Higher Order Triangular Distance Delaunay Graphs

Ahmad Biniaz, Anil Maheshwari, Michiel Smid

Carleton University

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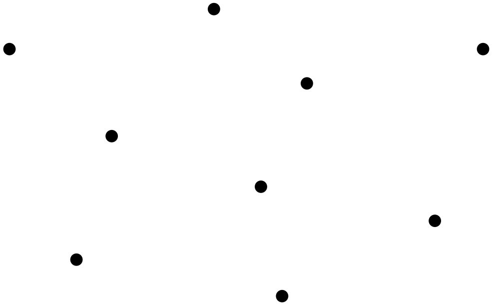
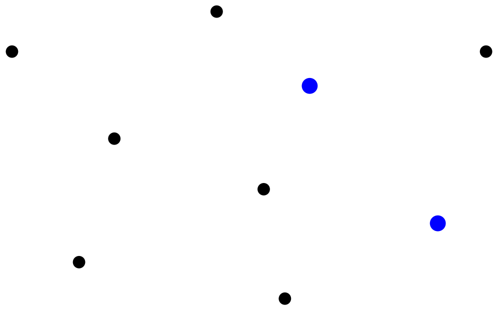
Outline

- Higher-Order Triangular-Distance Delaunay Graphs
  - Definition
  - Connectivity
  - Bottleneck Biconnected Graph
  - Bottleneck Hamiltonian Cycle
  - Bottleneck Perfect Matching
  - Maximum Matching
  - Blocking

Definition



Definition



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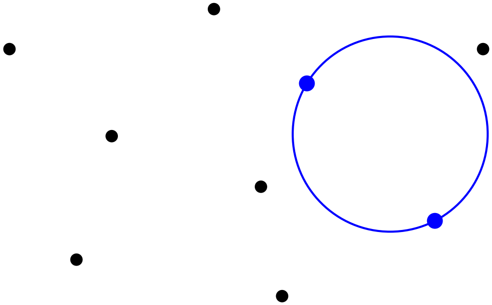

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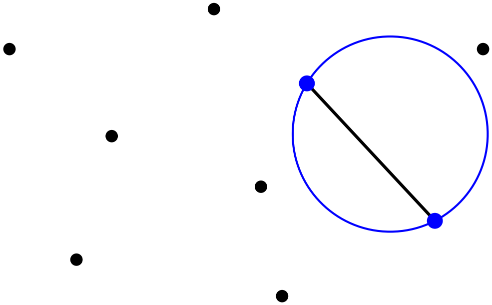

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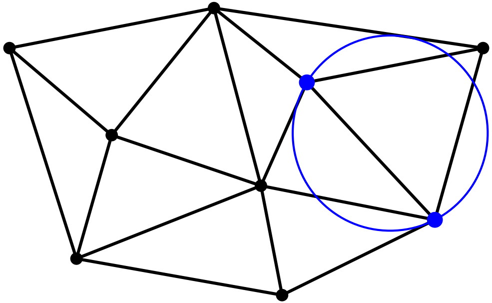

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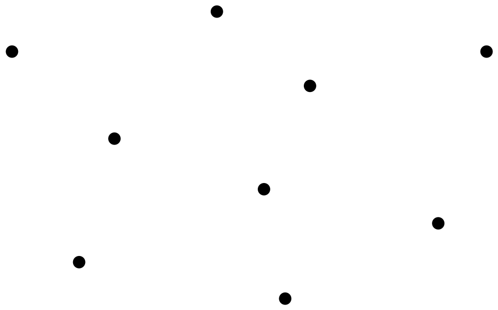

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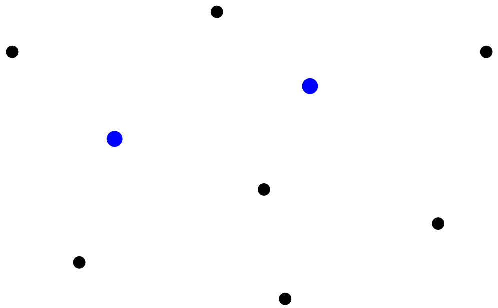

