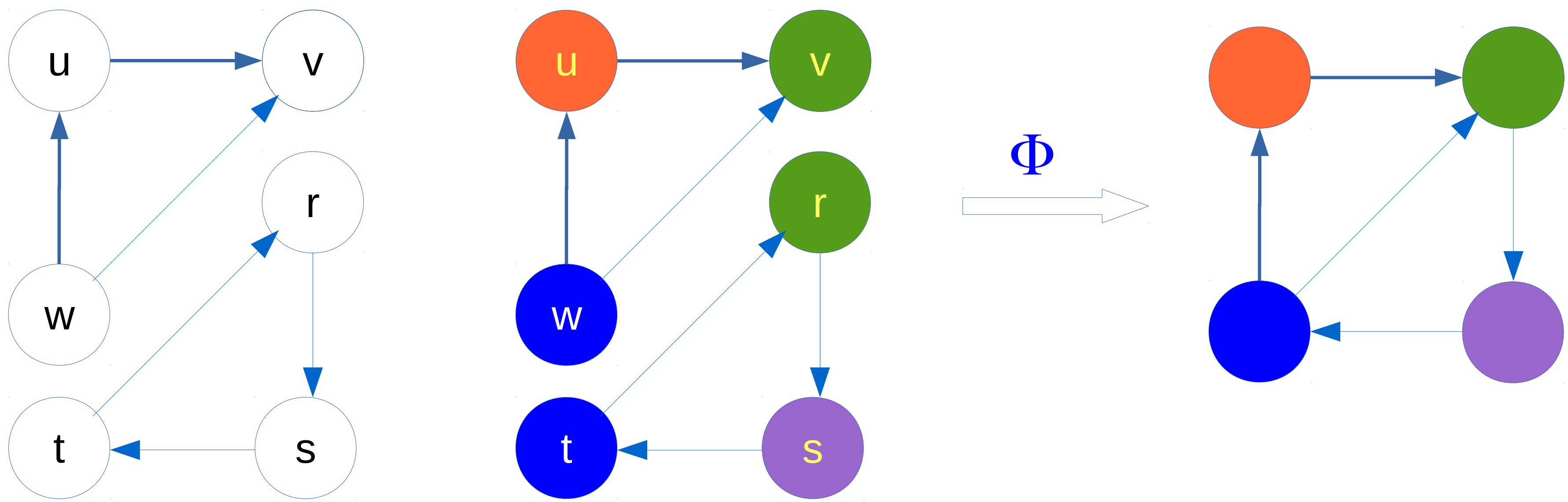

- Ocliques are oriented graphs having $\chi_o(G) = |V(G)|$

Note: Non adjacent nodes in an Oclique are in 2-dipath

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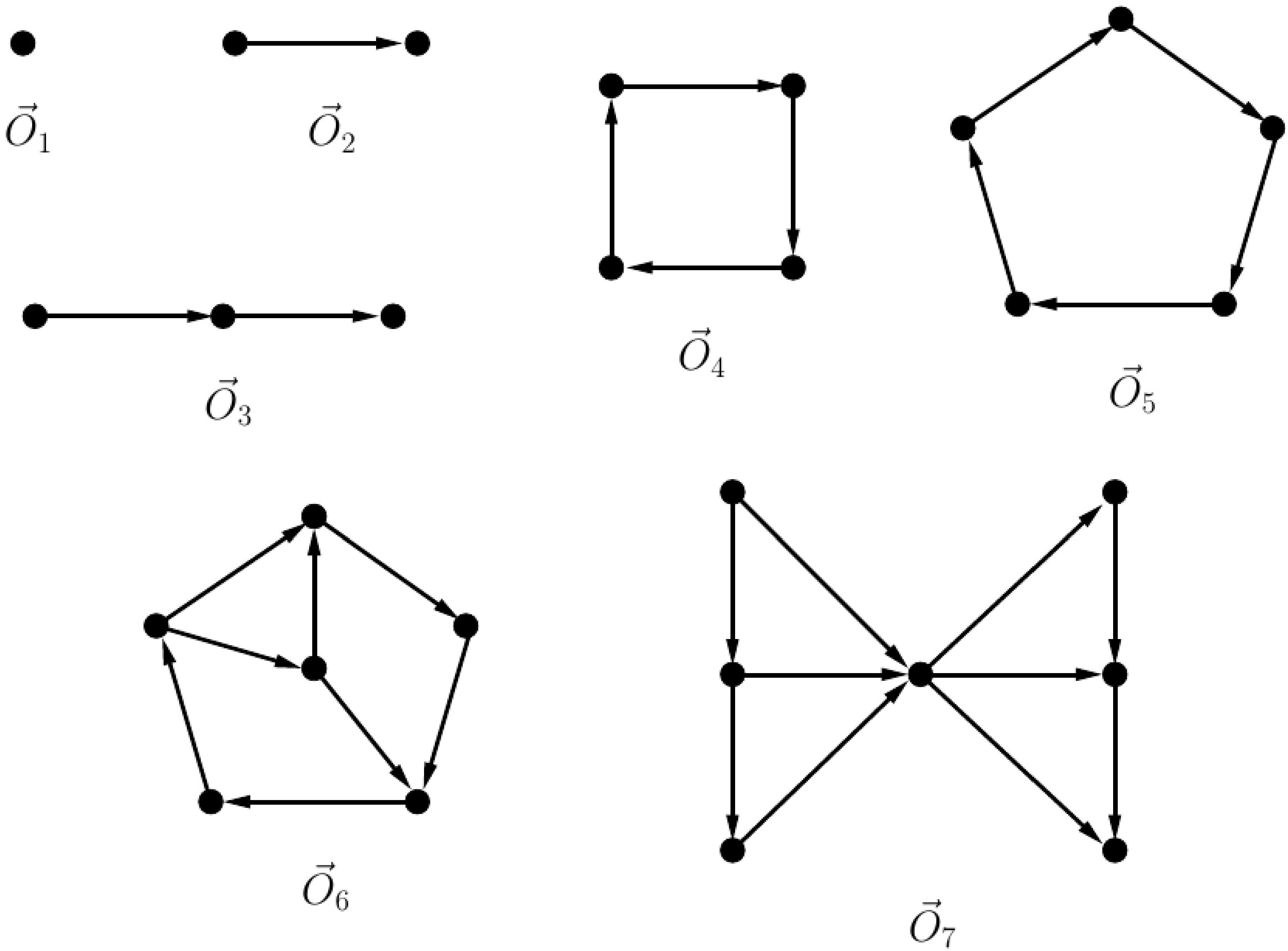

Sample o-cliques

Eg: Oriented Coloring- Outerplanar graphs – *Proof design*

- Fact 1 – $\mathcal{O}(G)$ has a degree 2-vertex, say *v*

Éric Sopena: 1997

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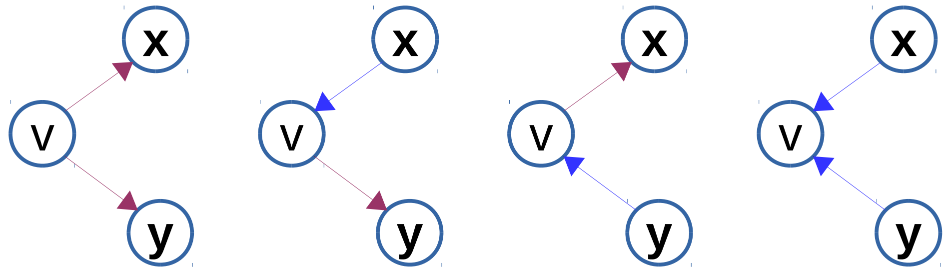

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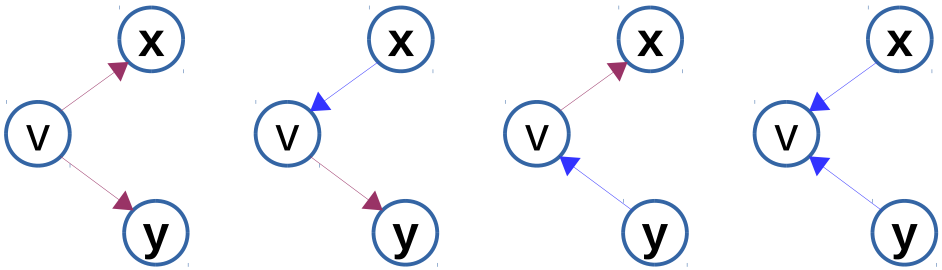

- Task - Find a target graph that mimics *x,y* pair.

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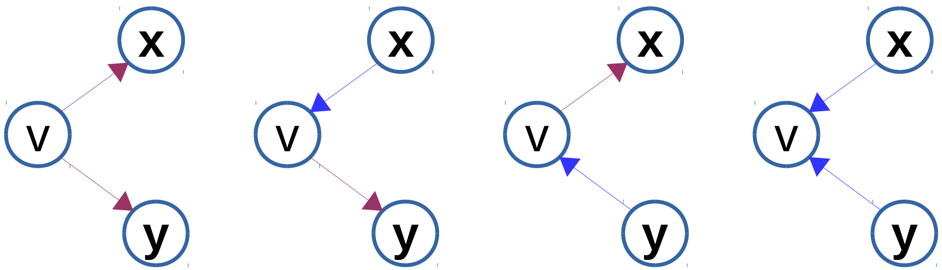

- Task - Find a target graph that mimics *x,y* pair.
- Fact 3 – Paley tournament QR_7 is the target

Éric Sopena: 1997

Eg: Oriented Coloring- Outerplanar graphs – *Proof design*

QR_n: *Tournament based on Quadratic Residues*

$$n \in \texttt{Prime}^k$$

$$n = 3 \bmod 4$$

$$S_n = \texttt{Non-zero squares of } [n]$$

$$S_7 = \{1, 2, 4\}$$

$$b - a \in S_n \implies \overrightarrow{ab} \in QR_n$$

Properties of the tournament

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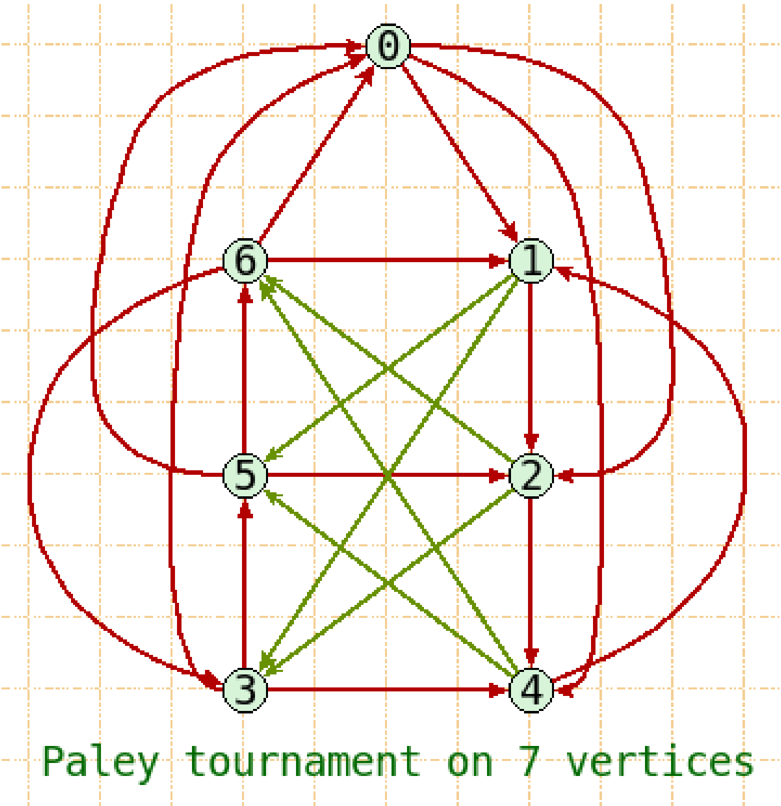

