Domination in Some Subclasses of Bipartite Graphs

Arti Pandey, B.S. Panda

Department of Mathematics
Indian Institute of Technology, Delhi

February 10, 2015

Outline

1. Introduction to Domination Concepts

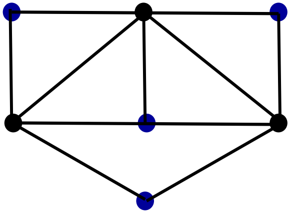
2. Complexity Status

3. Our Results
   - Polynomial time algorithm for Circular-convex bipartite graphs
   - Polynomial time algorithm for Triad-convex bipartite graphs
   - Hardness results for Star-convex bipartite graphs

4. Future Aspects

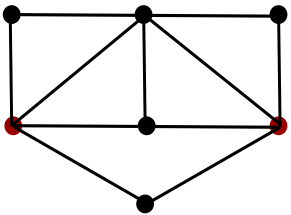
Domination

- The concept of domination was first introduced by **Berge** and **Ore** in 1962.
- A vertex v is said to **dominate** a vertex w in G if $v \in N_G[w]$.
- A set $D \subseteq V$ is called a **dominating set** of the graph $G = (V, E)$ if every vertex of G is dominated by some vertex in D.
- The **domination number** of G, denoted by γ_G is the cardinality of a minimum dominating set of G.



Domination

- The concept of domination was first introduced by **Berge** and **Ore** in 1962.
- A vertex v is said to **dominate** a vertex w in G if $v \in N_G[w]$.
- A set $D \subseteq V$ is called a **dominating set** of the graph $G = (V, E)$ if every vertex of G is dominated by some vertex in D.
- The **domination number** of G, denoted by γ_G is the cardinality of a minimum dominating set of G.



Problem Statement

Minimum Domination Problem (MDP)

Instance: A graph $G = (V, E)$.

Solution: A dominating set D of G.

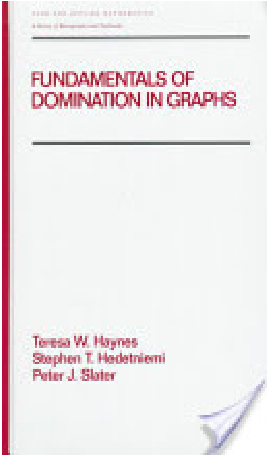
Measure: Cardinality of the D.

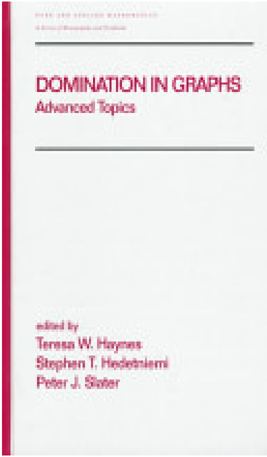
Domination Decision Problem (DDP)

Instance: A graph $G = (V, E)$ and a positive integer $k \leq |V|$.

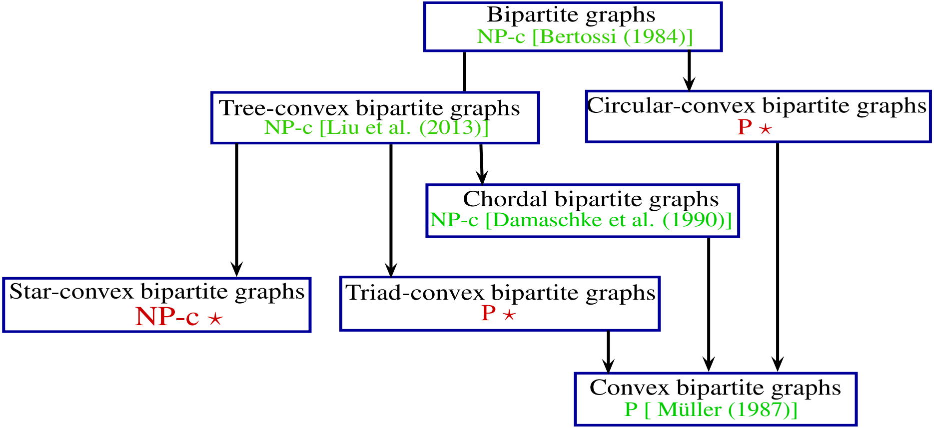
Question: Does there exist a dominating set D of G such that $|D| \leq k$?

Two Books on Domination





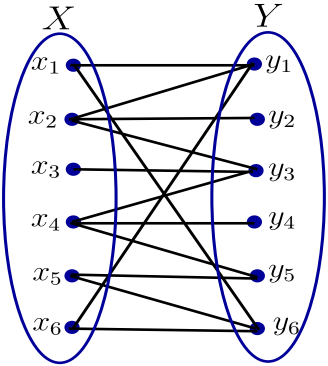
Hierarchy Relationship Between Some Graph Classes

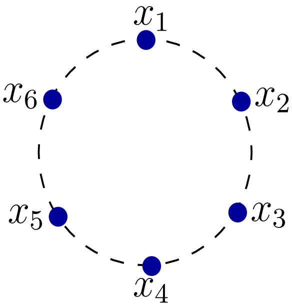


Some References for Graph Classes

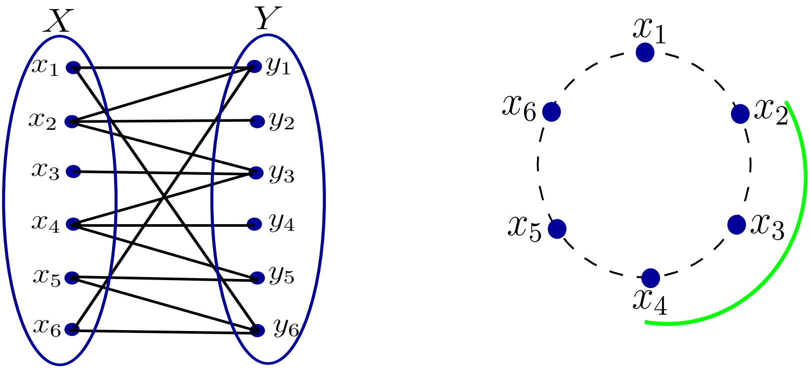
1. Liang, Y.D., Blum, N.: Circular convex bipartite graphs: maximum matching and hamiltonian circuits. Inform. Process. Lett. 56 (1995) 215-219.
2. Liu, T., Lu, M., Lu, Z., Xu, K.: Circular convex bipartite graphs: Feedback vertex sets. Theoret. Comput. Sci. DOI: 10.1016/j.tcs.2014.05.001 (2014).
3. Liu, W.J.T.,Wang, C., Xu, K.: Feedback vertex sets on restricted bipartite graphs. Theoret. Comput. Sci. 507 (2013) 41-51.
4. Lu, M., Liu, T., Xu, K.: Independent domination: reductions from circular- and triad-convex bipartite graphs to convex bipartite graphs. in: Proc. of FAW-AAIM (2013) 142-152.
5. Lu, Z., Liu, T., Xu, K.: Tractable connected domination for restricted bipartite graphs. in: Proc. of COCOON (2013) 721-728.
6. Song, Y., Liu, T., Xu, K.: Independent domination on tree-convex bipartite graphs. in: Proc. of FAW-AAIM (2012) 129-138.