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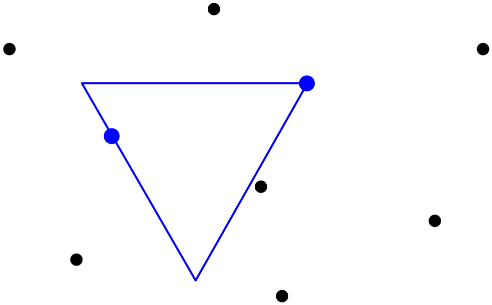

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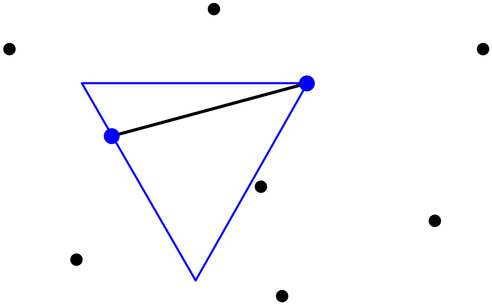

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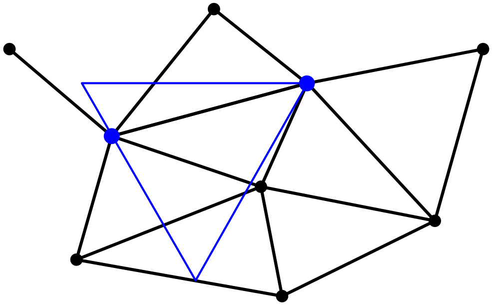

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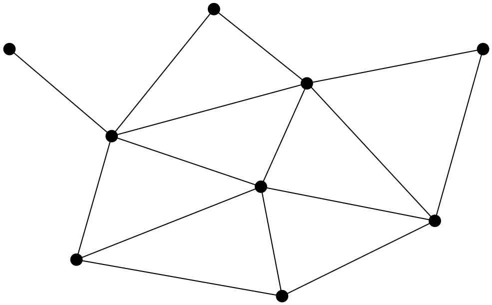

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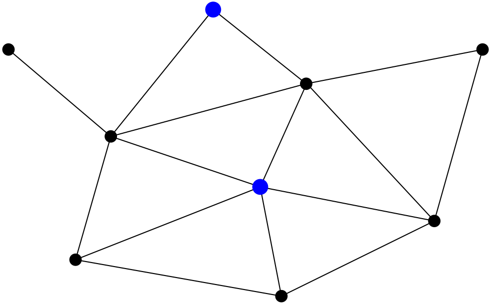

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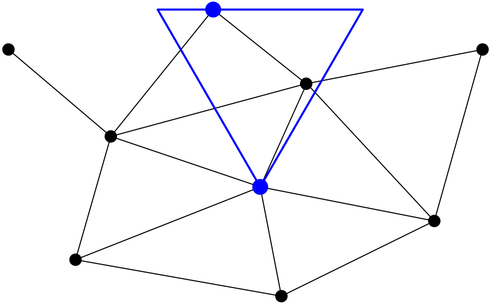

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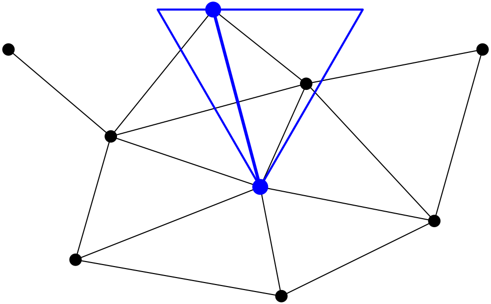

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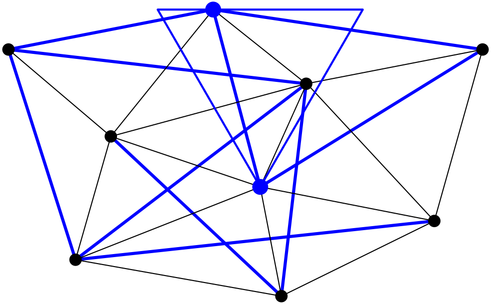

