New Polynomial Case for Efficient Domination in P_6-free Graphs

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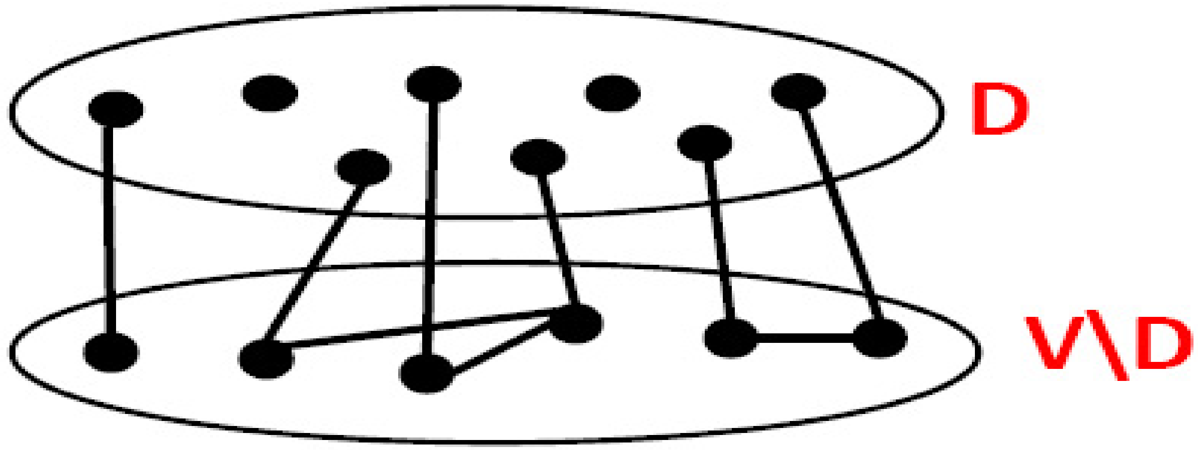
Efficient Dominating Sets in Graphs

Efficient Dominating Set (EDS):

A subset D of vertices such that D is an independent set and each vertex outside D has exactly one neighbor in D.



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A subset D of vertices such that D is an independent set and each vertex outside D has exactly one neighbor in D.

Examples

- Path P_n, for all n.
- Cycle C_n if and only of $n \equiv$ mod 3.
- Complete graph K_n, for all n.

WEIGHTED EFFICIENT DOMINATING SET (WED) Problem

If $S \subseteq V(G)$, then $w(S)$:= Total weight of vertices in S.

INSTANCE: Weighted graph (G, w), a positive integer k.

QUESTION: Does there exists an efficient dominating set D of G such that $w(D) \leq k$?

WED vs Graph Classes

- WED remains NP-complete on restricted classes of graphs such as
  - bipartite graphs [31]
  - planar bipartite graphs [25]
  - chordal bipartite graphs [25]
  - chordal graphs [31]
  - etc.,

WED vs Graph Classes

- WED is solvable in polynomial time for the graph classes such as
  - trees [2, 14]
  - co-comparability graphs [8, 11]
  - split graphs [9]
  - interval graphs [10, 11]
  - circular-arc graphs [9, 19]
  - permutation graphs [20]
  - trapezoid graphs [20], etc.,

Graphs defined by forbidden induced subgraphs

Let $\mathcal{F} = \{H_1, H_2, H_3, \ldots\}$ be a family of graphs.

A graph G is said to be **$\mathcal{F}$-free** if no induced subgraph of G is isomorphic to H_i, for every i.

Examples

Bipartite graphs : C_{2k+1}-free, $k \geq 1$ (König's Theorem).

Chordal graphs : C_k-free, $k \geq 4$.

Line graphs : A family of 9 forbidden subgraphs (Beineke).

Perfect graphs [1] : $\{C_{2k+1}, C_{2k+1}^c\}$-free, $k \geq 2$ (SPGT [12]).

[1] Graph G with $\chi(H) = \omega(H), \forall\ H \sqsubseteq G$.

WED vs H-free Graphs, H is a path

- Corneil et al. [13]: WED on P_4-free graphs (or cographs) can be solved in polynomial time.

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- Smart and Slater [29]: WED remains NP-complete for P_7-free graphs.

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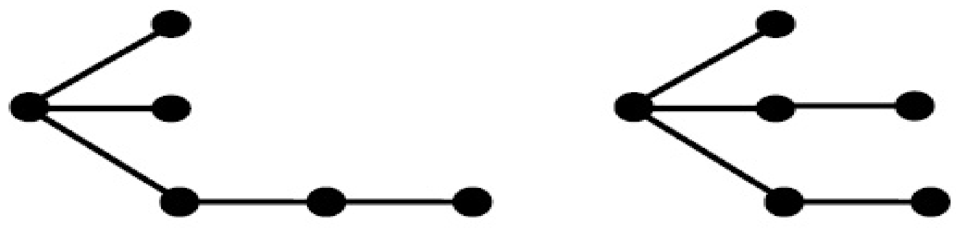
- Smart and Slater [29]: WED remains NP-complete for P_7-free graphs.
- The complexity of WED is unknown for P_6-free graphs.