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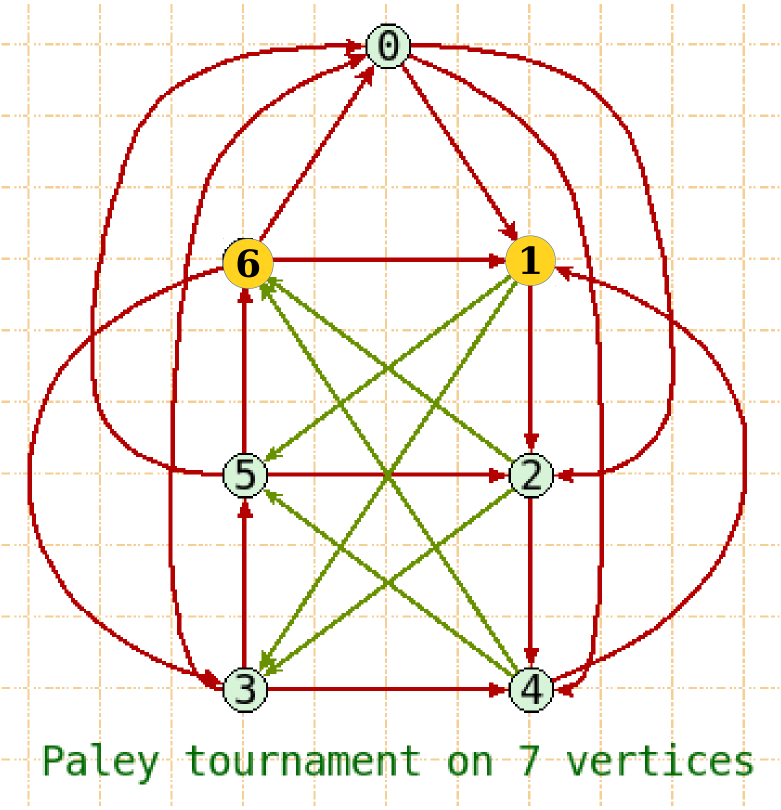

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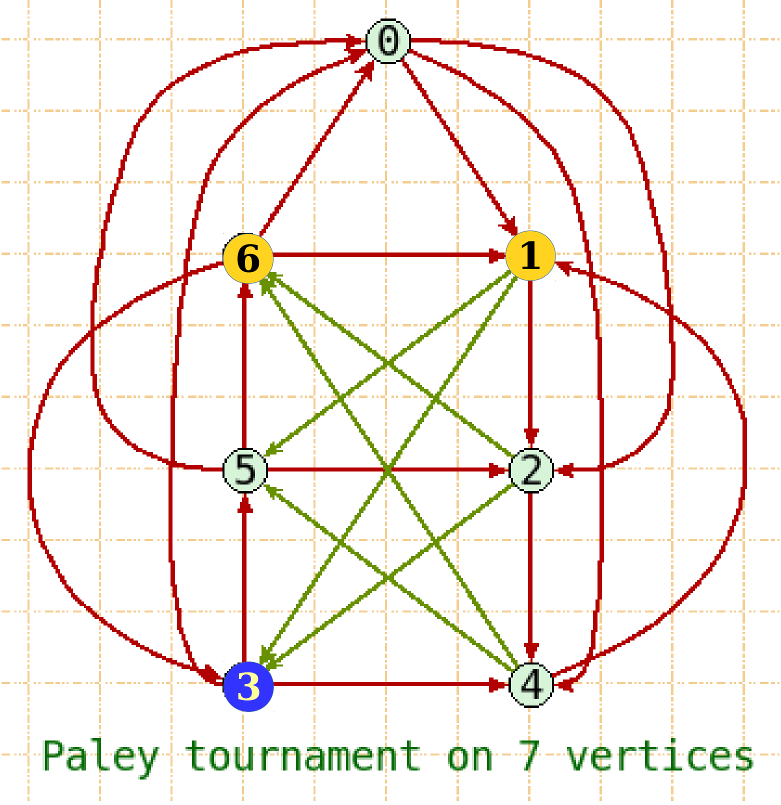

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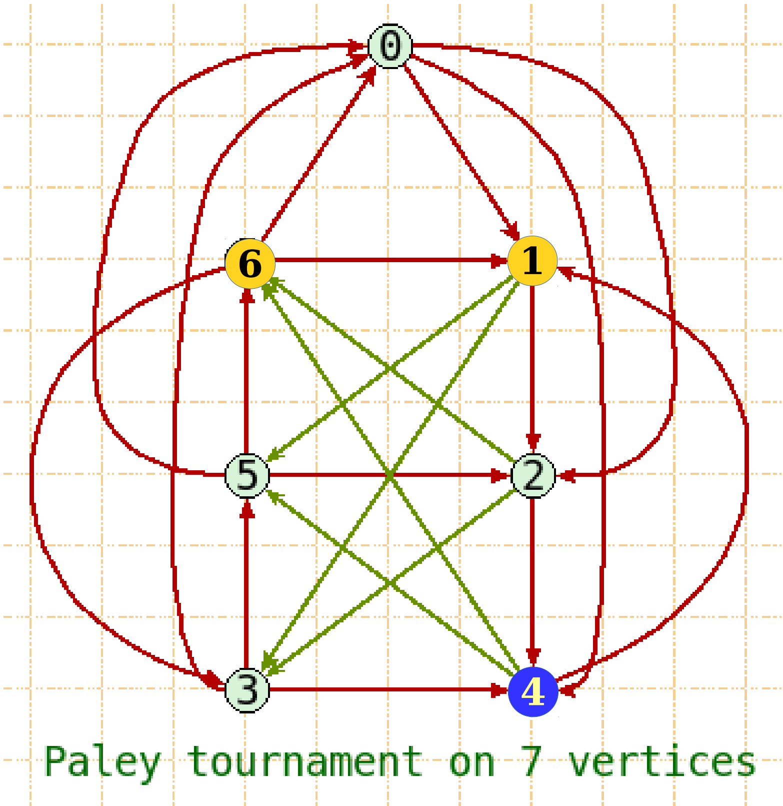

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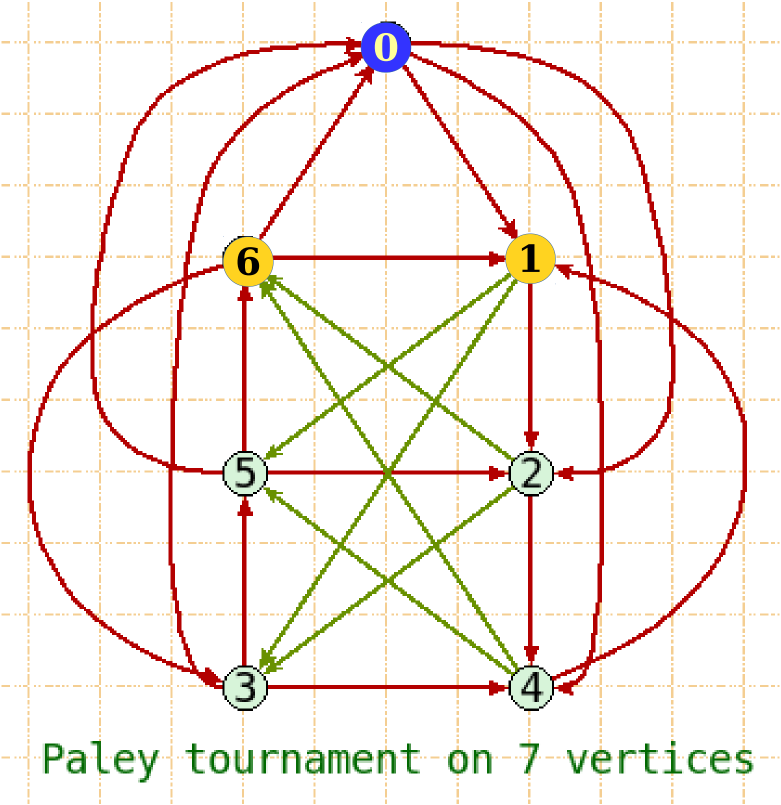

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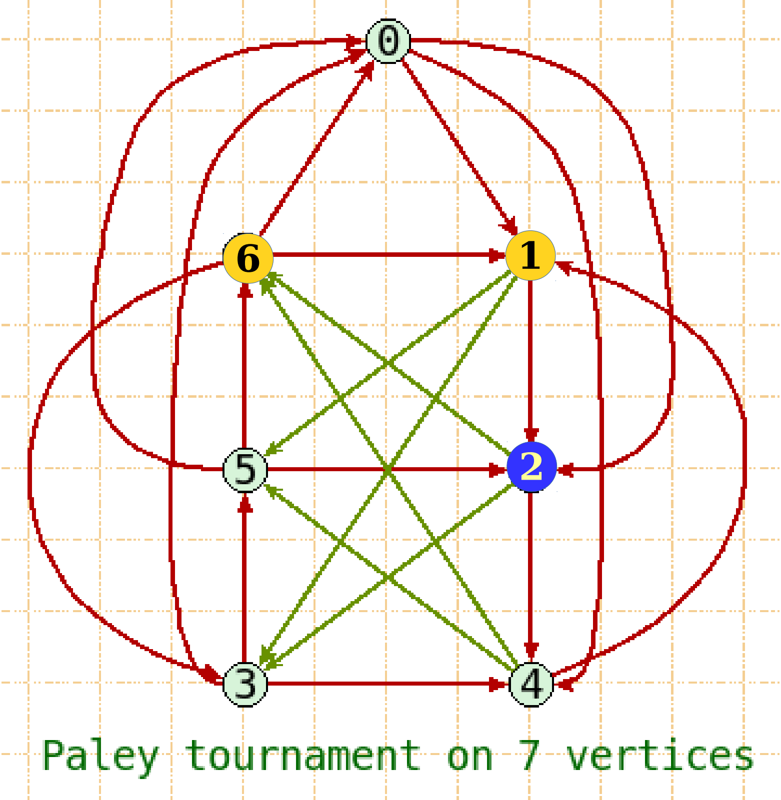

Properties of the tournament

Éric Sopena: 1997

1: Oriented Coloring- Basic Results

- vs chromatic number of graph G

$$\chi(G) \leq \chi_o(G)$$

André Raspaud, Éric Sopena: 1994

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- Therefore for the class of planar graphs

$$\chi_a(G) = 5, \quad \chi_o(G) \leq 80$$

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Best known bound till date

André Raspaud, Éric Sopena: 1994

3: Oriented Arc Coloring

- An oriented arc coloring of graph G is a mapping Φ from A(G) to A(H_k) such that,

i) $\phi(e_1) \neq \phi(e_2)$

whenever e_1, e_2 are in 2-dipath

ii) for 4 arcs e1 preceding e2,

a1 preceding a2,

$$\phi(e_1) = \phi(a_2) \implies \phi(e_2) \neq \phi(a_1)$$

Pascal Ochem, Alexandre Pinlou, Éric Sopena: 2007

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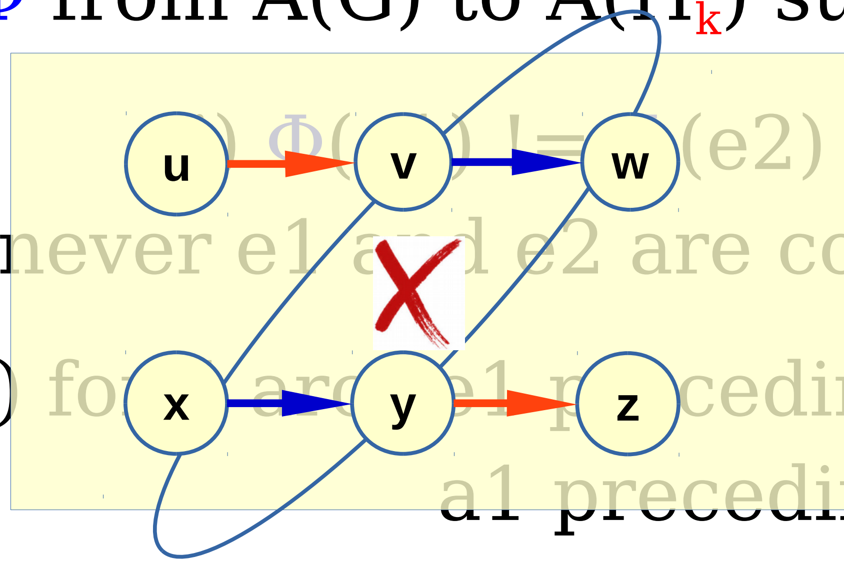