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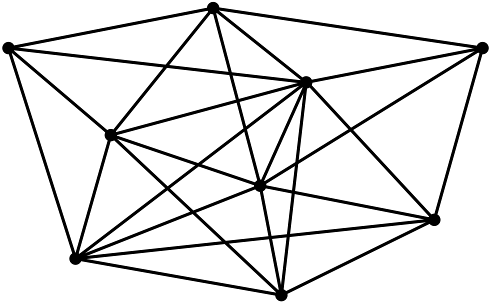
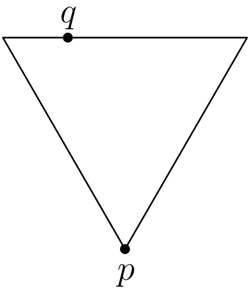


Distance (p, q)



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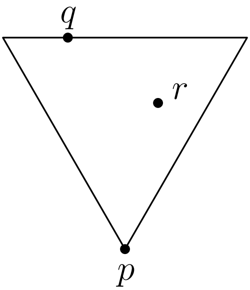

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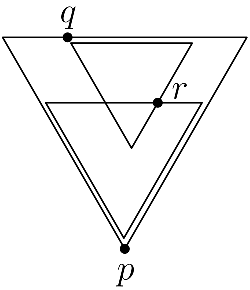

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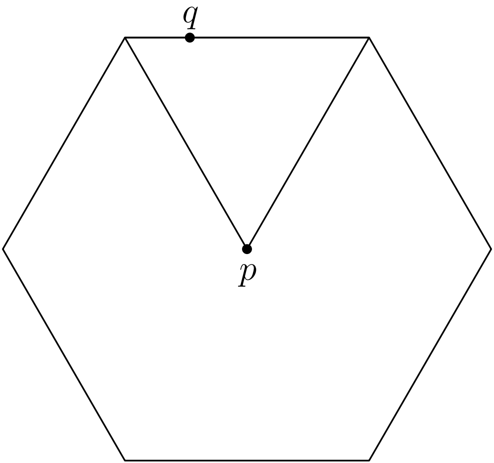

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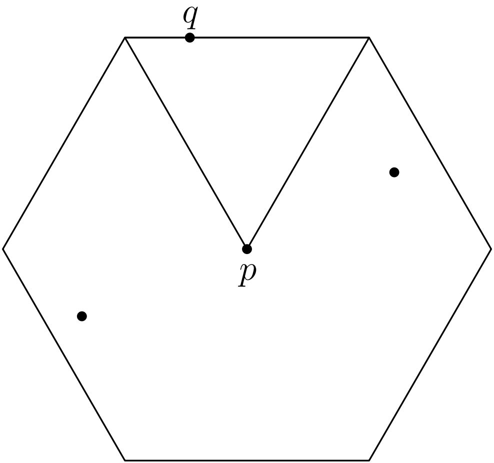

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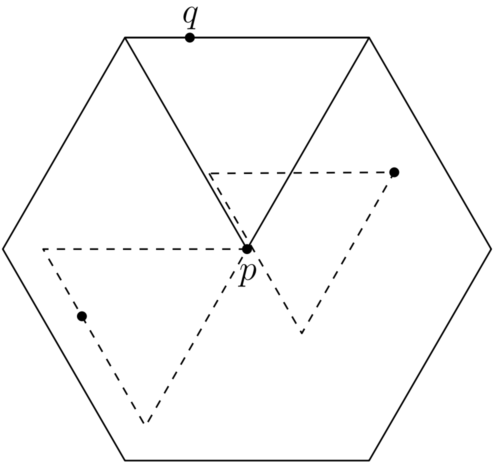
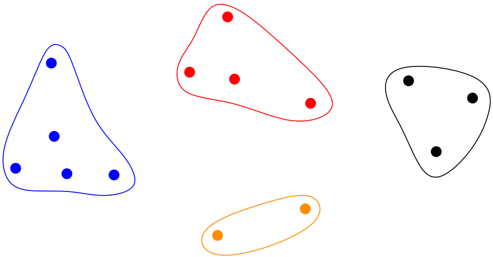
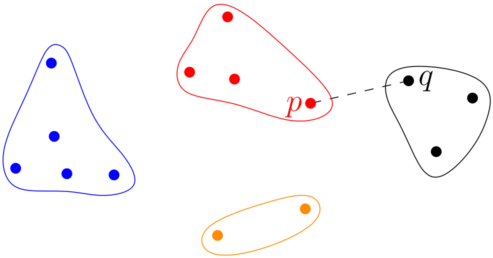


k-TD Connectivity



- k-TD is $(k+1)$-connected.