WED vs P_6-free Graphs

- The complexity of WED is unknown for P_6-free graphs.
- WED is shown to be solvable in polynomial time for the following subclasses of P_6-free graphs:
  - $(P_6, S_{1,1,3})$-free graphs [16]
  - $(P_6, S_{1,2,2})$-free graphs [4]
  - (P_6, bull)-free graphs [16]

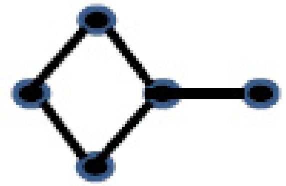


$S_{1,1,3}$ $S_{1,2,2}$

WED for P_6-free Graphs

In this talk, we prove the following:

WED can be solved in polynomial time for

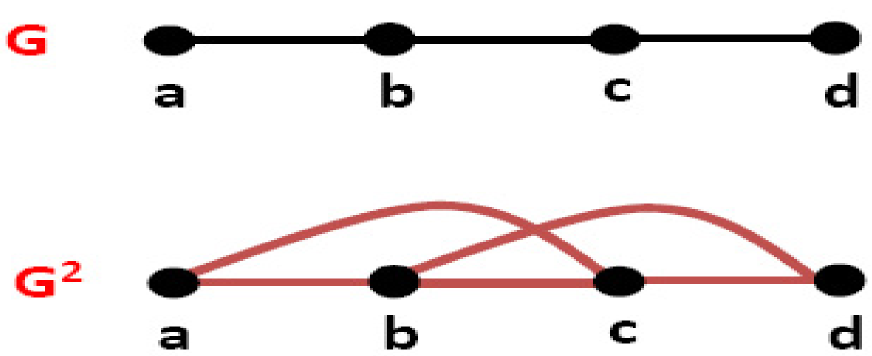
(P_6, )-free graphs

Square Graph G^2 from G

Given graph $G = (V, E)$.

Square graph $G^2 = (V, E')$,
where $E' = \{uv | u, v \in V$ and $\text{dist}_G(u, v) \in \{1, 2\}\}$.

Example



Brandstädt, Fičur, Leitert, and Milanič

Information Processing Letters, 115 (2015) 256-262

Theorem 1:

Let $\mathcal{G}$ be a graph class for which the Maximum Weight Independent Set problem is solvable in time $T(|G|)$ on squares of graphs from $\mathcal{G}$. Then, WED problem is solvable on graphs in $\mathcal{G}$ in time $O(\min\{nm + n, n^\omega\} + T(|G^2|)$, where $\omega < 2.3727$ is the matrix multiplication exponent [33].

P_5-free Graphs

WED can be solved in polynomial time [26]

- Milanič [26]: Let G be a P_5-free graph. If G has an EDS, then G^2 is P_4-free.

- Corneil et al. [13]: MWIS can be solved in linear-time for P_4-free graphs.

- So, the results follows by Theorem 1.

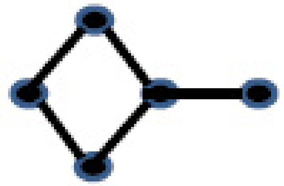
($P_6, S_{1,1,3}$)-free Graphs

WED can be solved in polynomial time [16]

- Karthick [16]: Let G be a ($P_6, S_{1,1,3}$)-free graph. If G has an EDS, then G^2 is P_5-free.

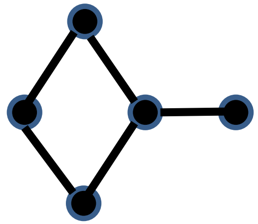
- Lokshantov et al. [22]: MWIS can be solved in polynomial time for P_5-free graphs.

- So, the result follows by Theorem 1.

(P_6, )-free Graphs

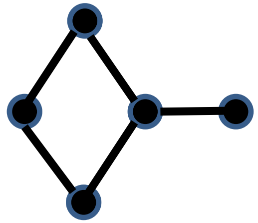
WED can be solved in polynomial time

- Let G be a (P_6, banner)-free graph. If G has an EDS, then G^2 is (P_6, banner)-free.