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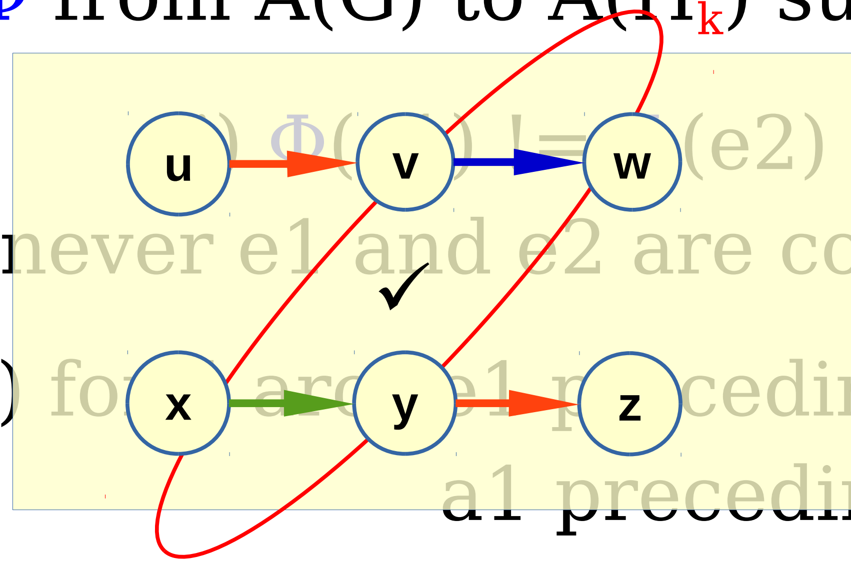

ii) for arc e1 preceding e2,

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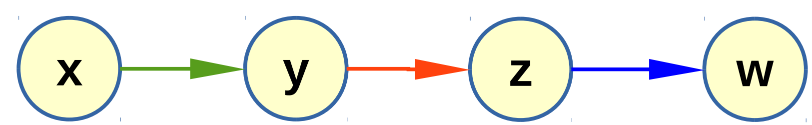
Φ(e1)=Φ(a2) ---> Φ(a1) != Φ(e2)

- Immediate consequence: P_4 needs three colors



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3: Oriented Arc Coloring- A Restatement

- Observation:

Oriented arc Coloring of G → Oriented vertex Coloring of LD(G)

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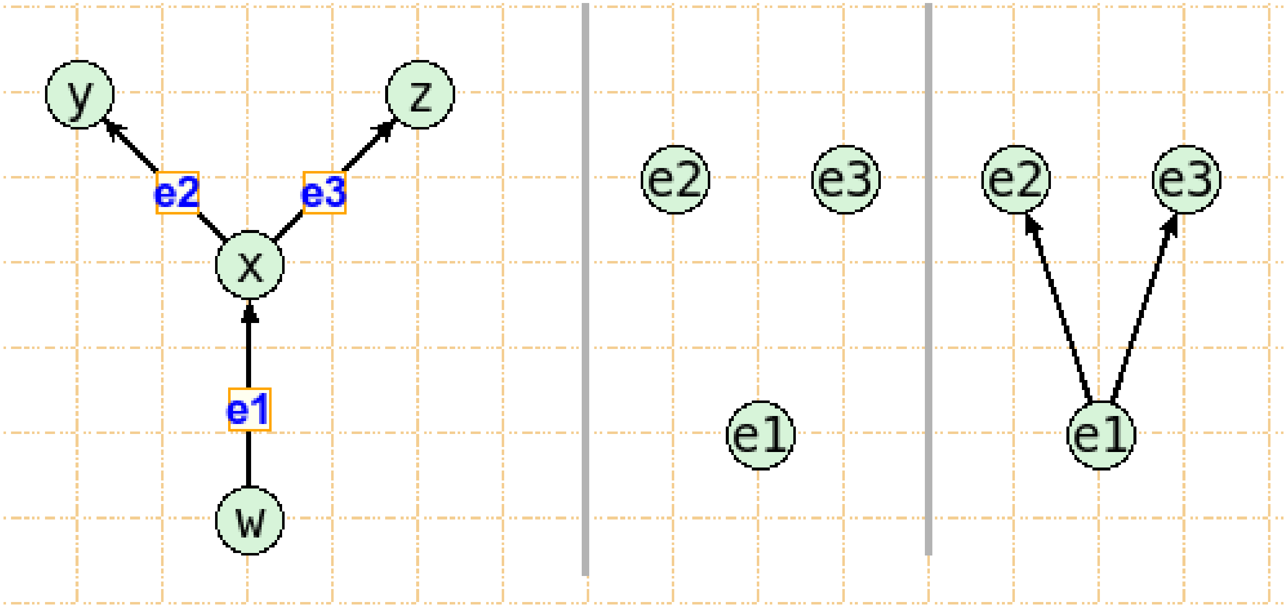

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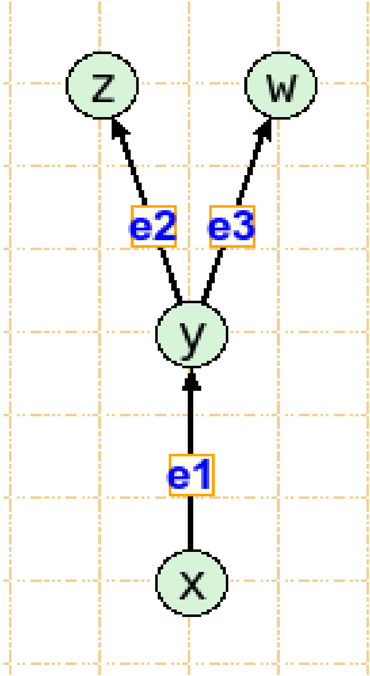
3: Oriented Arc Coloring- A Restatement

- Example

G



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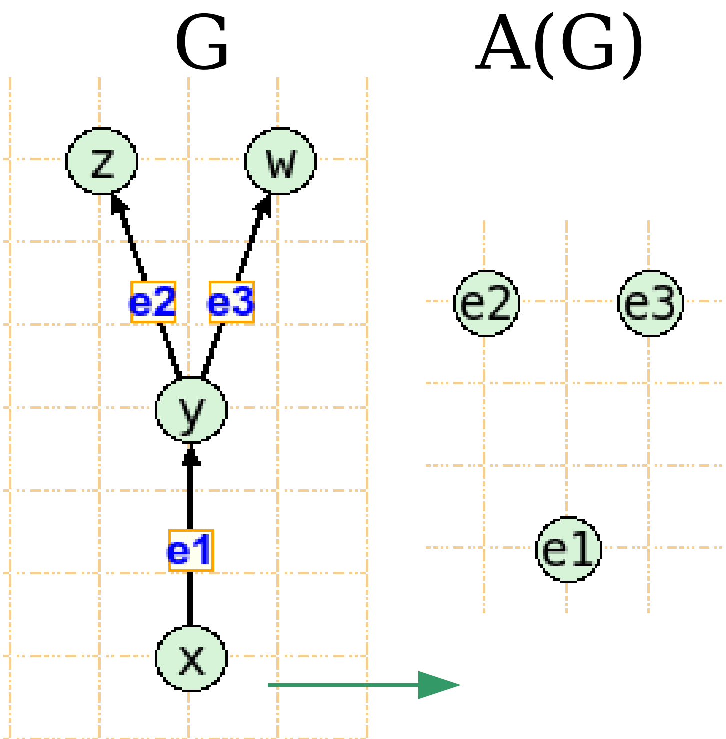

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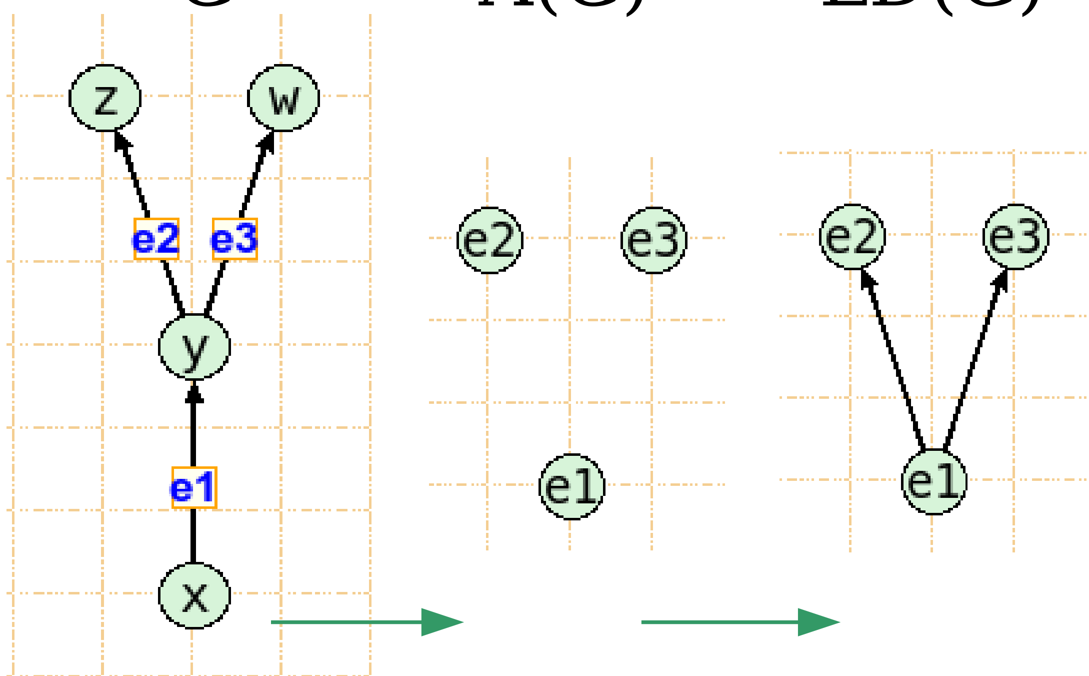
3: Oriented Arc Coloring- A Restatement

- Example

G A(G) LD(G)