k-TD Connectivity



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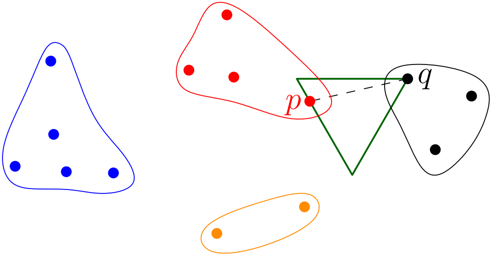

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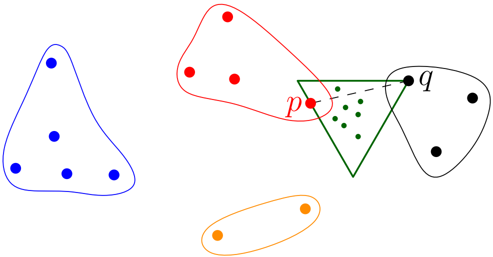

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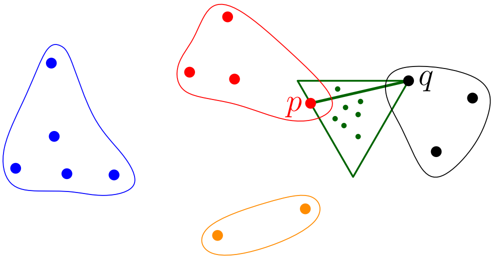
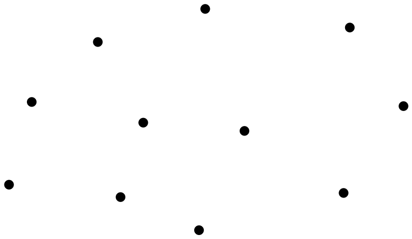
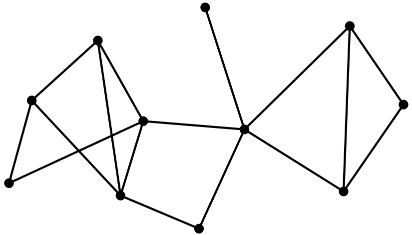


- k-TD is $(k+1)$-connected.

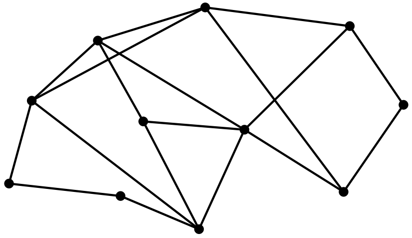
Bottleneck Biconnected Graph



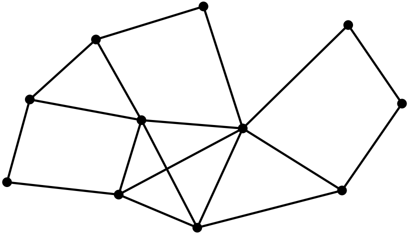
Bottleneck Biconnected Graph



Bottleneck Biconnected Graph

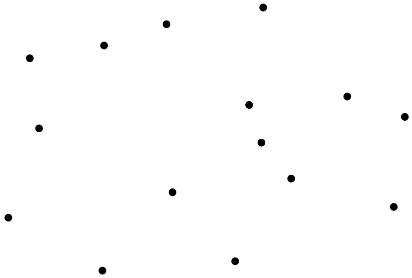


Bottleneck Biconnected Graph

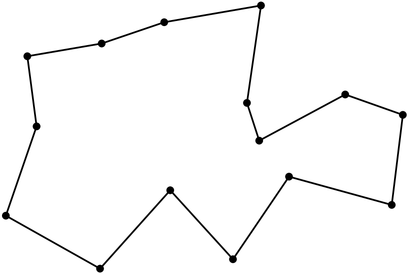


- 1-TD contains a bottleneck biconnected graph.

