Special classes of Boxicity-2 graphs

CALDAM 2015

Dibyayan Chakraborty

Indian Statistical Institute,Kolkata

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** Joint work with Sujoy K. Bhore, Sandip Das, Sagnik Sen

Motivation

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Rectangle intersection graphs

Intersection graph of axes-parallel rectangles on the plane.

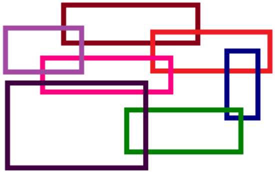

Figure: Rectangle intersection graph.

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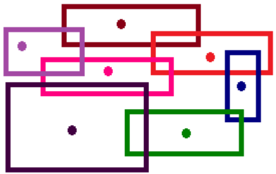

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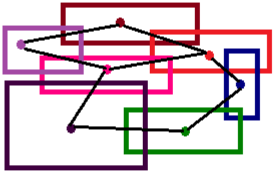

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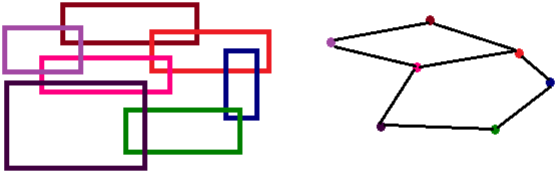

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Gap between Boxicity-2 and Boxicity-1 graphs

	Boxicity-2 graphs	**Boxicity-1 graphs (Interval graphs)**
Recognition	NP-Hard (Kratochvil, 1994)	Linear (McConnel et al., 2000, Booth et al. 1976)
Coloring	NP-Hard (Gulombic et al., 2012)	Polynomial (Stacho 2010, Garey et al., 1980)
Clique Number	Polynomial (Asano et al., 1983)	Polynomial
Clique cover	NP-hard (Asano et al., 1983)	Linear (Hsu et al., 1991)
Independent set	NP-hard (Kratochvil et al., 1990)	Polynomial (Gavril, 1974)

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Objective-1:

Generalisation of interval graphs which have boxicity 2.

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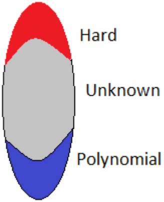

Figure: Can we reduce the grey area?

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Hardness of partitioning

Partitioning a graph into unit interval graphs is NP-Hard [Farrugia, 2004].

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For which subclasses of boxicity-2 graphs, partitioning is polynomial time solvable?

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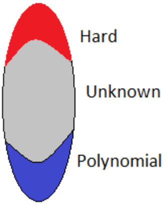

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Definitions

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2SIG

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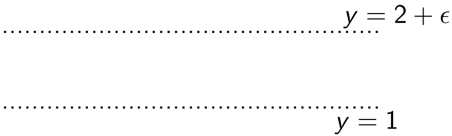
2SIG

- Two stab lines in $1 + \epsilon$ distance apart.
-
-



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2SIG

- Two stab lines in $1+\epsilon$ distance apart.
- Axes-parallel rectangles with unit height.
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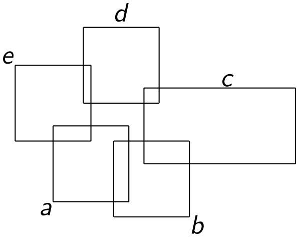

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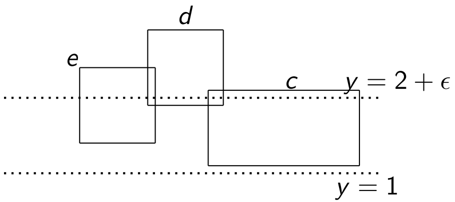
- Two stab lines in $1 + \epsilon$ distance apart.
- Axes-parallel rectangles with unit height.
- Each rectangle intersects exactly one stab line.



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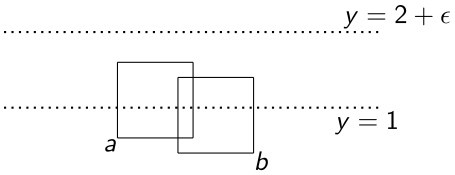

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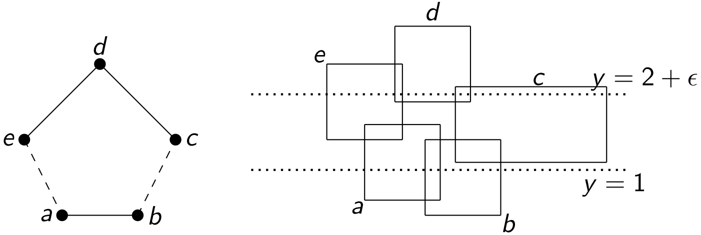

Figure: A representation (right) of a 2*SIG* graph (left).