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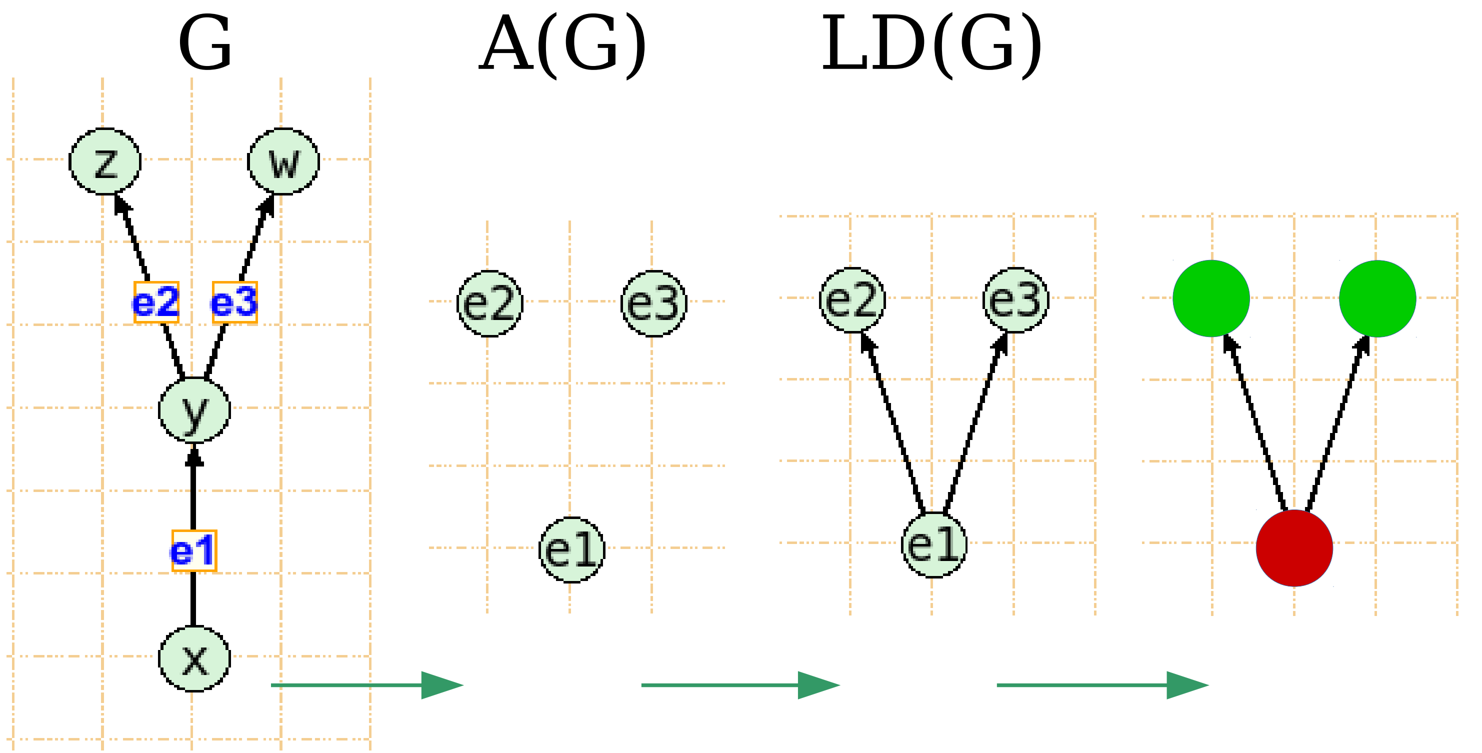

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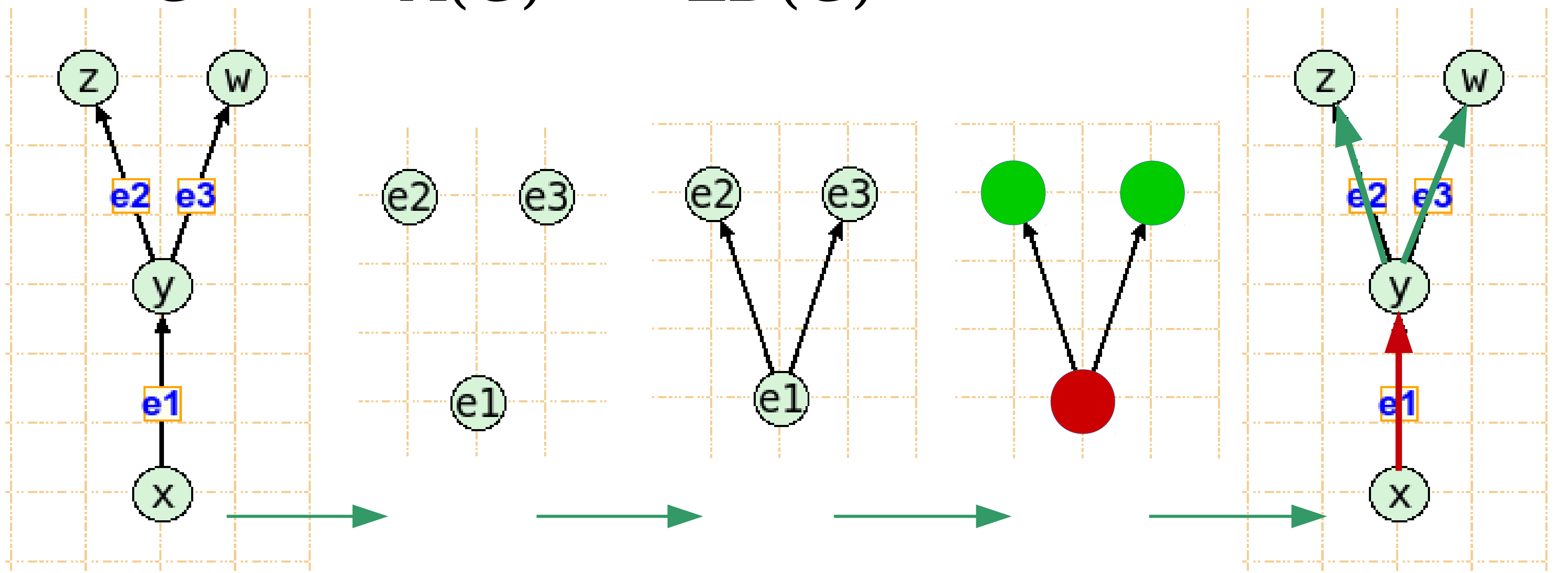

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3: Oriented Arc Coloring- Basic Results

- vs Oriented chromatic number for graph G

$$\chi'_o(G) \leq \chi_o(G)$$

- Also, for a graph G such that

$$\chi'_o(G) = k, \quad \chi_o(G) \leq f(k)$$

- vs Acyclic chromatic number k

$$2k(k-1) + \lfloor \frac{k}{2} \rfloor$$

Pascal Ochem, Alexandre Pinlou, Éric Sopena: 2007

Sample Problem Instances

- Various graph classes

  Outerplanar graphs

  Bipartite graph

  Series-Parallel graphs

  Cubic graphs

  Triangle free graphs

  Partial 2-trees

  Grids

  Sparse plane graphs

  Halin graphs

  Dense graphs

- Graph parameters

  Bounded degree

  Large girth

  Maximum avg degree

- Hardness Result
- Parameterized complexity

4: Total Oriented Coloring

- Searching for a definition
  - Input oriented graph G(V,A).
  - Color vertices + arcs
    1) coloring restricted to vertices is OVC
    2) coloring restricted to arcs is OAC
    3)

4: Total Oriented Coloring

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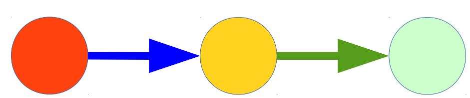
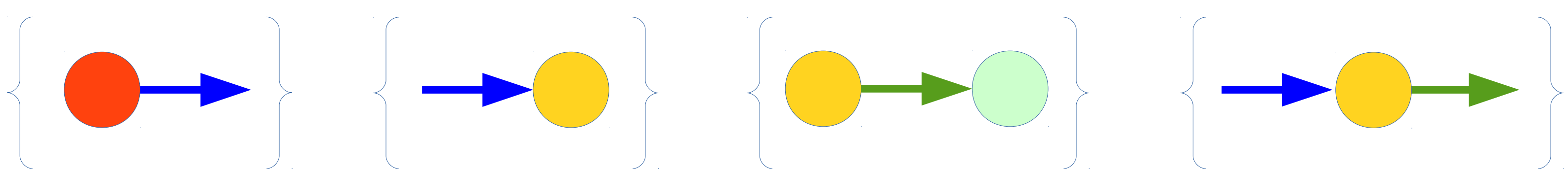
    3) 



V → A **A → V** **V → V** **A → A**

4 different proximity relations

4: Total Oriented Coloring

- Searching for a definition

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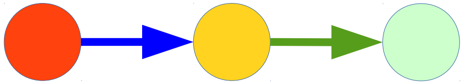

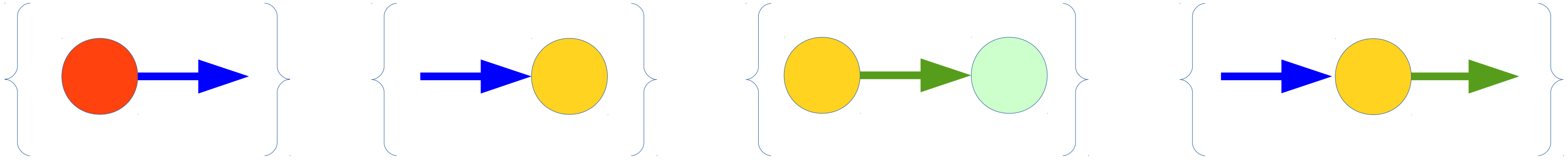

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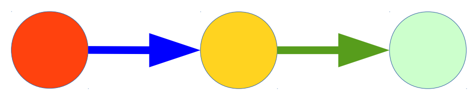