A bipartite graph $G = (X, Y, E)$ is called circular-convex bipartite graph if we can define an ordering $\alpha = (x_1, x_2, \ldots, x_p)$ on the vertices of X such that for every vertex $y \in Y$, either $N_G(y) = \{x_i, x_{i+1}, \ldots, x_j\}$ or $N_G(y) = \{x_j, x_{j+1}, \ldots, x_p, x_1, \ldots, x_i\}$ for some i, j, where $1 \leq i \leq j \leq p$.

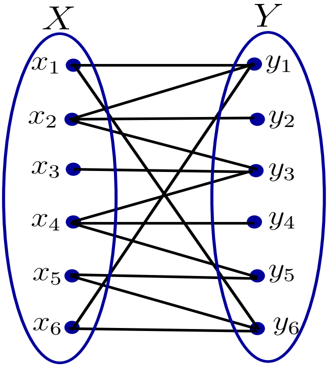


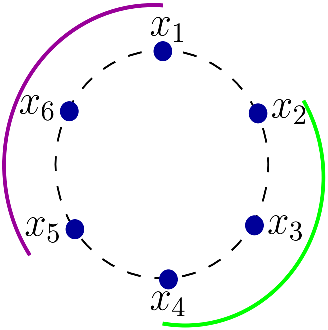


A bipartite graph $G = (X, Y, E)$ is called circular-convex bipartite graph if we can define an ordering $\alpha = (x_1, x_2, \ldots, x_p)$ on the vertices of X such that for every vertex $y \in Y$, either $N_G(y) = \{x_i, x_{i+1}, \ldots, x_j\}$ or $N_G(y) = \{x_j, x_{j+1}, \ldots, x_p, x_1, \ldots, x_i\}$ for some i, j, where $1 \leq i \leq j \leq p$.

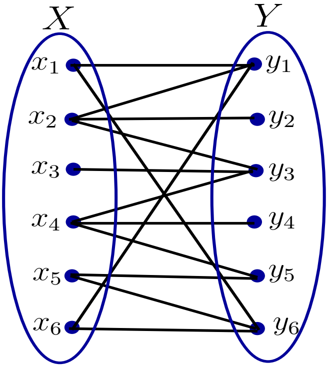


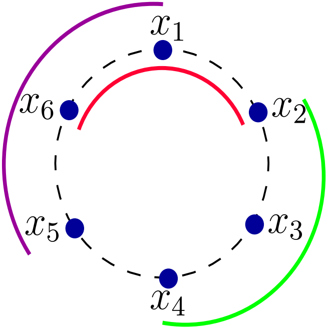
A bipartite graph $G = (X, Y, E)$ is called circular-convex bipartite graph if we can define an ordering $\alpha = (x_1, x_2, \ldots, x_p)$ on the vertices of X such that for every vertex $y \in Y$, either $N_G(y) = \{x_i, x_{i+1}, \ldots, x_j\}$ or $N_G(y) = \{x_j, x_{j+1}, \ldots, x_p, x_1, \ldots, x_i\}$ for some i, j, where $1 \leq i \leq j \leq p$.





A bipartite graph $G = (X, Y, E)$ is called circular-convex bipartite graph if we can define an ordering $\alpha = (x_1, x_2, \ldots, x_p)$ on the vertices of X such that for every vertex $y \in Y$, either $N_G(y) = \{x_i, x_{i+1}, \ldots, x_j\}$ or $N_G(y) = \{x_j, x_{j+1}, \ldots, x_p, x_1, \ldots, x_i\}$ for some i, j, where $1 \leq i \leq j \leq p$.





Minimum Domination in Circular-convex Bipartite Graphs

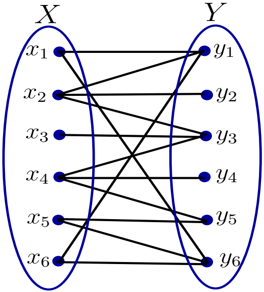
Let D^* be a minimum dominating set of a circular convex bipartite graph $G = (X, Y, E)$. Then the following two cases arise:

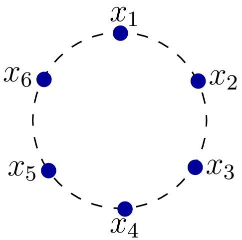
- $D^* \cap X = \emptyset$

  $D^* = Y$

- $D^* \cap X \neq \emptyset$

  $x_i \in D^* \cap X$ for some i, $1 \leq i \leq n$

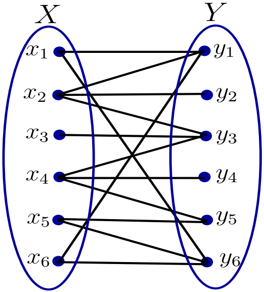


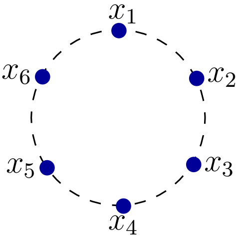


Minimum Domination in Circular-convex Bipartite Graphs

Let D^* be a minimum dominating set of a circular convex bipartite graph $G = (X, Y, E)$. Then the following two cases arise:

- $D^* \cap X = \emptyset$
  $D^* = Y$
- $D^* \cap X \neq \emptyset$
  $x_i \in D^* \cap X$ for some i, $1 \leq i \leq n$

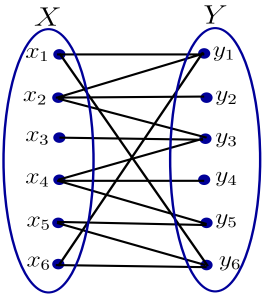


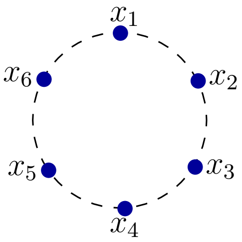


Minimum Domination in Circular-convex Bipartite Graphs

Let D^* be a minimum dominating set of a circular convex bipartite graph $G = (X, Y, E)$. Then the following two cases arise:

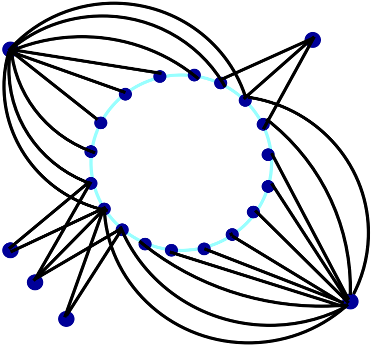
- $D^* \cap X = \emptyset$
  $D^* = Y$
- $D^* \cap X \neq \emptyset$
  $x_i \in D^* \cap X$ for some i, $1 \leq i \leq n$





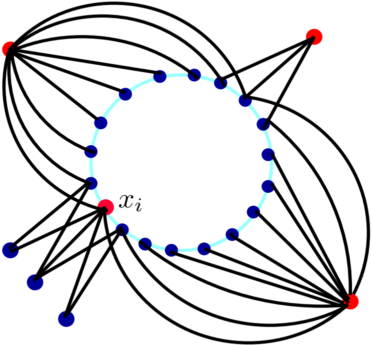
First Approach

- Define $G' = G \setminus N_G[x_i]$.
- Find minimum dominating set D' of G'.
- Output $D' \cup \{x_i\}$ as a dominating set of G.



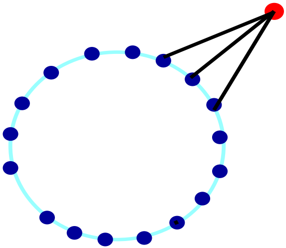
An Example for Our Approach

- Define $G' = G \setminus N_G[x_i]$.
- Find minimum dominating set D' of G'.
- Output $D' \cup \{x_i\}$ as a dominating set of G.