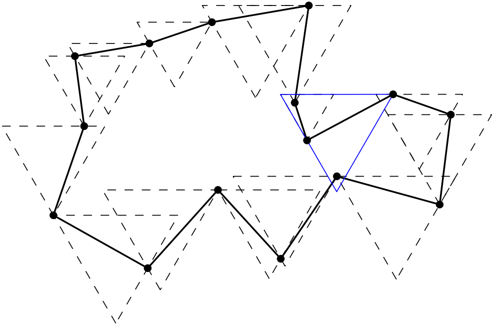
Bottleneck Hamiltonian Cycle



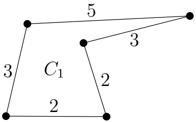
Bottleneck Hamiltonian Cycle



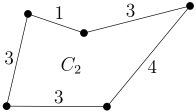
Bottleneck Hamiltonian Cycle



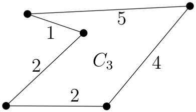
7-TD Contains a Bottleneck Hamiltonian Cycle



$ws(C_1) = 5, 3, 3, 2, 2$



$ws(C_2) = 4, 3, 3, 3, 1$

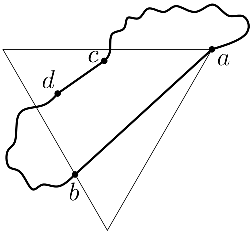


$ws(C_3) = 5, 4, 2, 2, 1$

$ws(C_2) < ws(C_1) < ws(C_3)$

- We show that a minimal Hamiltonian cycle is in 7-TD.
- We prove that all edges of a minimal cycle are in 7-TD.

7-TD Contains a Bottleneck Hamiltonian Cycle



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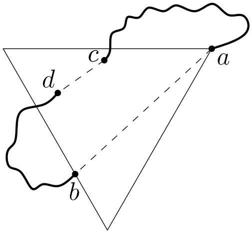

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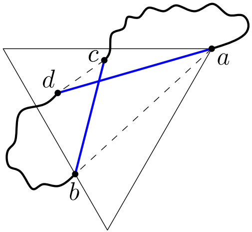

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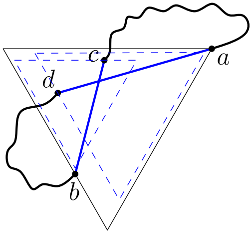

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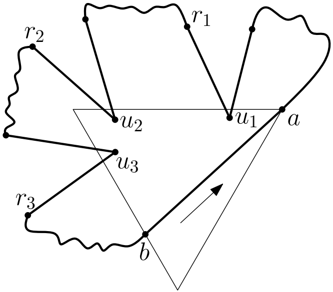

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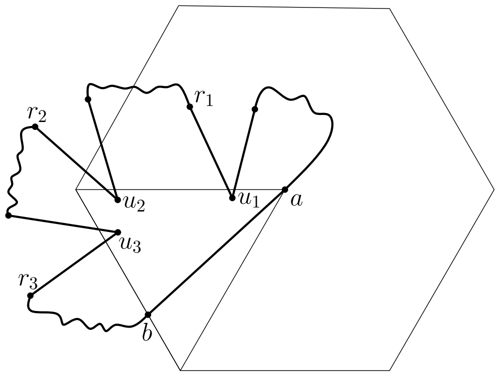

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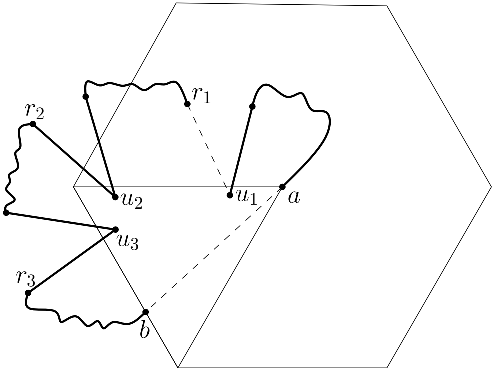

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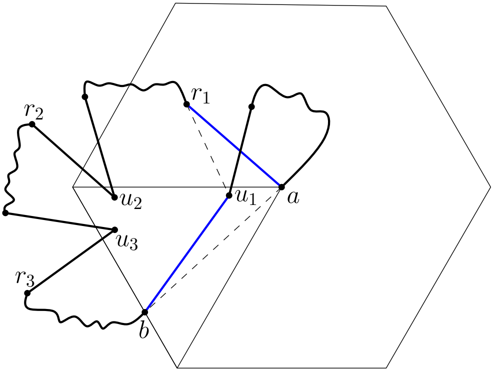