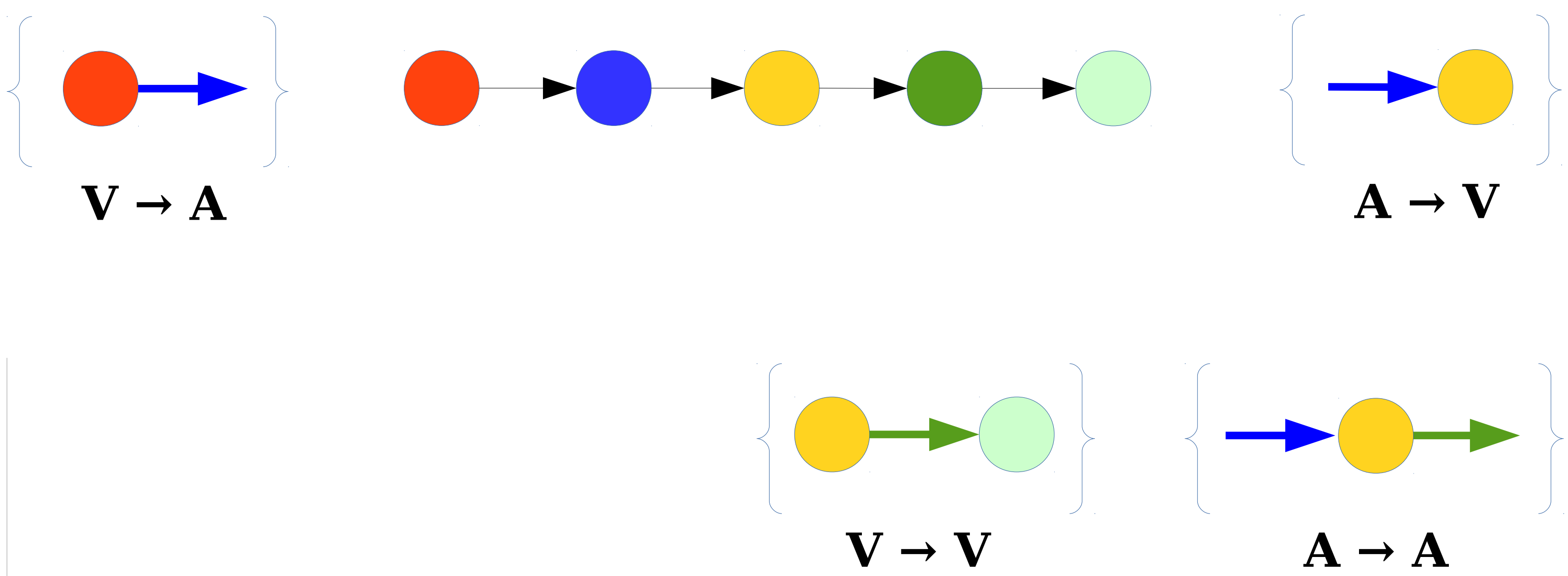

4 different proximity relations

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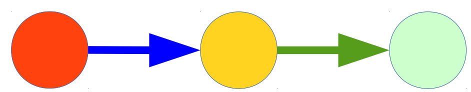
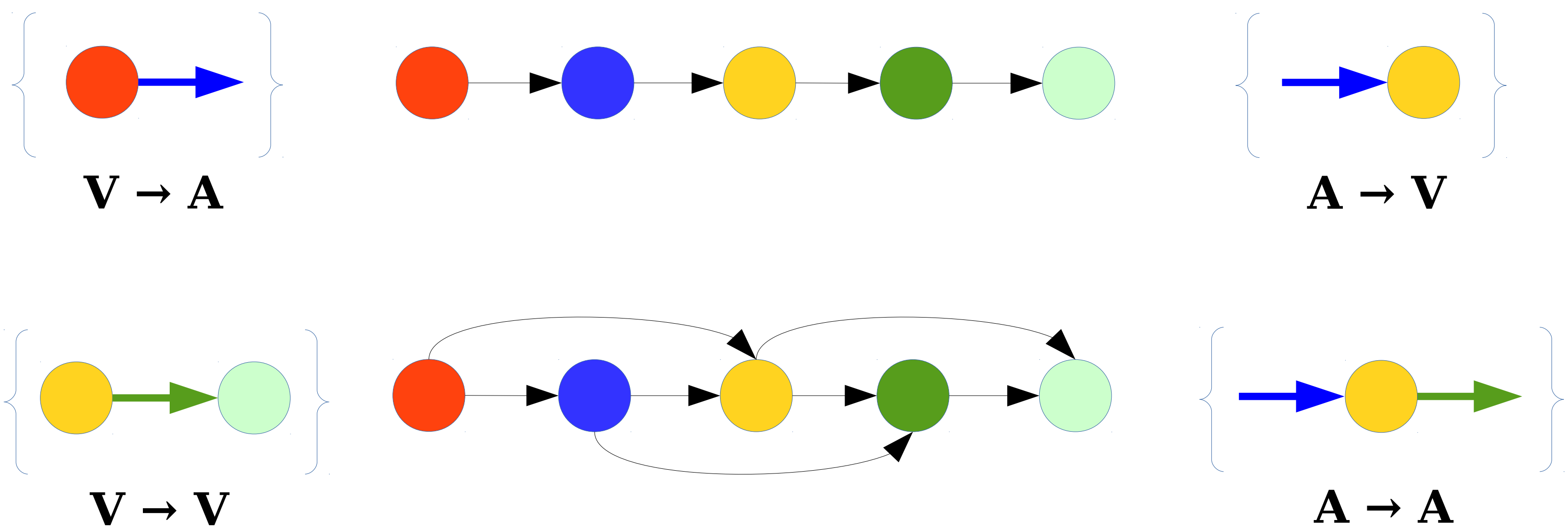
4: Total Oriented Coloring

- Searching for a definition

3)





A.Nandy, S.Sen, S.Das,
S.Nandi, SP: 2017

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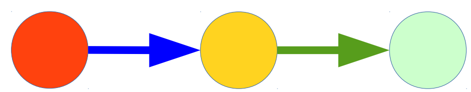
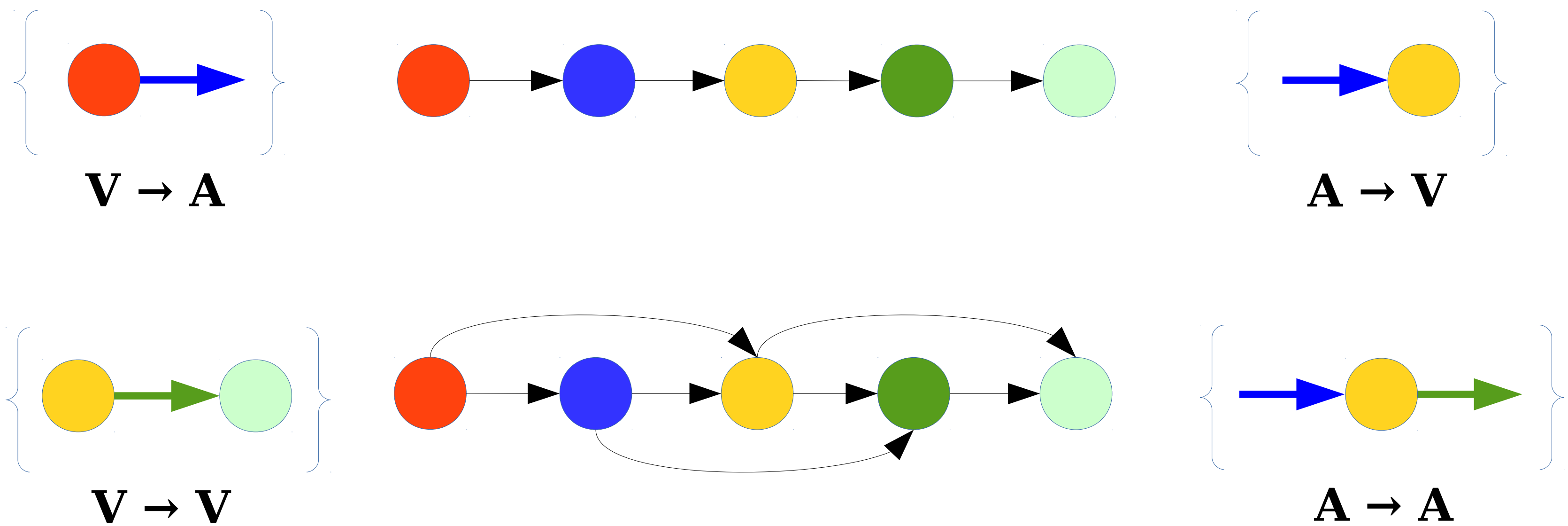


G



$S(G)$

$S(G)^2$

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4: Total Oriented Coloring

- Complete Definition

  Graph G is k-total oriented colorable if there exists a homomorphism

$$f : S(G)^2 \to H_k$$

an oclique on k vertices.

A.Nandy, S.Sen, S.Das, S.Nandi, SP: 2017

4: Total Oriented Coloring

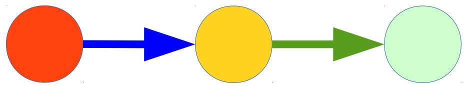
- Complete Definition

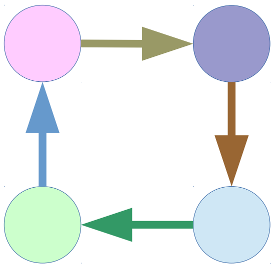
  Graph G is k-total oriented colorable if there exists a homomorphism $f : S(G)^2 \rightarrow H_k$ an oclique on k vertices.

- Immediate consequence:

  P_3 needs five colors

  C_4 needs eight colors





4: Total Oriented Coloring- Basic Results

- vs Oriented chromatic number for graph G

$$\chi''_o(G) \leq 2 \cdot \chi_o(G)$$

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  - Tightness yet to be investigated!!!

4: Total Oriented Coloring- Immediate Conclusions

- vs Oriented chromatic number for graph G

$$\chi''_o(G) \leq 2 \cdot \chi_o(G) \implies \chi''_o(\mathcal{P}(G)) \leq 160$$

-

4: Total Oriented Coloring- Immediate Conclusions

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$$\chi_o''(G) \leq 2 \cdot \chi_o(G) \implies \chi_o''(\mathcal{P}(G)) \leq 160$$

- vs Acyclic chromatic number k for graph G

$$\chi_o''(G) \leq k \cdot 2^{k-1} + 2k(k-1) + \lfloor \frac{k}{2} \rfloor$$

$$\implies \chi_o''(\mathcal{P}(G)) \leq 122$$

4: Total Oriented Coloring- Outerplanar graphs - *Schema*

- Fact 1 – Homomorphism $f : \mathcal{O}(G)_i \rightarrow QR_7$

4: Total Oriented Coloring- Outerplanar graphs - *Schema*

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QR_n : *Tournament based on Quadratic Residues*

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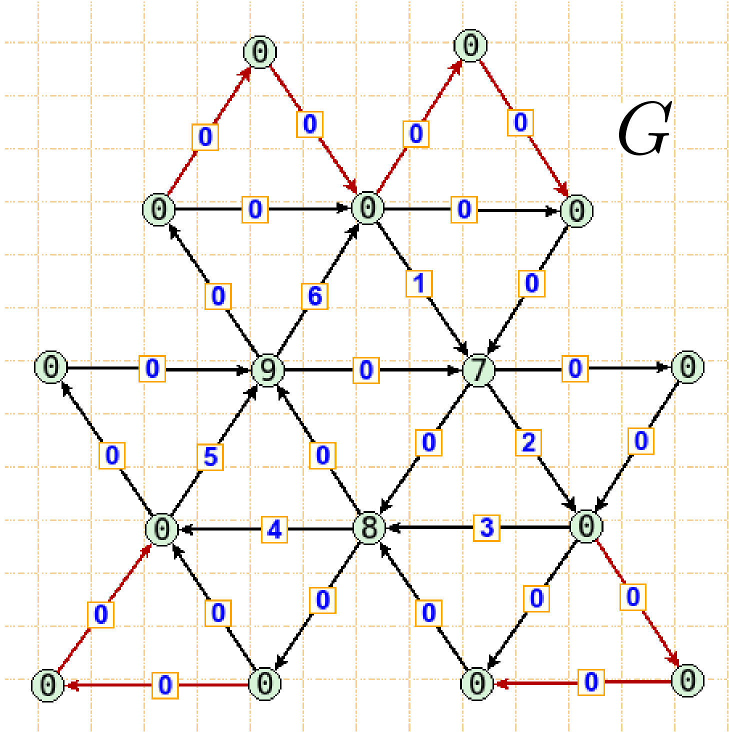
- Fact 5 – $\chi_o''(G) = 12$



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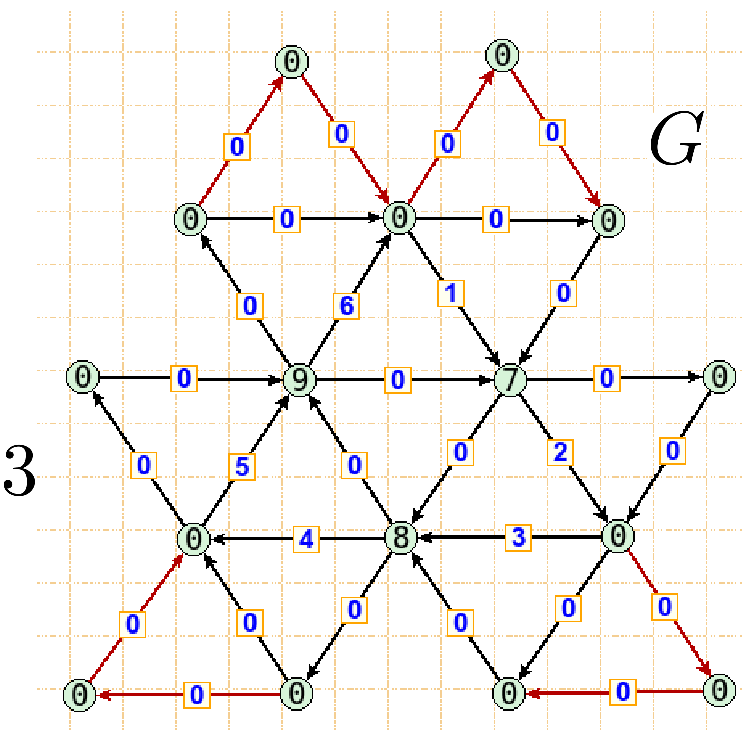
- Result – $12 \leq \chi''_o(\mathcal{O}(G)) \leq 13$