Definitions

2SUIG

- Two stab lines in $1 + \epsilon$ distance apart.
- Axes-parallel unit squares.
- Each unit squares intersects exactly one stab line.

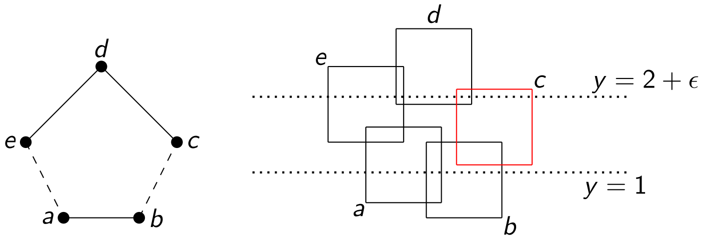

Figure: A representation (right) of a $2SUIG$ graph (left).

Our Contributions

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Main results

- Given the partition we can list all the maximal cliques of a 2SUIG in polynomial time.
  Hence, the clique number can be computed in polynomial time.
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✓ Clique number = order of the biggest complete subgraph

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✓ Clique number = order of the biggest complete subgraph

✓ To understand the properties of the cliques without using the intersection model.

Proof (sketch)

Theorem

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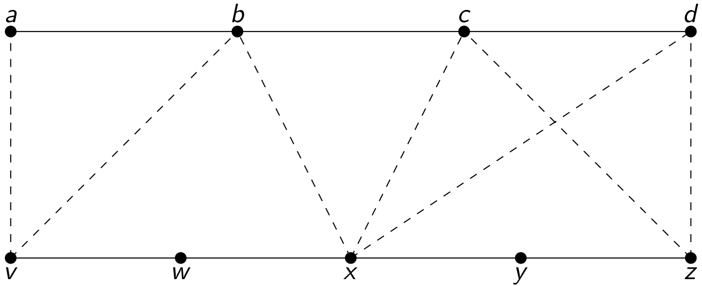

Figure: A labeling scheme of the bridge edges: a useful tool.

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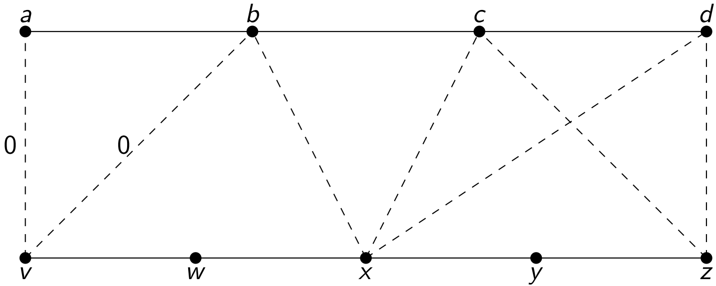

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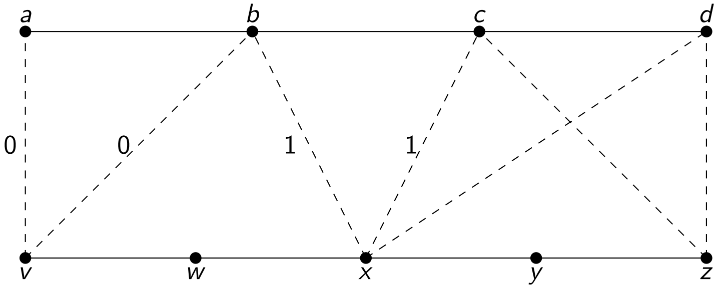

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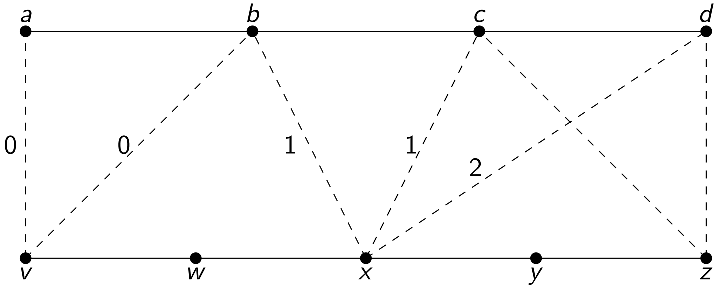

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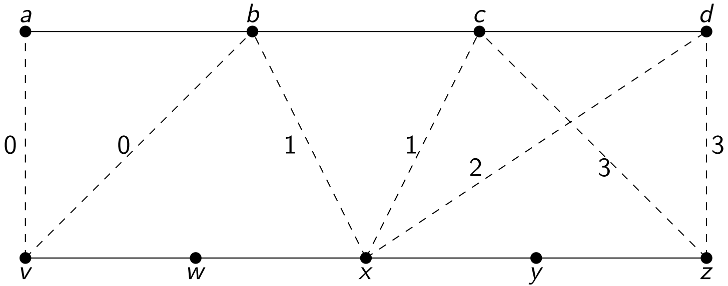

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Lemma

Bridge edges with different labels are not part of the same maximal clique in a 2SUIG.