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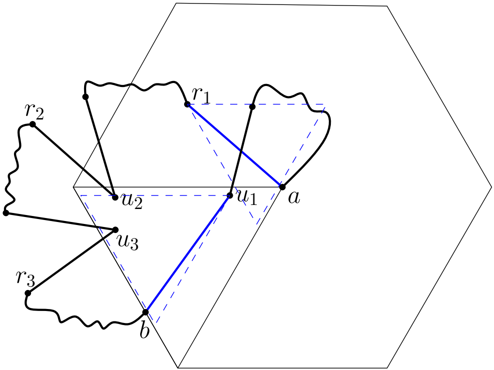

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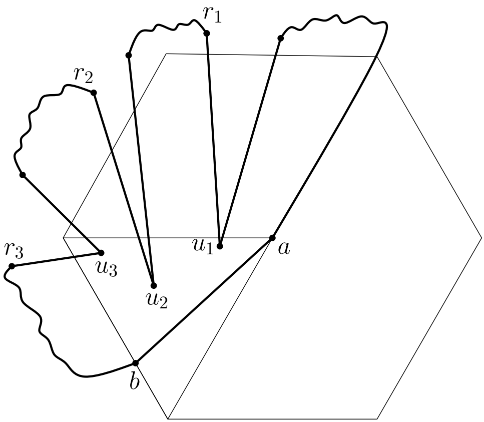

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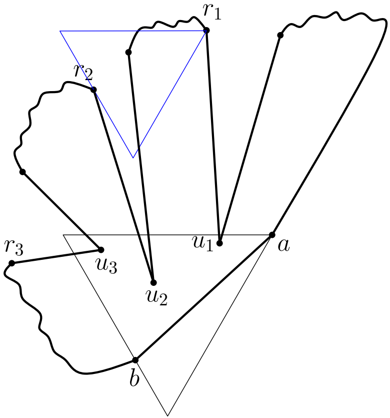

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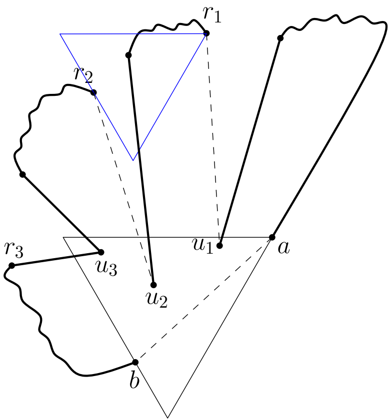

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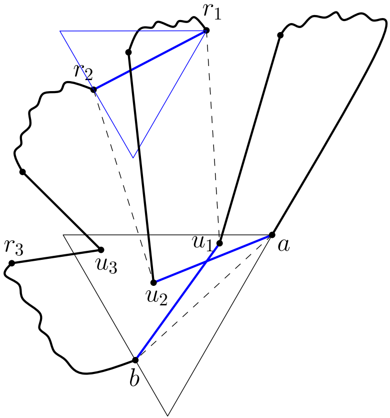

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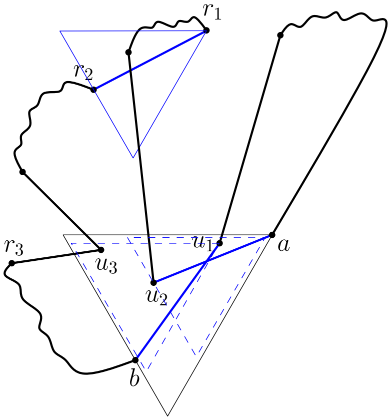

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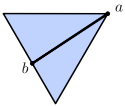
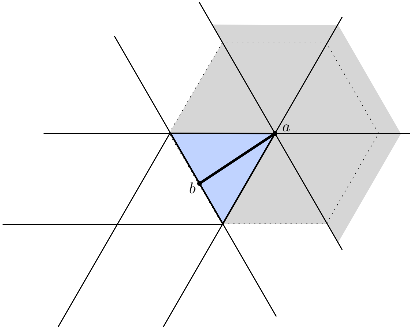


- 7-TD contains a bottleneck Hamiltonian cycle.