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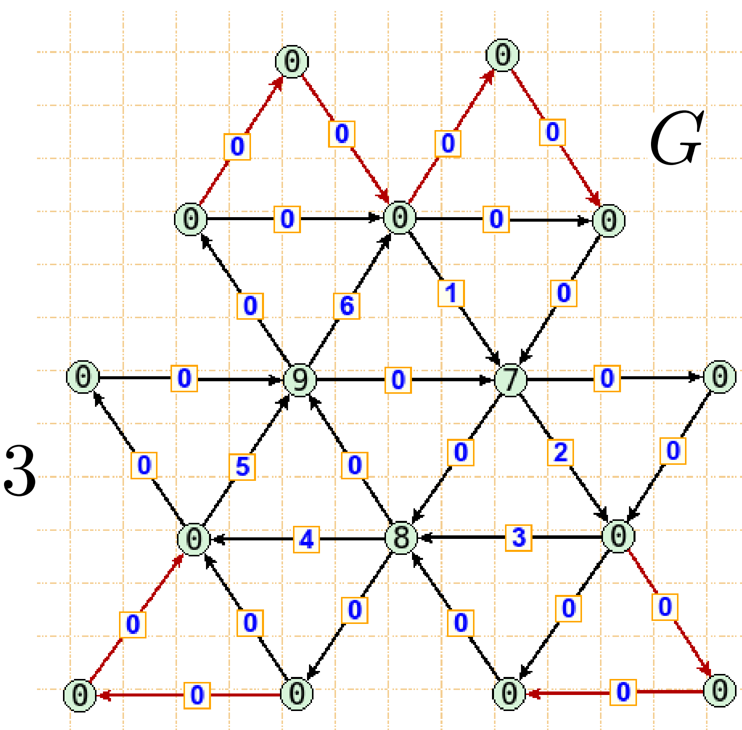

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Fractional Graph Theory

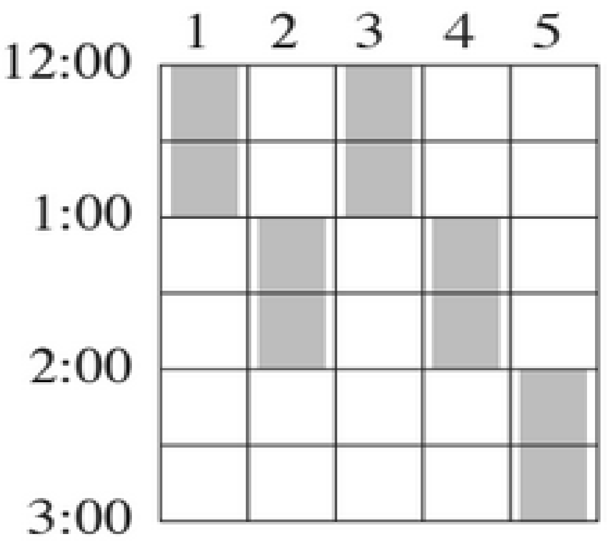
"Perfectness" (perfect matching)

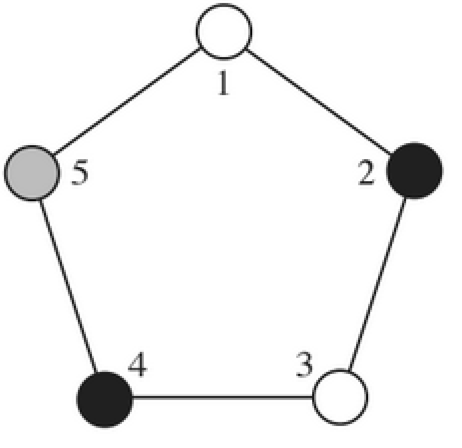
↘ Maximal (maximal matching)

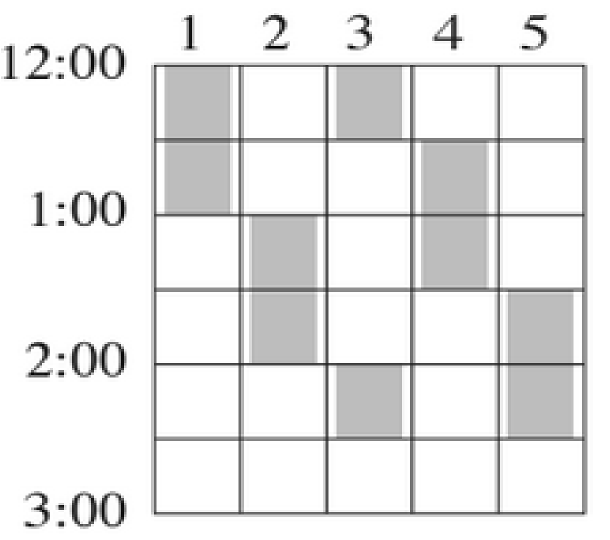
↘ Fractional (fractional matching)

Example with coloring

Schedule 5 committees in shortest possible duration
given each runs for 1 hr.







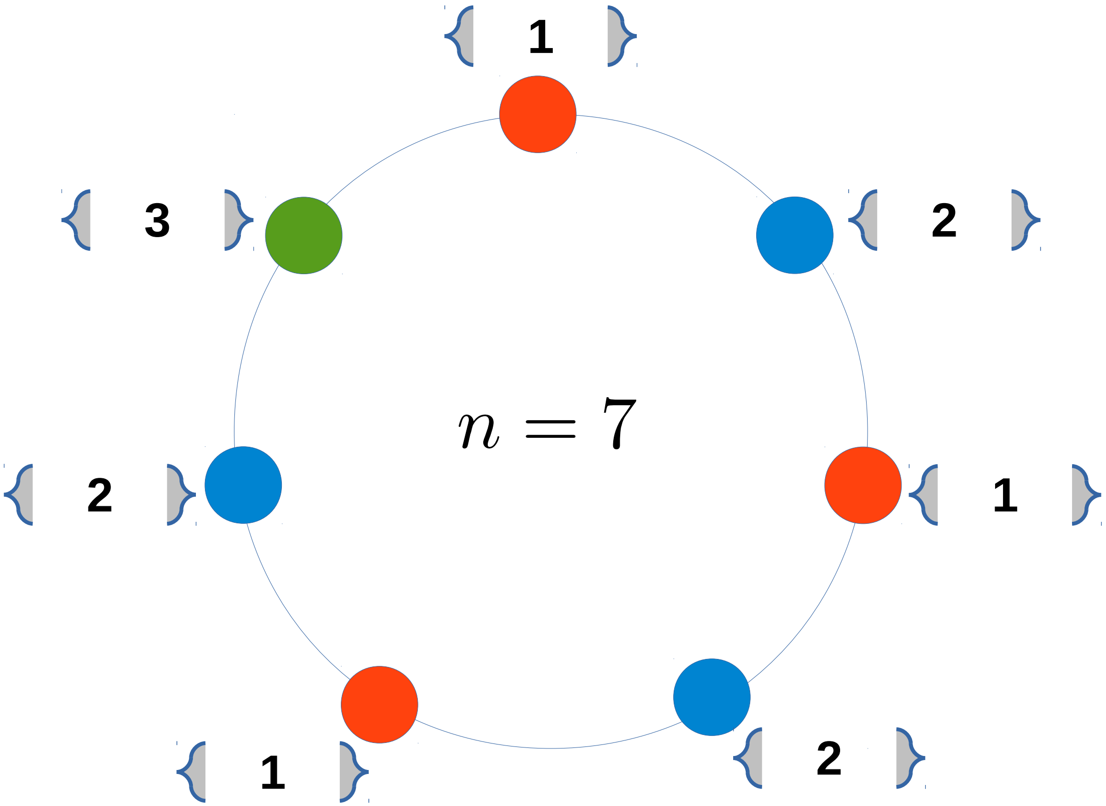
3 Hour schedule for 5 committees

2.5 Hour schedule for 5 committees

Illustration from the book “Fractional graph theory” by Scheinerman and Ullman

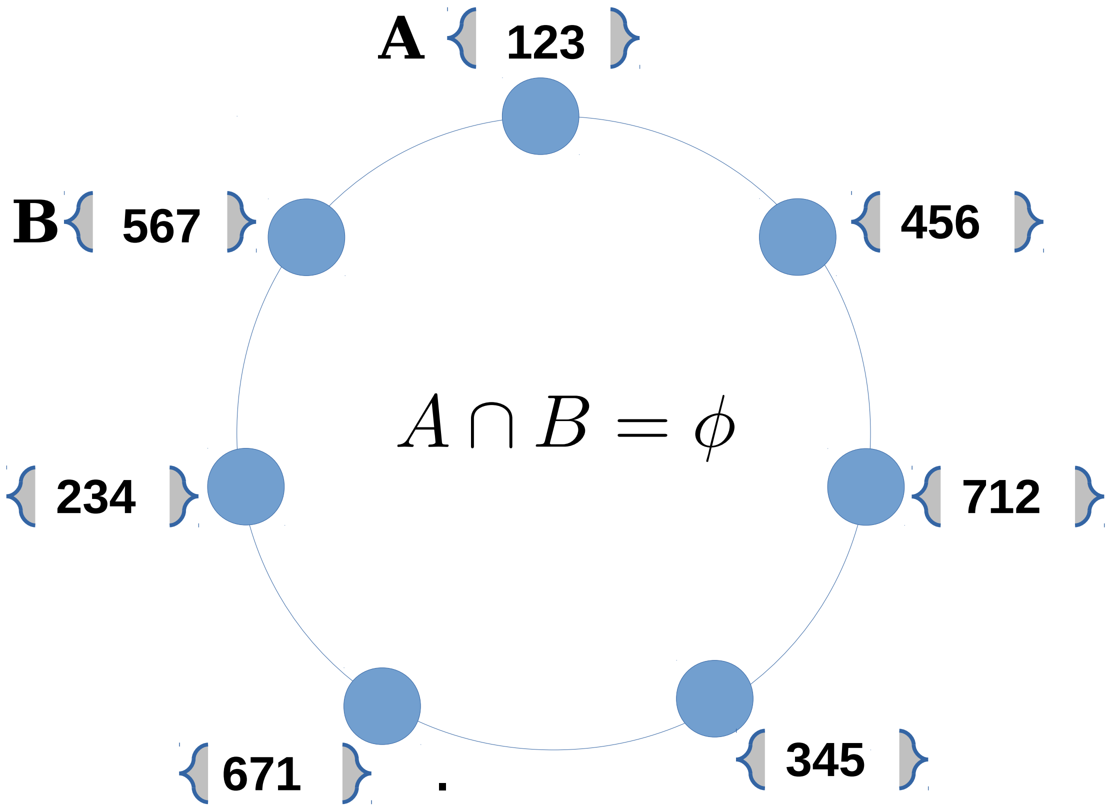
Fractional Coloring

- A simple odd cycle is 3-colorable where each node receives one color.



Fractional Coloring

- Now, we assign a color-tuple of fixed length to each node with the condition that color-tuples of adjacent nodes are non-intersecting.



Fractional Coloring

- Now, we assign a color-tuple of fixed length to each node with the condition that color-tuples of adjacent nodes are non-intersecting.
- Optimization Problem:
  - How many colors (a) are needed?
  - What is the length of the tuple (b)?