An Example for Our Approach

- Define $G' = G \setminus N_G[x_i]$.
- Find minimum dominating set D' of G'.
- Output $D' \cup \{x_i\}$ as a dominating set of G.



Minimum Domination in Circular-Convex Bipartite Graphs

Lemma

Let $G = (X, Y, E)$ be a circular-convex bipartite graph, and D^* be a minimum dominating set of G. Then, for a vertex $x_i \in D^* \cap X$, $|N_G(x_i) \cap D^*| \leq 2$.

Proof.

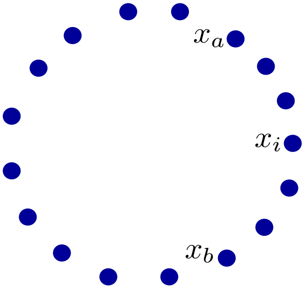
□

Minimum Domination in Circular-Convex Bipartite Graphs

Lemma

Let $G = (X, Y, E)$ be a circular-convex bipartite graph, and D^* be a minimum dominating set of G. Then, for a vertex $x_i \in D^* \cap X$, $|N_G(x_i) \cap D^*| \leq 2$.

Proof.

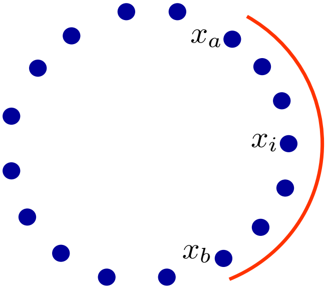


Minimum Domination in Circular-Convex Bipartite Graphs

Lemma

Let $G = (X, Y, E)$ be a circular-convex bipartite graph, and D^* be a minimum dominating set of G. Then, for a vertex $x_i \in D^* \cap X$, $|N_G(x_i) \cap D^*| \leq 2$.

Proof.

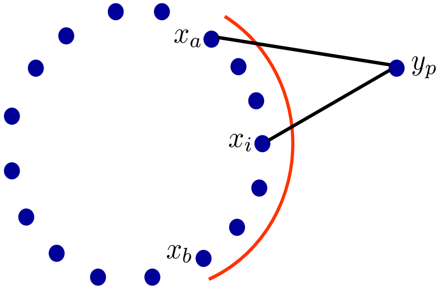


Minimum Domination in Circular-Convex Bipartite Graphs

Lemma

Let $G = (X, Y, E)$ be a circular-convex bipartite graph, and D^* be a minimum dominating set of G. Then, for a vertex $x_i \in D^* \cap X$, $|N_G(x_i) \cap D^*| \leq 2$.

Proof.

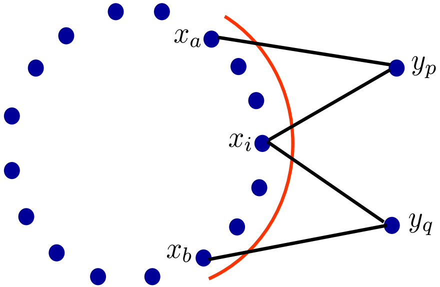


Minimum Domination in Circular-Convex Bipartite Graphs

Lemma

Let $G = (X, Y, E)$ be a circular-convex bipartite graph, and D^* be a minimum dominating set of G. Then, for a vertex $x_i \in D^* \cap X$, $|N_G(x_i) \cap D^*| \leq 2$.

Proof.

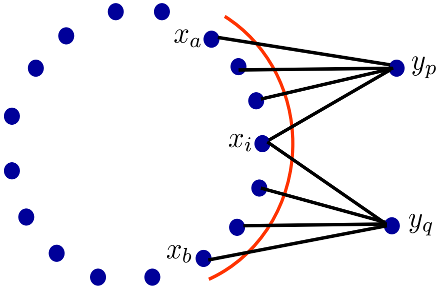


Minimum Domination in Circular-Convex Bipartite Graphs

Lemma

Let $G = (X, Y, E)$ be a circular-convex bipartite graph, and D^* be a minimum dominating set of G. Then, for a vertex $x_i \in D^* \cap X$, $|N_G(x_i) \cap D^*| \leq 2$.

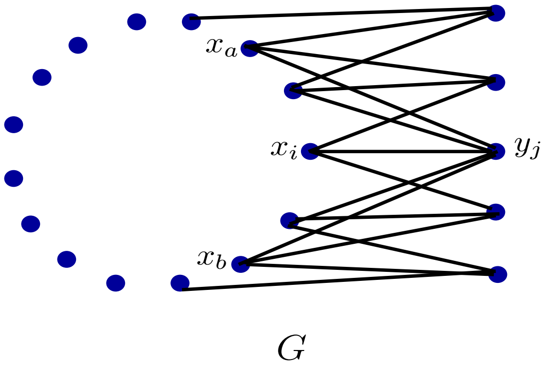
Proof.



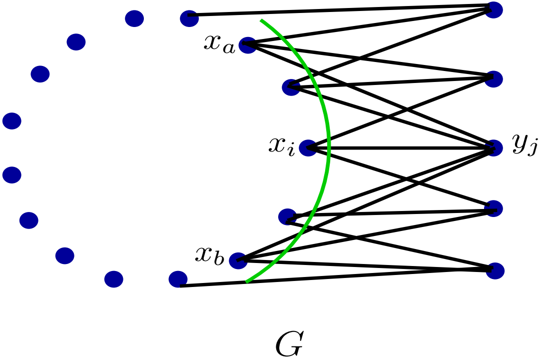
Let D^* be a minimum dominating set of G.

- If D^* does not contain any vertex from X, then $D^* = Y$.
- If D^* contains at least one vertex say x_i from X, then we have the following cases:
  1. None of the neighbor of x_i belongs to D^*.
  2. Exactly one neighbor of x_i belongs to D^*.
  3. Exactly two neighbors of x_i belong to D^*.

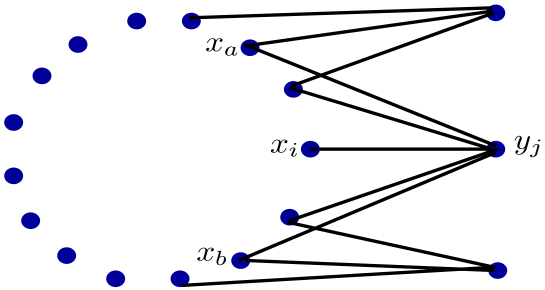
Case 2: Exactly one neighbor y_j of x_i belongs to D^*



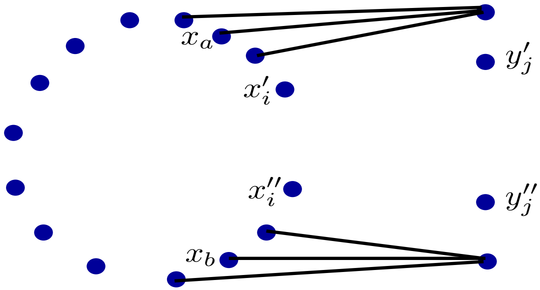
Case 2: Exactly one neighbor y_j of x_i belongs to D^*