7-TD Contains a Bottleneck Hamiltonian Cycle



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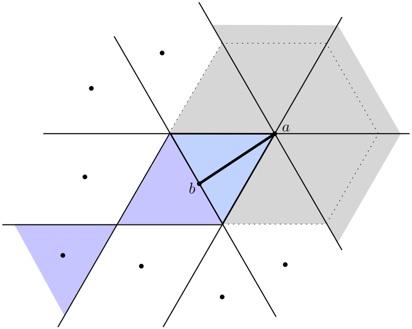
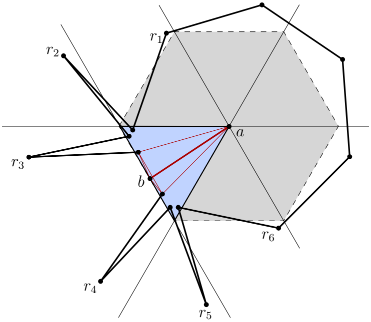


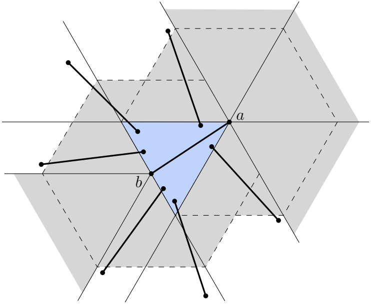
- 7-TD contains a bottleneck Hamiltonian cycle.

Lower Bound



- 5-TD may not contain any bottleneck Hamiltonian cycle.
- 6-TD? (open problem)

Bottleneck Perfect Matching



- 6-TD contains a bottleneck perfect matching.
- 5-TD may not contain any bottleneck perfect matching.

Tutte-Berge Formula for Maximum Matching

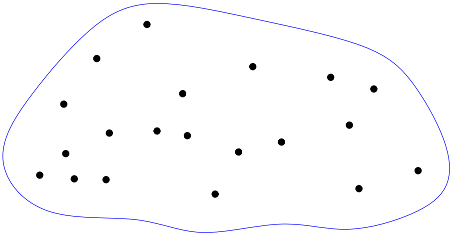
- The size of a maximum matching in G is $\frac{1}{2}(n - d(G))$, where $d(G) = \max\{o(G - S) - |S|\}$, for all $S \subset V(G)$.

Tutte-Berge Formula for Maximum Matching

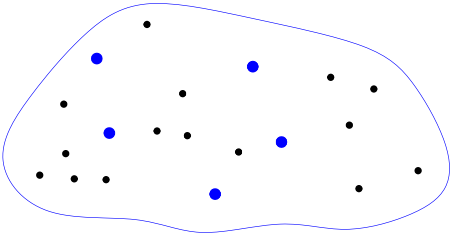
- The size of a maximum matching in G is $\frac{1}{2}(n - d(G))$, where $d(G) = \max\{o(G - S) - |S|\}$, for all $S \subset V(G)$.

- We remove a set S from TD and present an upper bound on the number of the resulting components.

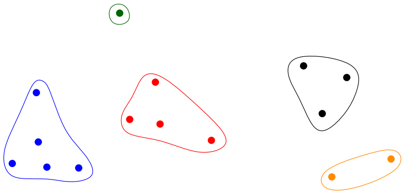
Remove a set S from TD



Remove a set S from TD

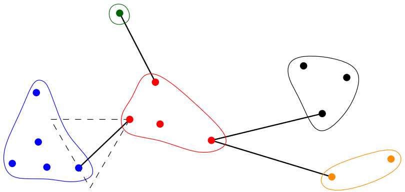


MST of TD minus S



- m: the number of components after removing vertices in S.