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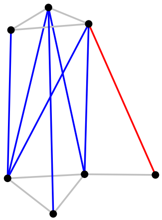

Figure: Set containing edges with different labels.

Proof (sketch)

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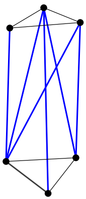

Figure: Edges with same label creates maximal cliques.

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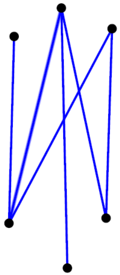

Figure: Bipartite graph created by the endpoints in different partition.

Enumerate the maximal bi-cliques using algorithm of Alexe, 2004.

Our contributions

Main results

- Given the partition we can list all the maximal cliques of a 2SUIG in polynomial time.
  Hence, the clique number can be computed in polynomial time.
- For any 2SIG graph G we have $\chi(G) \leq 2\omega(G)$.
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✓ Bridge-triangle-free = no triangle having a bridge edge (for some partition)

About bridge-triangle-free 2SUIG graphs

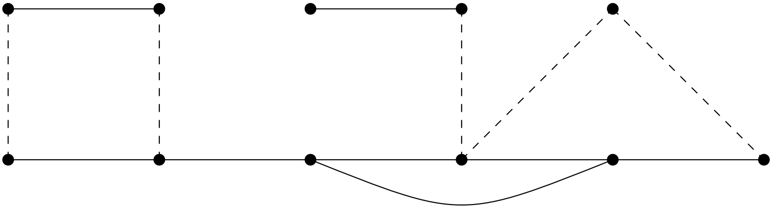

Figure: A bridge-triangle-free 2SUIG graph.

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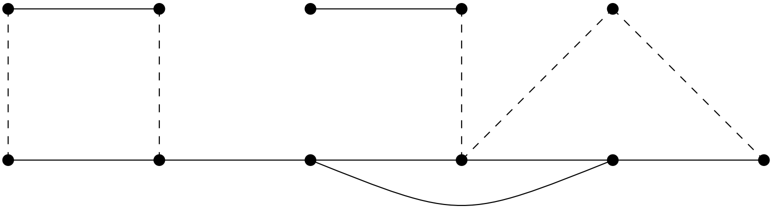

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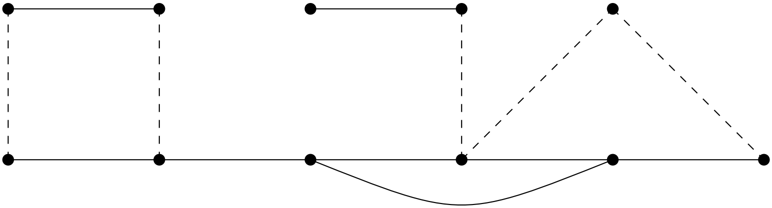

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✓ "Nearly perfect" graph, Gyárfás, 1987, 2013; Kostochka, 2004.

✓ Subclass of 2-interval graphs. (Colouring:- NP-Hard in general, Kratochvil et al., 2001).

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- A graph is a 2SUI^{2}G graph G if and only if it can be represented by an ISSR adjacency matrix.

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✓ 2SUI^{2}G = a 2SUIG graph with the upper stab line inducing an independent set (for some partition)

Example of a 2SUI^{2}G graph

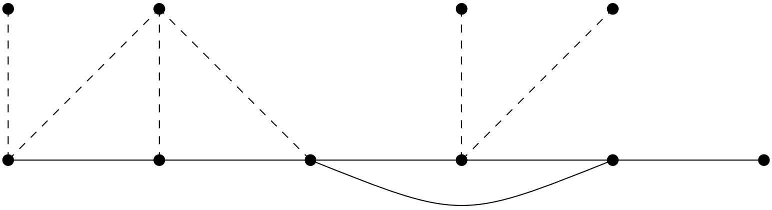

Figure: A $2SUI^2G$ graph.

Definition of ISSR matrix

$$ISSR = \left[\begin{array}{c|c} [SNIR] & [PSA] \\ \hline [PSA]^t & 0 \end{array}\right]$$

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$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure: Example of a SNIR matrix

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Conclusion with open problems

Extensions

- Chromatic number of $2SUIG$ graphs.

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- Chromatic number of $2SUIG$ graphs.
- Nice extensions of matrix characterisation are possible.

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Conjecture

Recognising 2SIG is NP hard.

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- Consider **circles** as the intersecting objects.

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- Consider **circles** as the intersecting objects.
- Consider parallel stab lines with **arbitrary slope**.
- Consider two **concentric circles** as the stabbing objects.
- Generalize the results of unit interval graphs.


THANKYOU