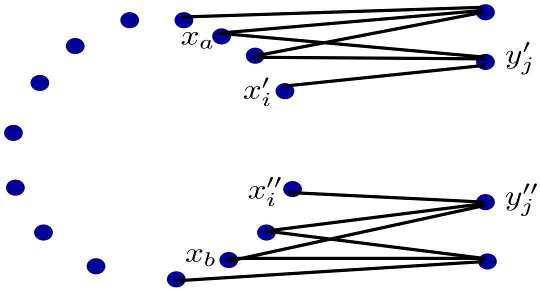
Case 2: Exactly one neighbor y_j of x_i belongs to D^*



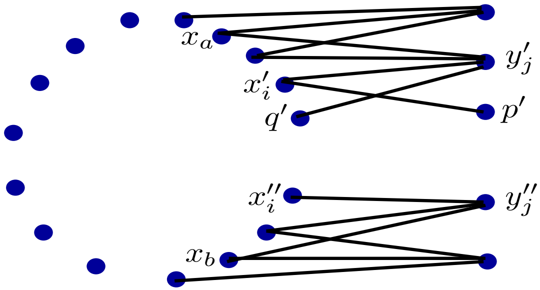
Case 2: Exactly one neighbor y_j of x_i belongs to D^*



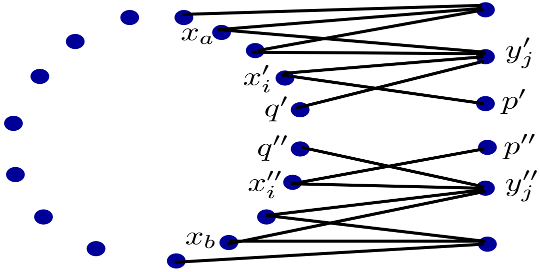
Case 2: Exactly one neighbor y_j of x_i belongs to D^*



Case 2: Exactly one neighbor y_j of x_i belongs to D^*

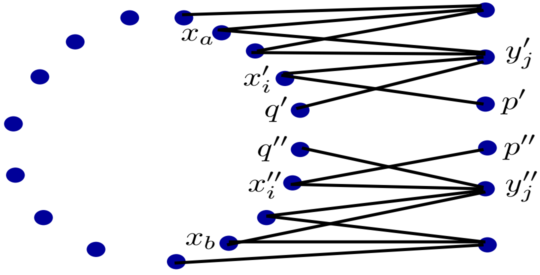


Case 2: Exactly one neighbor y_j of x_i belongs to D^*



$G_{x_i}^{y_j}$

Case 2: Exactly one neighbor y_j of x_i belongs to D^*

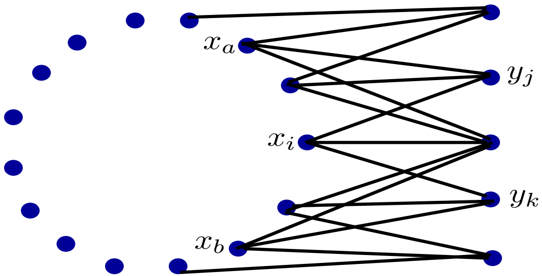


$G_{x_i}^{y_j}$

Lemma

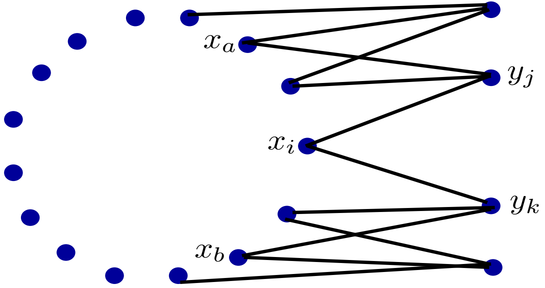
If D is a minimum dominating set of $G_{x_i}^{y_j}$ containing all the support vertices of $G_{x_i}^{y_j}$, that is, $\{x'_i, x''_i, y'_j, y''_j\} \subseteq D$, then $D_{x_i}^{y_j} = (D \setminus \{x'_i, x''_i, y'_j, y''_j\}) \cup \{x_i, y_j\}$ is a minimum dominating set of G.

Case 3: Exactly two neighbors y_j, y_k of x_i belongs to D^*

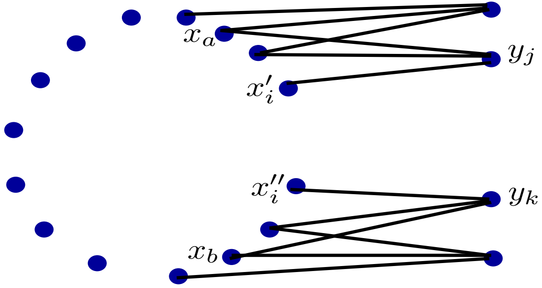


G

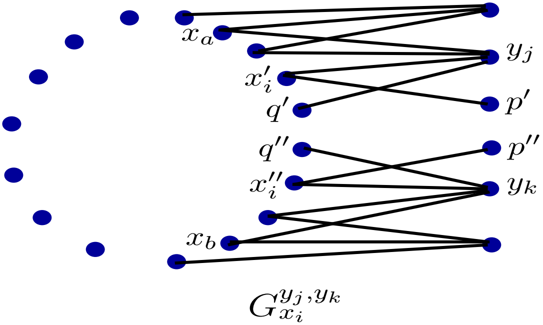
Case 3: Exactly two neighbors y_j, y_k of x_i belongs to D^*



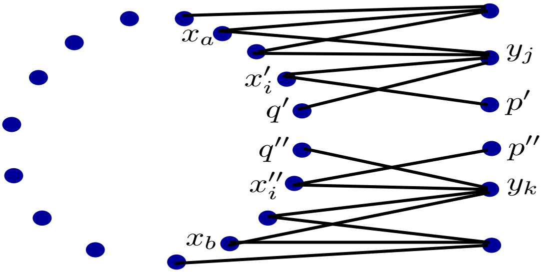
Case 3: Exactly two neighbors y_j, y_k of x_i belongs to D^*



Case 3: Exactly two neighbors y_j, y_k of x_i belongs to D^*



Case 3: Exactly two neighbors y_j, y_k of x_i belongs to D^*



$G_{x_i}^{y_j,y_k}$

Lemma

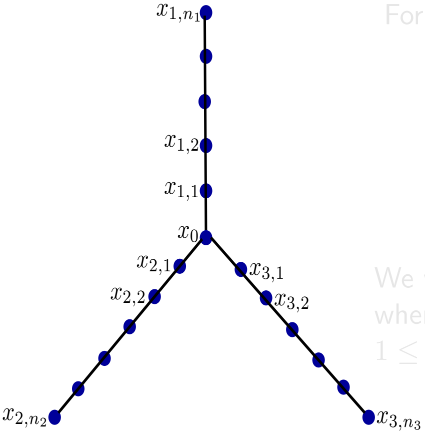
If D is a minimum dominating set of $G_{x_i}^{y_j,y_k}$ containing all the support vertices of $G_{x_i}^{y_j,y_k}$, that is, $\{x_i', x_i'', y_j, y_k\} \subseteq D$, then $D_{x_i}^{y_j,y_k} = (D \setminus \{x_i', x_i''\}) \cup \{x_i\}$ is a minimum dominating set of G.

Theorem

A minimum dominating set of circular-convex bipartite graph can be computed in $O(n^5)$ *time.*

Minimum Domination in Triad-Convex Bipartite Graphs

Let $G = (X, Y, E)$ be a triad-convex bipartite graph.