“such that a / b is minimized” ?

Fractional Coloring

Defining Fractional Chromatic Number:

- A **b**-fold coloring of a graph G assigns to each vertex of G a set of **b** colors so that adjacent vertices receive disjoint sets of colors.

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Fractional Coloring

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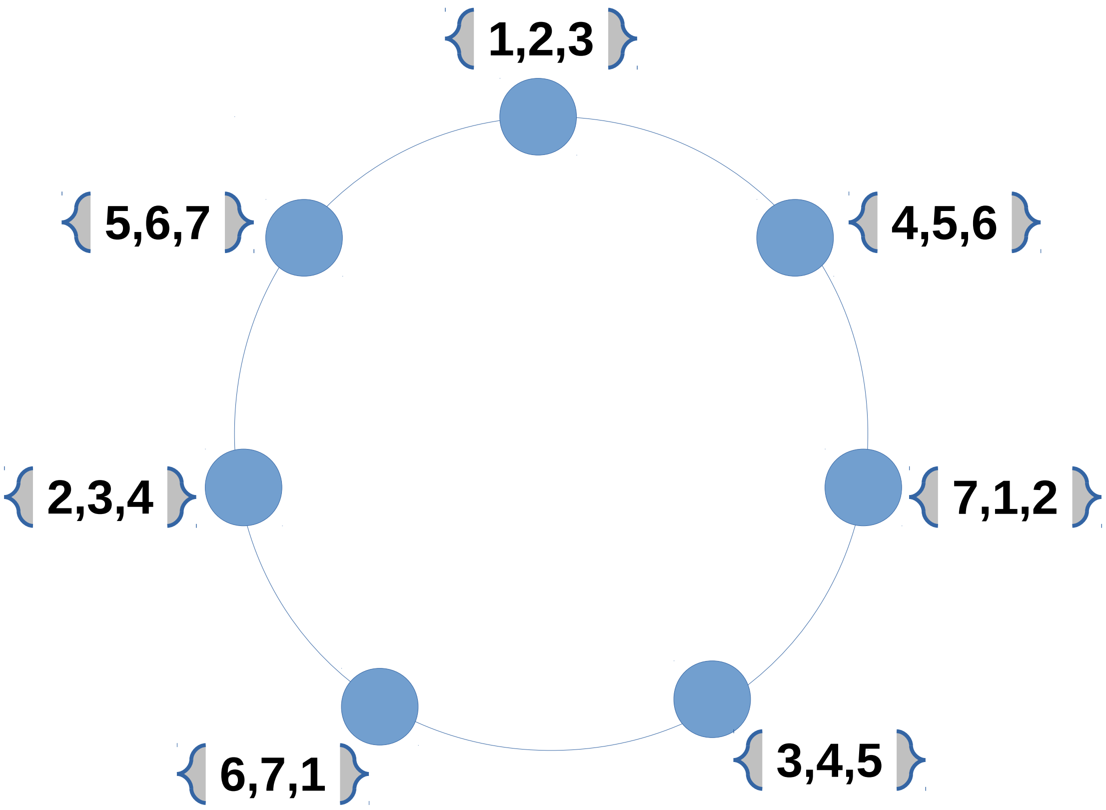
- G is **a** : **b**-colorable if it has a **b**-fold coloring in which the colors are drawn from a palette of **a** colors.
- Fractional chromatic number

$$\chi_f(G) = \lim_{b\to\infty} \frac{\chi_b(G)}{b} = \inf_b \frac{\chi_b(G)}{b}$$

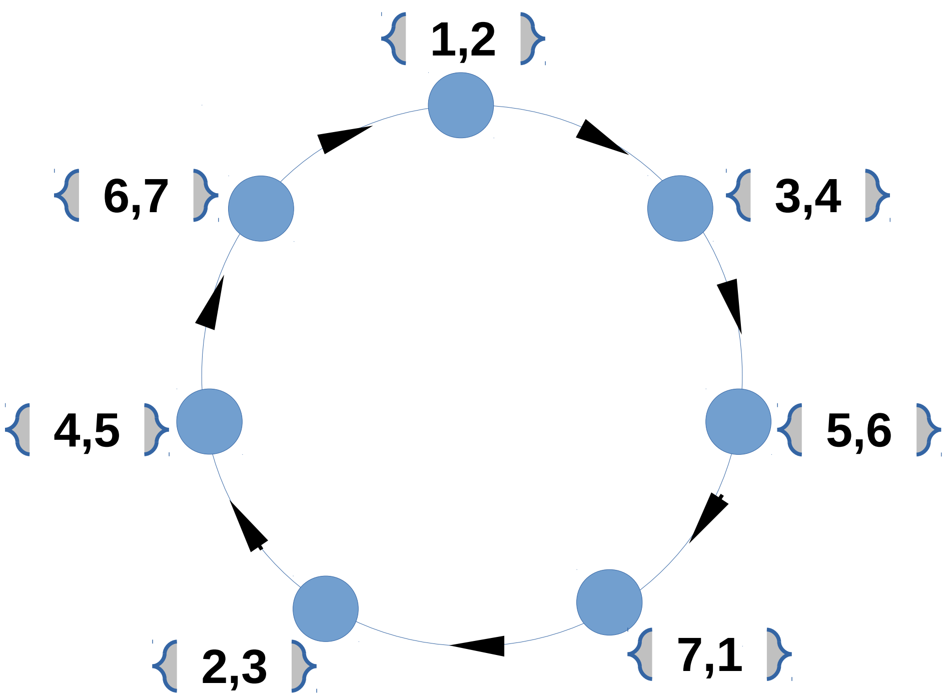
Fractional coloring of C_7



- a = 7, b = 3
- This is 7:3 fold coloring
- $\chi_f(G) = 2.34$
- Natural bound $\frac{n}{\alpha}$

Oriented fractional coloring of $\vec{C}_7$ - A revisit

$$\chi_{of}(G) = \lim_{b\to\infty} \frac{\chi_{ob}(G)}{b} = \inf_{b} \frac{\chi_{ob}(G)}{b}$$



- a = 7, b = 2
- This is 7:2 fold coloring
- $\chi_{of}(\overrightarrow{C}_7) = 3.5$
- $\chi_o(\overrightarrow{C}_7) = 4$

S.Das, S. Sen, SP: 2017

Oriented Fractional coloring of Directed Cycles

Main Result 1: Given a directed cycle $\vec{C}_n$ of length n, the oriented fractional chromatic number,

$$\chi_{of}(\vec{C}_n) = \begin{cases} 4 & \text{if n is not a multiple of (4k-1) kind of prime} \\ 4-1/k & \text{if n is a multiple of smallest (4k-1) kind of prime} \end{cases}$$

eg., $\vec{C}_{7=4*2-1} = 3.5$, $\vec{C}_{77=7*11}$ also 3.5 as we can repeat the $\vec{C}_7$ coloring 11 times, instead $\vec{C}_{11}$ coloring 7 times

S.Das, S. Sen, SP: 2017

Set Theoretic Proof sketch –
(centered around minimal ofc cycle)

Distinct color tuples A, B, C assigned to u,v,w in uvw. Let D be such that $|D| < |A|=|B|=|C|$

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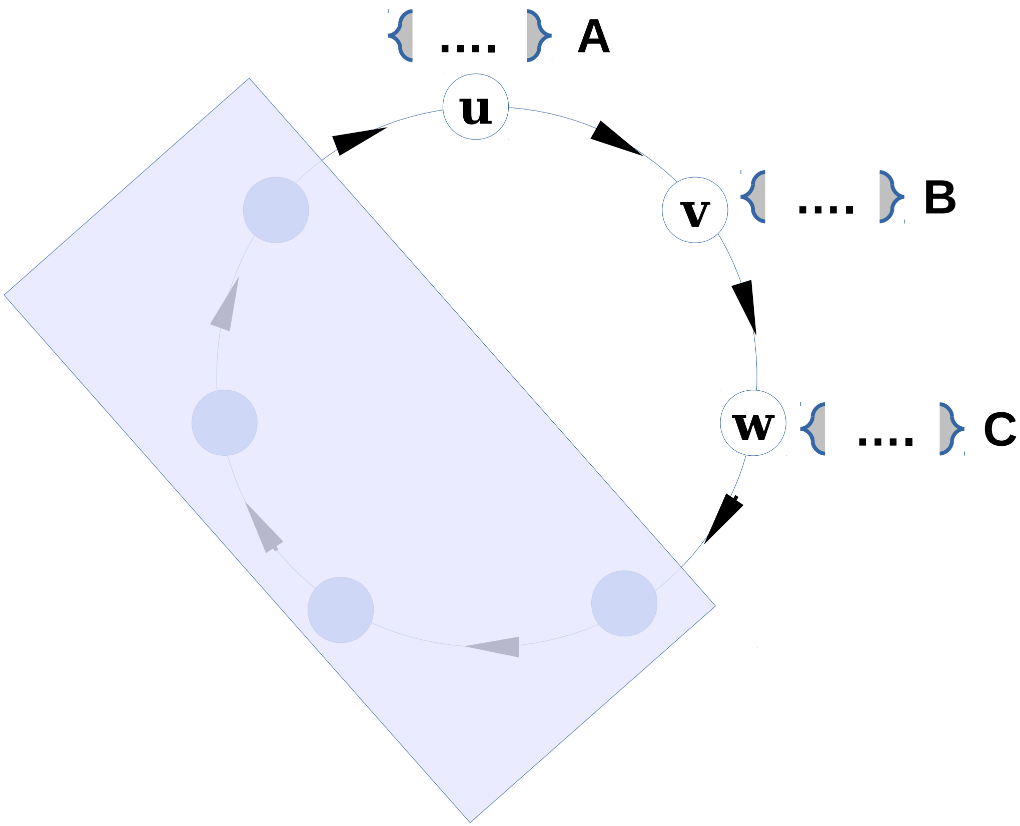


$$A = \{1, \cdots, k\}$$

$$B = \{k+1, \cdots, 2k\}$$

$$C = \{2k+1, \cdots, 3k\}$$

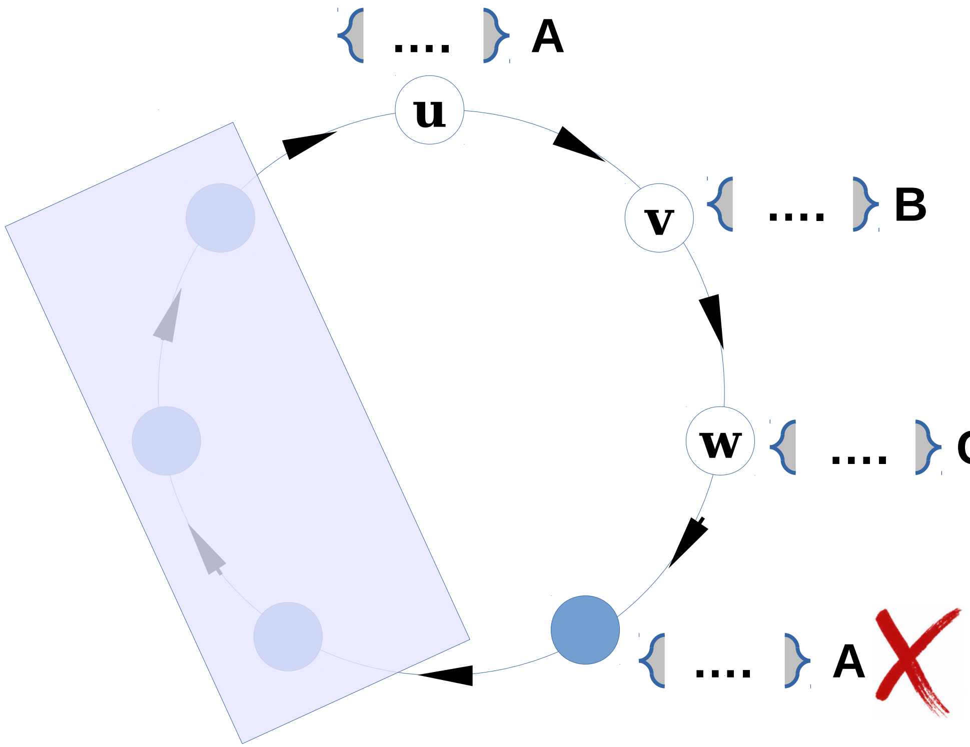
$$D = \{3k+1, \cdots, 4k-\delta\}$$

S.Das, S. Sen, SP: 2017

Proof sketch –
(centered around minimal ofc cycle)