MST of TD minus S



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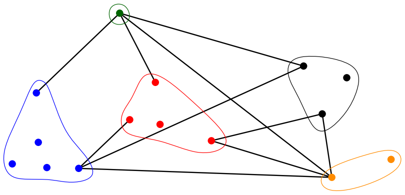

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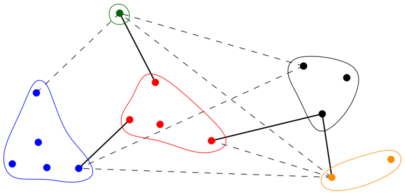
MST of TD minus S



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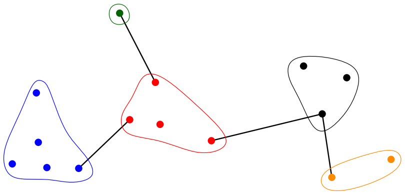
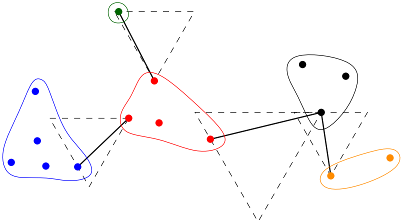
MST of TD minus S



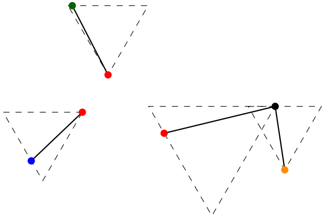
- m: the number of components after removing vertices in S.
- $|MST| = m - 1$

MST of TD minus S



- m: the number of components after removing vertices in S.
- $|MST| = m - 1$, and every triangle in MST is empty.

MST of TD minus S



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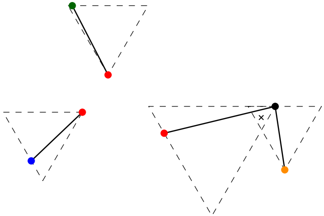
- $|MST| = m - 1$, and every triangle in MST is empty.

MST of *TD* minus *S*



- m: the number of components after removing vertices in S.
- $|MST| = m - 1$, and every triangle in MST is empty.
- Each point in S is in at most three triangles of MST.

Maximum Matching

- Each point in S is in at most three triangles of MST.

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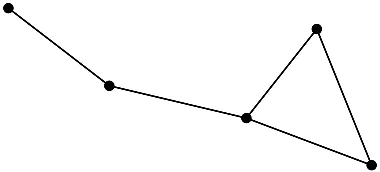
- open problem: better lower bound for 1-TD.

Blocking k-TD



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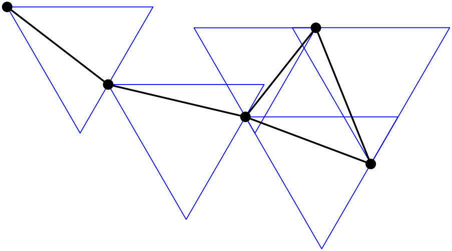

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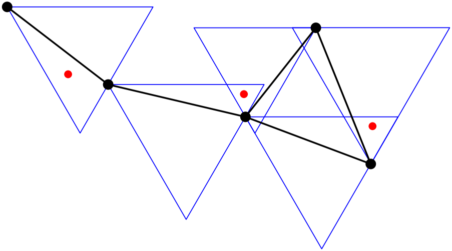

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Summary of Results

bottleneck	**TD**	**GG**	**RNG**
biconnected	1-TD	1-GG	1-RNG [Chang et al.]
matching	6-TD	17-GG	17-RNG [Chang et al.]
Hamiltonicity	7-TD	10-GG [Kaiser et al.]	19-RNG [Chang et al.]

- k-TD is $(k+1)$ connected.
- $\frac{(k+1)(n-1)}{3}$ points necessary, $(k+1)(n-1)$ points sufficient to block k-TD.

matching	**TD**	**GG**
2-order	$\frac{n}{2}$	$\frac{n}{2}$ [Biniaz et al.]
1-order	$\frac{2(n-1)}{5}$	$\frac{2(n-1)}{5}$ [Biniaz et al.]
0-order	$\frac{n-1}{3}$ [Babu et al.]	$\frac{n-1}{4}$ [Biniaz et al.]

Open problems

- What is a tight lower bound for the size of maximum matching in 1-TD?
- Does 6-TD contain a bottleneck Hamiltonian cycle?
- For which values of $k = 1, \ldots, 6$, is the graph k-TD Hamiltonian?

Thank you

Any question?

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