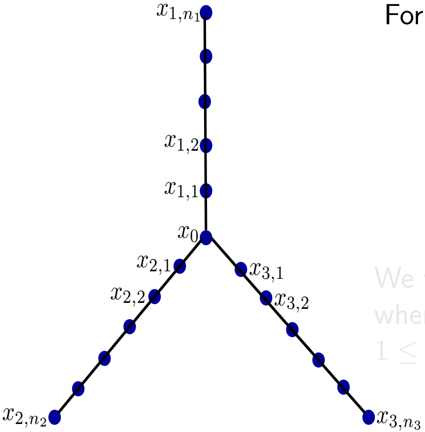


For each $y \in Y$, $T[N_G(y)]$ is a subtree of T.

We write $X = \{x_0\} \cup X_1 \cup X_2 \cup X_3$, where $X_i = \{x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}\}$ for each i, $1 \leq i \leq 3$.

Minimum Domination in Triad-Convex Bipartite Graphs

Let $G = (X, Y, E)$ be a triad-convex bipartite graph.

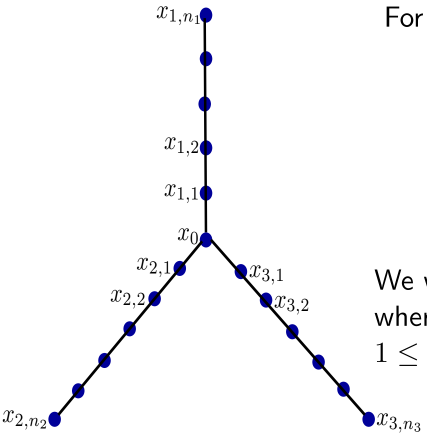


For each $y \in Y$, $T[N_G(y)]$ is a subtree of T.

We write $X = \{x_0\} \cup X_1 \cup X_2 \cup X_3$, where $X_i = \{x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}\}$ for each i, $1 \leq i \leq 3$.

Minimum Domination in Triad-Convex Bipartite Graphs

Let $G = (X, Y, E)$ be a triad-convex bipartite graph.

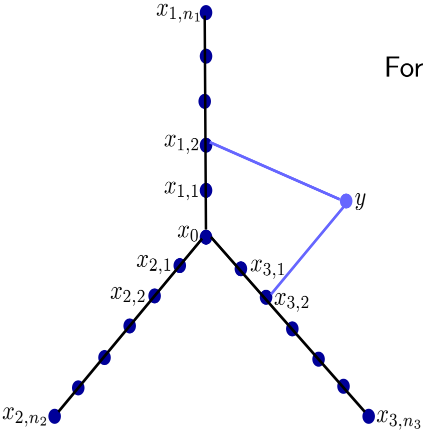


For each $y \in Y$, $T[N_G(y)]$ is a subtree of T.

We write $X = \{x_0\} \cup X_1 \cup X_2 \cup X_3$, where $X_i = \{x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}\}$ for each i, $1 \leq i \leq 3$.

Minimum Domination in Triad-Convex Bipartite Graphs

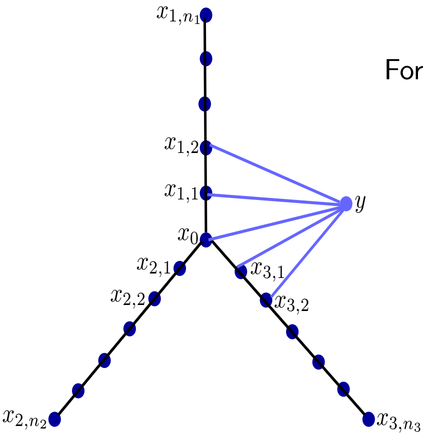
Let $G = (X, Y, E)$ be a triad-convex bipartite graph.



For each $y \in Y$, $T[N_G(y)]$ is a subtree of T.

Minimum Domination in Triad-Convex Bipartite Graphs

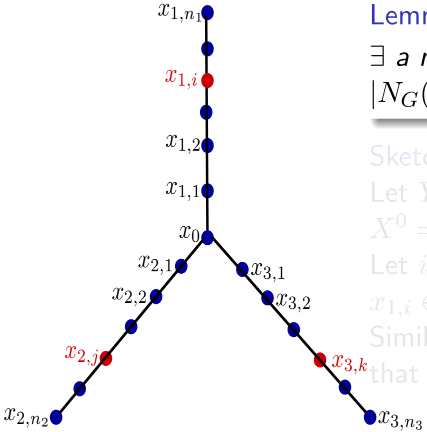
Let $G = (X, Y, E)$ be a triad-convex bipartite graph.



For each $y \in Y$, $T[N_G(y)]$ is a subtree of T.

Minimum Domination in Triad-Convex Bipartite Graphs

Let $G = (X, Y, E)$ be a triad-convex bipartite graph.



Lemma

∃ a minimum dominating set D of G such that $|N_G(x_0) \cap D| \leq 3$.

Sketch of the Proof:

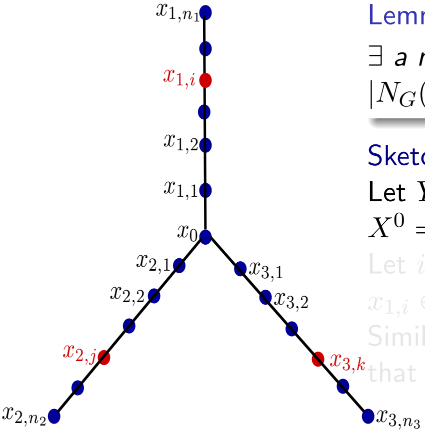
Let $Y^0 = N_G(x_0)$ and $X^0 = N_G(Y_0) = N_G(N_G(x_0))$.

Let i be the maximum index such that $x_{1,i} \in X_0$.

Similarly, j, k be the maximum indices such that $x_{2,j}, x_{3,k} \in X_0$.

Minimum Domination in Triad-Convex Bipartite Graphs

Let $G = (X, Y, E)$ be a triad-convex bipartite graph.



Lemma

$\exists$ a minimum dominating set D of G such that $|N_G(x_0) \cap D| \leq 3$.

Sketch of the Proof:

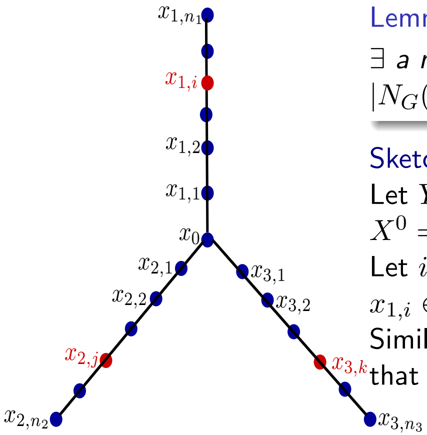
Let $Y^0 = N_G(x_0)$ and $X^0 = N_G(Y_0) = N_G(N_G(x_0))$.

Let i be the maximum index such that $x_{1,i} \in X_0$.

Similarly, j, k be the maximum indices such that $x_{2,j}, x_{3,k} \in X_0$.

Minimum Domination in Triad-Convex Bipartite Graphs

Let $G = (X, Y, E)$ be a triad-convex bipartite graph.