Distinct color tuples A, B, C assigned to u,v,w in uvw. Let D be such that |D| < |A|=|B|=|C|

1. Around the cycle: A-B-C-A doesn't occur



S.Das, S. Sen, SP: 2017

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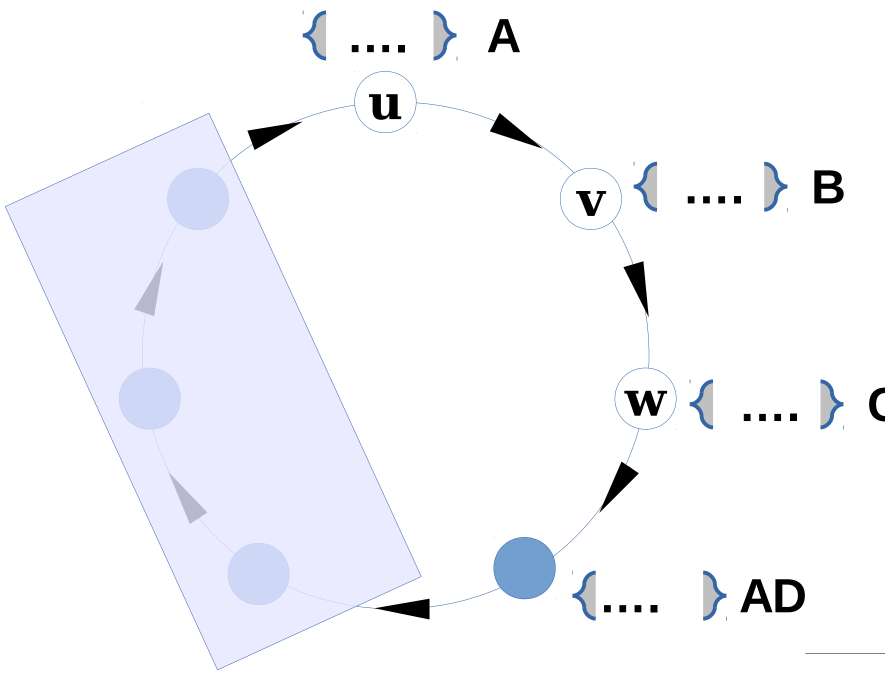
Distinct color tuples A, B, C assigned to u,v,w in uvw. Let D be such that |D| < |A|=|B|=|C|

1. Around the cycle: A-B-C-A doesn't occur
2. Define Triple △ A-B-C, AD-BD-CD, and Quad □ AD-AB-BC-CD



S.Das, S. Sen, SP: 2017

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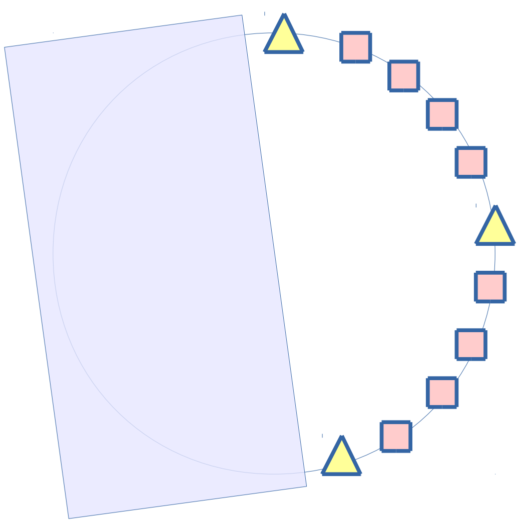
4. Arc-distance between two successive triples is same.



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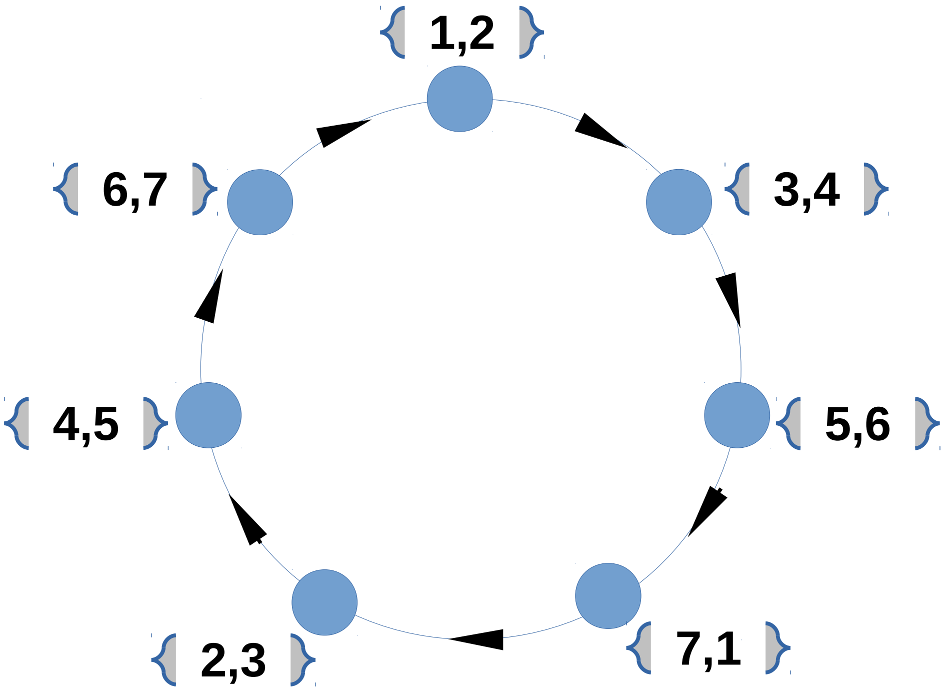

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Fractional coloring of oriented cycles

Corollary result 2. Given an oriented cycle $\vec{C}_n$ of length n, such that the difference of forward and reverse arcs is m, then the oriented fractional chromatic number,

$$\chi_{of}(\overrightarrow{C}_n) = \begin{cases} 2 & \text{if } m = 0 \text{ w/o 2-dipath} \\ 3 & \text{if } m = 0 \text{ with 2-dipath} \\ 4 & \text{if } m \text{ is not a multiple of (4k-1) kind of prime} \\ 4-1/k & \text{if } m \text{ is a multiple of smallest (4k-1) kind of prime} \end{cases}$$

S.Das, S. Sen, SP: 2017

6. Oriented Chromatic Polynomial

A word of caution ...

- Not every structural property/concept can be lifted in the oriented domain.

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- For instance, Compliment of a graph.

A word of caution ...

- Not every structural property/concept can be lifted in the oriented domain.
- For instance, Compliment of a graph.
- Quick answer: ‘Reverse the arcs’ wont help. Why?

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Each soul is potentially divine.The goal is to manifest this divinity within by controlling nature, external and internal.Do this either by work, or worship or psychic control or philosophy -by one or more or all of these and be free.This is the whole of religion. Doctrines and dogmas, rituals and forms, books and temples are but secondary details. -*Swami Vivekananda*

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Ashram im Mühlheim am Main: Pestalozzistraße 2, 63165 Mühlheim/Main. Tel.: 06108-823105, Fax: 06108-823107

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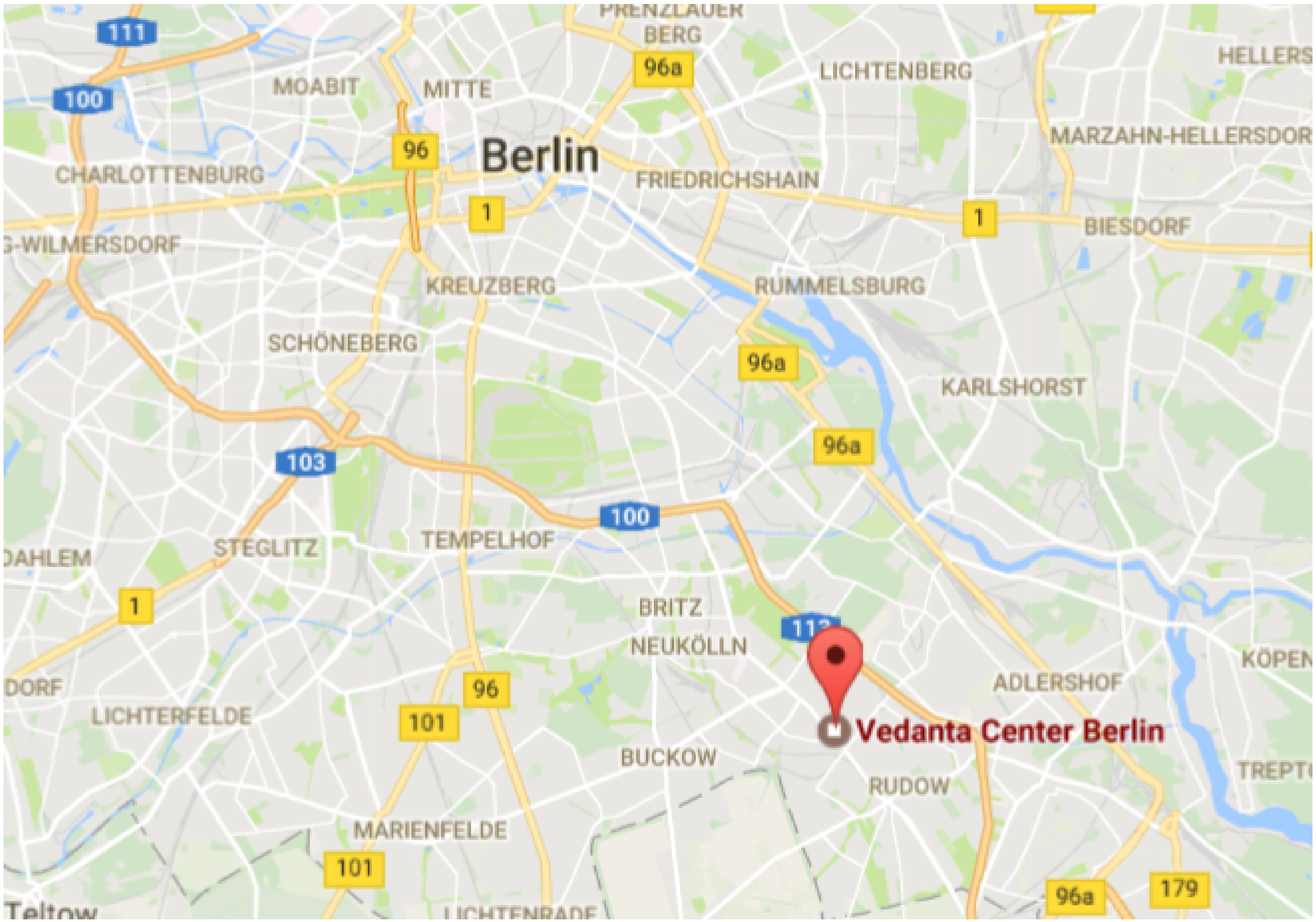

Thank you