Lemma

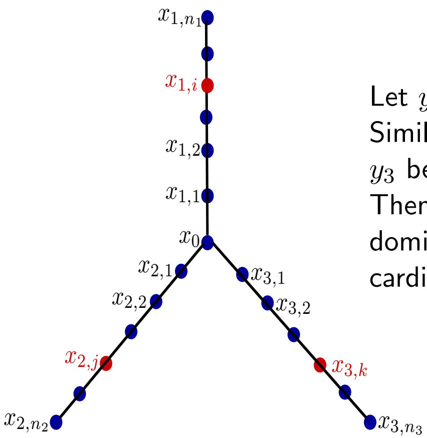
$\exists$ a minimum dominating set D of G such that $|N_G(x_0) \cap D| \leq 3$.

Sketch of the Proof:

Let $Y^0 = N_G(x_0)$ and $X^0 = N_G(Y_0) = N_G(N_G(x_0))$.
Let i be the maximum index such that $x_{1,i} \in X_0$.
Similarly, j, k be the maximum indices such that $x_{2,j}, x_{3,k} \in X_0$.

Minimum Domination in Triad-Convex Bipartite Graphs

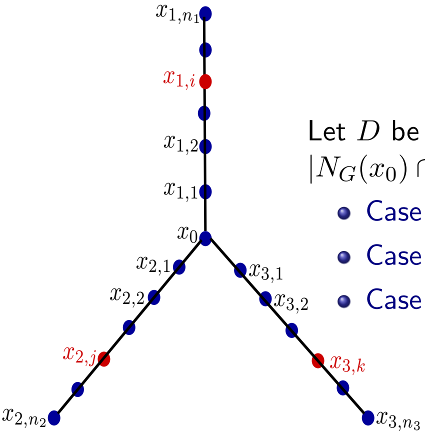
Let $G = (X, Y, E)$ be a triad-convex bipartite graph.



Let y_1 be a vertex in $N_G(x_{1,i}) \cap Y_0$,
Similarly y_2 be a vertex in $N_G(x_{2,j}) \cap Y_0$ and y_3 be a vertex in $N_G(x_{3,k}) \cap Y_0$.
Then $(D \setminus \{N_G(x_0)\}) \cup \{x_0, y_1, y_2, y_3\}$ is a dominating set of G of same or lesser cardinality than the cardinality of D.

Minimum Domination in Triad-Convex Bipartite Graphs

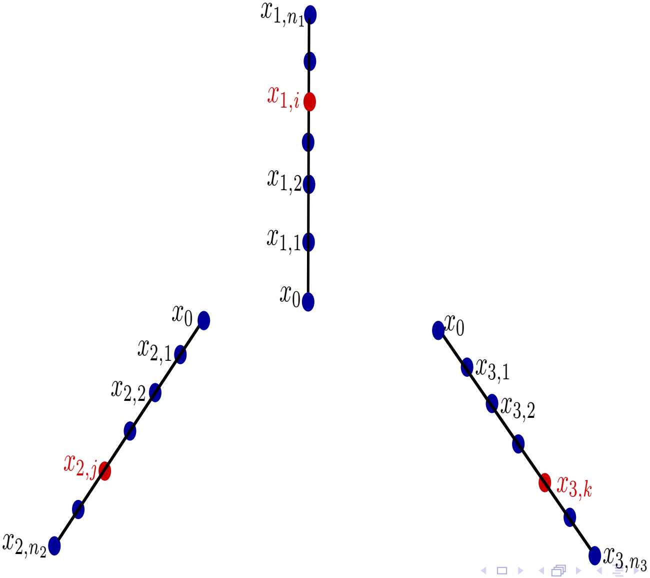
Let $G = (X, Y, E)$ be a triad-convex bipartite graph.



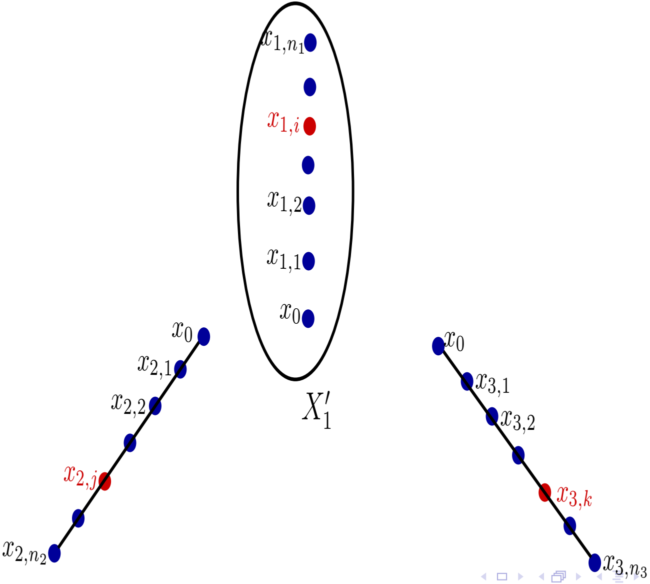
Let D be a minimum dominating set of G such that $|N_G(x_0) \cap D| \leq 3$. Then, the following cases arise:

- Case 1: $x_0 \in D$ and $N_G(x_0) \cap D = \emptyset$
- Case 2: $x_0 \in D$ and $1 \leq |N_G(x_0) \cap D| \leq 3$
- Case 3: $x_0 \notin D$ and $1 \leq |N_G(x_0) \cap D| \leq 3$

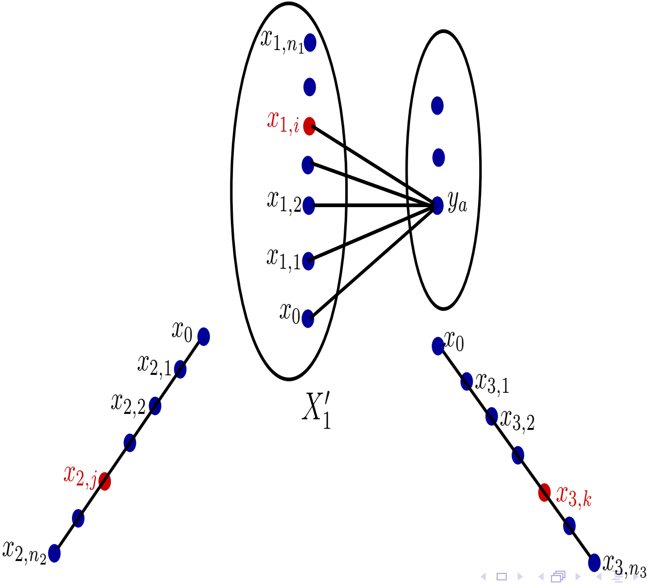
Case 2: $x_0 \in D$ and $1 \leq |N_G(x_0) \cap D| \leq 3$



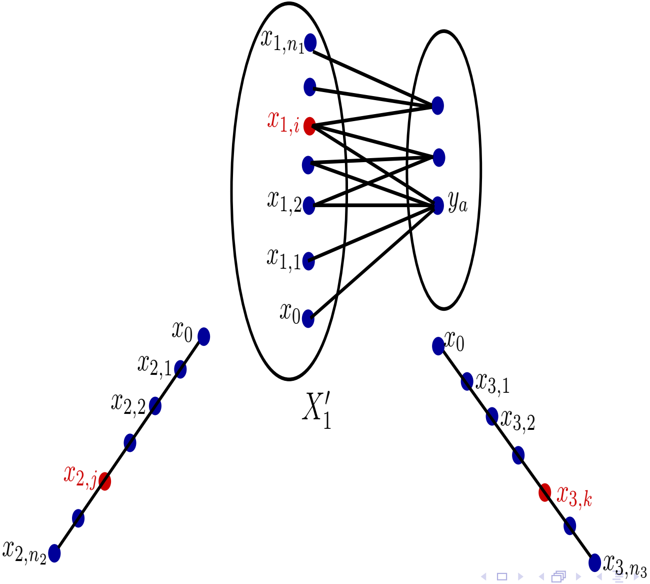
Case 2: $x_0 \in D$ and $1 \leq |N_G(x_0) \cap D| \leq 3$



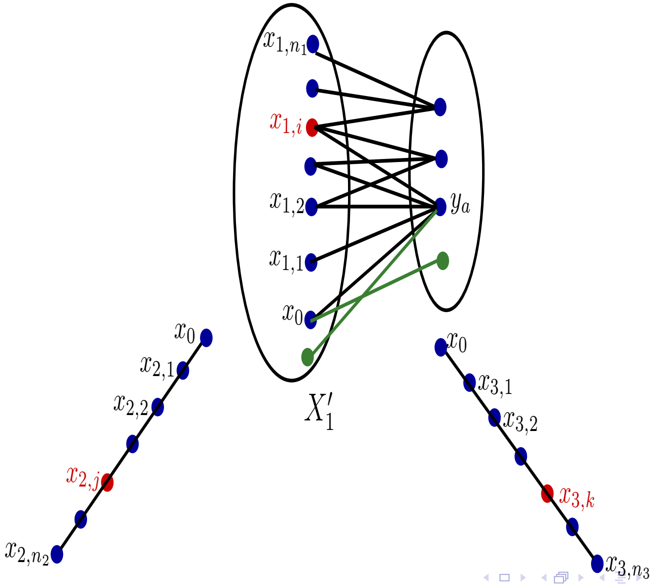
Case 2: $x_0 \in D$ and $1 \leq |N_G(x_0) \cap D| \leq 3$



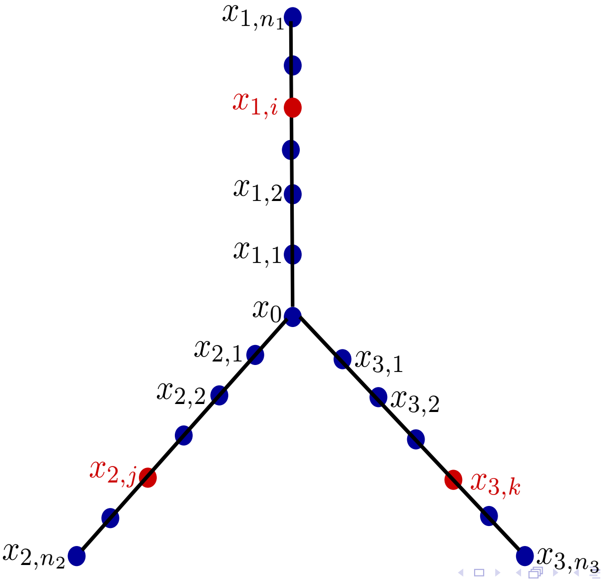
Case 2: $x_0 \in D$ and $1 \leq |N_G(x_0) \cap D| \leq 3$



Case 2: $x_0 \in D$ and $1 \leq |N_G(x_0) \cap D| \leq 3$



$Case3 : x_0 \notin D$ and $1 \leq |N_G(x_0) \cap D| \leq 3$

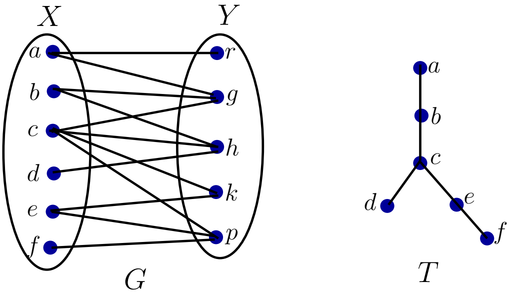


Theorem

A minimum dominating set of circular-convex bipartite graph can be computed in $O(n^8)$ *time.*

Definition

A bipartite graph $G = (X, Y, E)$ is called a tree-convex bipartite graph, if a tree $T = (X, E_X)$ can be defined on the vertices of X, such that for every vertex y in Y, the neighborhood of y induces a subtree of T.

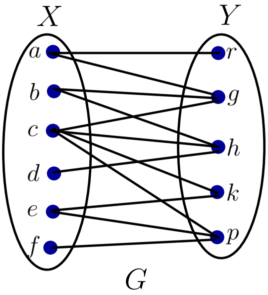


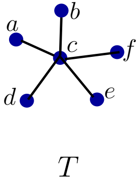
Definition

For T a star, G is called star-convex bipartite graph.

Definition

A bipartite graph $G = (X, Y, E)$ is called a tree-convex bipartite graph, if a tree $T = (X, E_X)$ can be defined on the vertices of X, such that for every vertex y in Y, the neighborhood of y induces a subtree of T.





Definition

For T a star, G is called star-convex bipartite graph.

Star-convex Bipartite Graphs

Lemma

A bipartite graph $G = (X, Y, E)$ is a star-convex bipartite graph if and only if there exists a vertex x in X such that every vertex y in Y is either a pendant vertex or is adjacent to x.

Set Cover Problem

- Let S be any non-empty set and F be a family of subsets of S. For the set system (S, F), a set $C \subseteq F$ is called a **cover** of S, if every element of S belongs to at least one element of C.
- The **Min Set Cover problem** is to find a minimum cardinality cover of S for a given set system (S, F).
- For a given positive integer k and a set system (S, F), the **Decide Set Cover problem** is to decide whether S has a cover of cardinality at most k.

Theorem

The DECIDE SET COVER *problem is NP-complete.*

Theorem

The MIN SET COVER *problem for input instance* (S, F) *does not admit a* $(1-\epsilon)\ln|S|$*-approximation algorithm for any* $\epsilon > 0$ *unless* *NP* $\subseteq$ *DTIME*$(|S|^{O(\log\log|S|)})$. *Furthermore, this inapproximability result holds for the case when the size of the input collection* F *is no more than the size of the set* S.

Reduction from Set Cover to Dominating Set

Given a set system (S, F), where $S = \{S_1, S_2, \ldots, S_p\}$ and $F = \{C_1, C_2, \ldots, C_q\}$, $q \leq p$, we construct the star-convex bipartite graph $G = (X, Y, E)$ as follows.

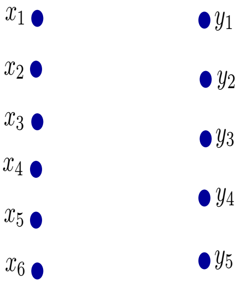
1. For each element S_i in the set S, add a vertex x_i in X.
2. For each set C_j in the collection F, add a vertex c_j in Y.
3. Add a vertex x_{p+1} in X, a vertex y_{q+1} in Y, and set of edges $\{x_{p+1}y_j \mid 1 \leq j \leq q+1\}$ in E.
4. If an element S_i belongs to set C_j, then add an edge between vertices x_i and c_j in graph G.

An example of Reduction

$$S = \{1, 2, 3, 4, 5, 6\}$$
$$F = \{C_1 = \{1, 2, 3\}, C_2 = \{2, 3, 4\}, C_3 = \{4, 5\}, C_4 = \{4, 5, 6\},$$
$$C_5 = \{3, 5, 6\}\}$$

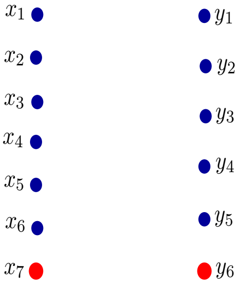
An example of Reduction

$S = \{1, 2, 3, 4, 5, 6\}$
$F = \{C_1 = \{1, 2, 3\}, C_2 = \{2, 3, 4\}, C_3 = \{4, 5\}, C_4 = \{4, 5, 6\},$
$C_5 = \{3, 5, 6\}\}$



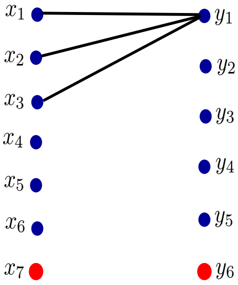
An example of Reduction

$S = \{1, 2, 3, 4, 5, 6\}$
$F = \{C_1 = \{1, 2, 3\}, C_2 = \{2, 3, 4\}, C_3 = \{4, 5\}, C_4 = \{4, 5, 6\},$
$C_5 = \{3, 5, 6\}\}$



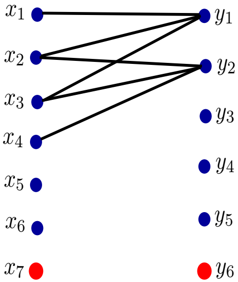
An example of Reduction

$S = \{1, 2, 3, 4, 5, 6\}$
$F = \{C_1 = \{1, 2, 3\}, C_2 = \{2, 3, 4\}, C_3 = \{4, 5\}, C_4 = \{4, 5, 6\},$
$C_5 = \{3, 5, 6\}\}$



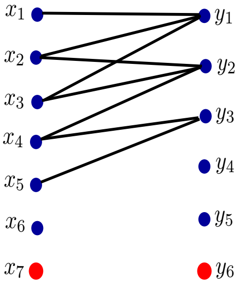
An example of Reduction

$S = \{1, 2, 3, 4, 5, 6\}$
$F = \{C_1 = \{1, 2, 3\}, C_2 = \{2, 3, 4\}, C_3 = \{4, 5\}, C_4 = \{4, 5, 6\},$
$C_5 = \{3, 5, 6\}\}$



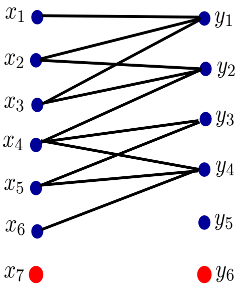
An example of Reduction

$S = \{1, 2, 3, 4, 5, 6\}$
$F = \{C_1 = \{1, 2, 3\}, C_2 = \{2, 3, 4\}, C_3 = \{4, 5\}, C_4 = \{4, 5, 6\},$
$C_5 = \{3, 5, 6\}\}$



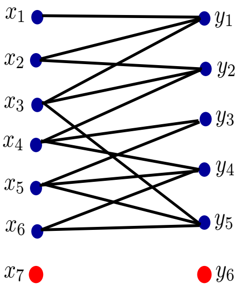
An example of Reduction

$S = \{1, 2, 3, 4, 5, 6\}$
$F = \{C_1 = \{1, 2, 3\}, C_2 = \{2, 3, 4\}, C_3 = \{4, 5\}, C_4 = \{4, 5, 6\},$
$C_5 = \{3, 5, 6\}\}$



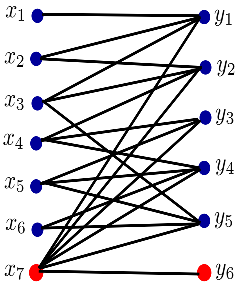
An example of Reduction

$S = \{1, 2, 3, 4, 5, 6\}$
$F = \{C_1 = \{1, 2, 3\}, C_2 = \{2, 3, 4\}, C_3 = \{4, 5\}, C_4 = \{4, 5, 6\},$
$C_5 = \{3, 5, 6\}\}$



An example of Reduction

$S = \{1, 2, 3, 4, 5, 6\}$
$F = \{C_1 = \{1, 2, 3\}, C_2 = \{2, 3, 4\}, C_3 = \{4, 5\}, C_4 = \{4, 5, 6\},$
$C_5 = \{3, 5, 6\}\}$



Claim

S has a cover of cardinality k if and only if G has a dominating set of cardinality $k + 1$.

Star-convex Bipartite Graphs: Results

Theorem

The DOMINATION DECISION *problem is NP-complete for star-convex bipartite graphs.*

Theorem

The MINIMUM DOMINATION *problem for a star-convex bipartite graph G with n vertices does not admit a $(1-\epsilon)\ln n$-approximation algorithm for any $\epsilon > 0$ unless NP $\subseteq$ DTIME$(n^{O(\log\log n)})$.*

Theorem

The MINIMUM DOMINATION *problem is linear-time solvable for bounded degree star-convex bipartite graphs.*

Star-convex Bipartite Graphs: Results

Theorem

The DOMINATION DECISION *problem is NP-complete for star-convex bipartite graphs.*

Theorem

The MINIMUM DOMINATION *problem for a star-convex bipartite graph G with n vertices does not admit a $(1-\epsilon)\ln n$-approximation algorithm for any $\epsilon > 0$ unless NP $\subseteq$ DTIME$(n^{O(\log \log n)})$.*

Theorem

The MINIMUM DOMINATION *problem is linear-time solvable for bounded degree star-convex bipartite graphs.*

Star-convex Bipartite Graphs: Results

Theorem

The DOMINATION DECISION *problem is NP-complete for star-convex bipartite graphs.*

Theorem

The MINIMUM DOMINATION *problem for a star-convex bipartite graph G with n vertices does not admit a $(1-\epsilon)\ln n$-approximation algorithm for any $\epsilon > 0$ unless NP $\subseteq$ DTIME$(n^{O(\log \log n)})$.*

Theorem

The MINIMUM DOMINATION *problem is linear-time solvable for bounded degree star-convex bipartite graphs.*

Future Aspects

- To propose algorithms with better time complexity for the MINIMUM DOMINATION problem for circular-convex bipartite graphs and triad-convex bipartite graphs.
- To study other interesting problems which are NP-complete for bipartite graphs, but polynomial time solvable for convex bipartite graphs.

Future Aspects

- To propose algorithms with better time complexity for the MINIMUM DOMINATION problem for circular-convex bipartite graphs and triad-convex bipartite graphs.
- To study other interesting problems which are NP-complete for bipartite graphs, but polynomial time solvable for convex bipartite graphs.

References

[1] Berge, C.: *Theory of Graphs and its Applications*. Methuen, London, 1962.

[2] Bertossi, A.A.: Dominating set for split and bipartite graphs, Inform. Process. Lett. 19 (1984) 37-40.

[3] Chlebík, M., , Chlebíková, J.: Approximation hardness of dominating set problems in bounded degree graphs, *Information and Computation*, 206 (11) (2008) 1264-1275.

[4] Garey, M.R., Johnson, D.S.: *Computers and Interactability: a guide to the theory of NP-completeness*, Freeman, New York, 1979.

[5] Ore, O.: *Theory of Graph*. American Mathematical Society, Providence, RI, 1962.

Thank you for your attention...

Questions?