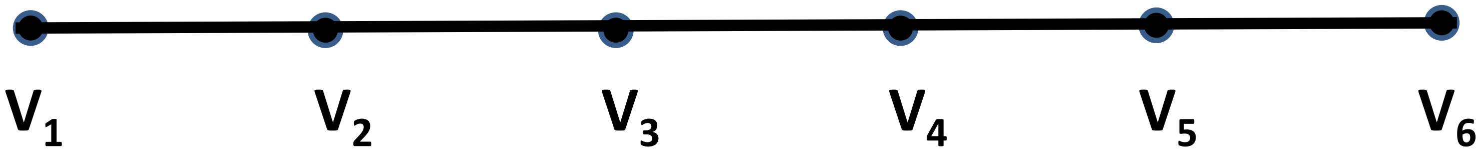
G:= $(P_6$, )-free

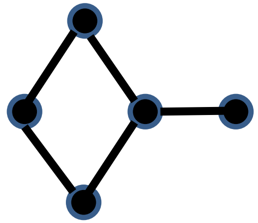
Suppose that G contains an EDS, say D

Claim : G^2 is P_6-free

G:= $(P_6$, )-free

Assume the contrary that G^2 contains an induced P_6



G:= (P_6, )-free

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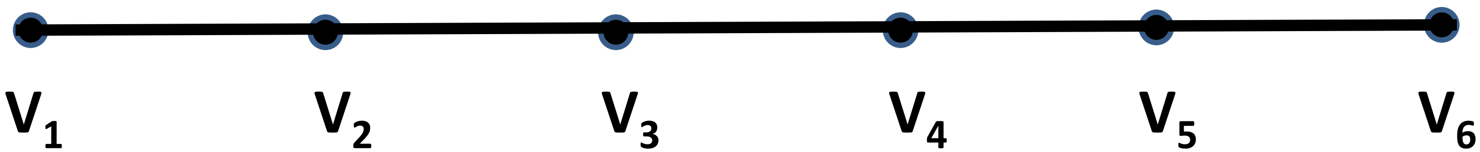
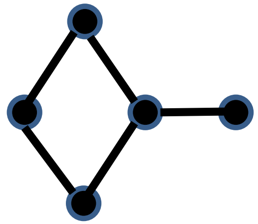
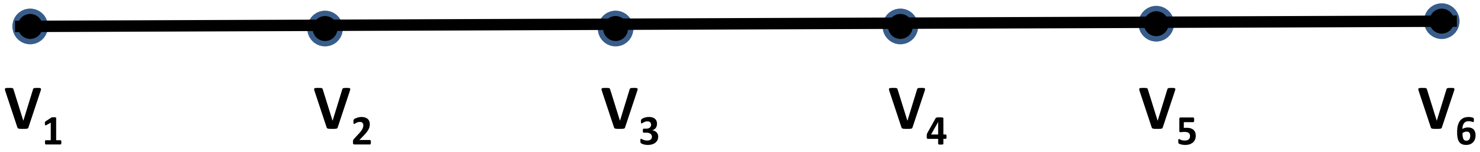


$dist_G(v_1, v_2) \le 2$, $dist_G(v_2, v_3) \le 2$, $dist_G(v_3, v_4) \le 2$

$dist_G(v_4, v_5) \le 2$, $dist_G(v_5, v_6) \le 2$

$G := (P_6,$  $)$-free

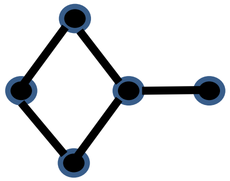


$dist_G(v_1, v_3) \geq 3$, $dist_G(v_1, v_4) \geq 3$, $dist_G(v_1, v_5) \geq 3$

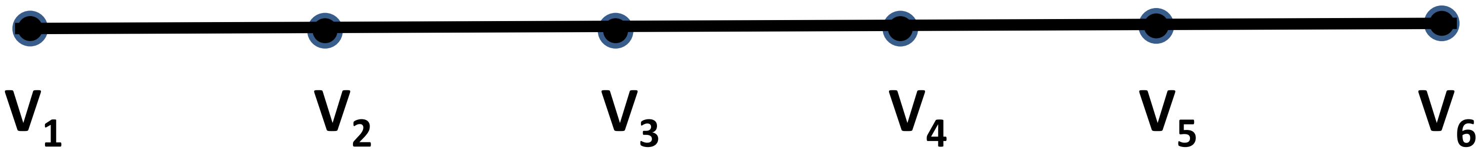
$dist_G(v_1, v_6) \geq 3$, $dist_G(v_2, v_4) \geq 3$, $dist_G(v_2, v_5) \geq 3$

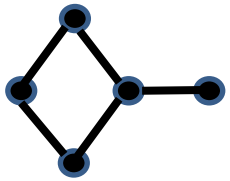
$dist_G(v_2, v_6) \geq 3$, $dist_G(v_3, v_5) \geq 3$, $dist_G(v_3, v_6) \geq 3$

$dist_G(v_4, v_6) \geq 3$

$G := (P_6,$  $)$-free

Case 1: $dist_G(v_5, v_6) = 1$



G:= (P_6 , )-free

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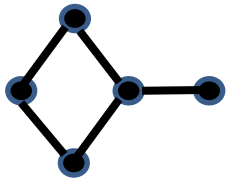
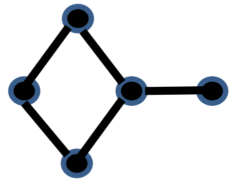
$dist_G(v_4, v_6) \geq 3$

$G := (P_6$, )-free

Case 1: $dist_G(v_5, v_6) = 1$

Case 1.1: $dist_G(v_3, v_4) = 1$

$dist_G(v_4, v_6) \geq 3$

G:= (P_6, )-free

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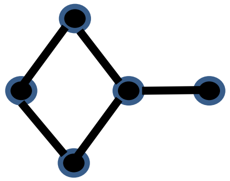
G:= (P_6 , )-free

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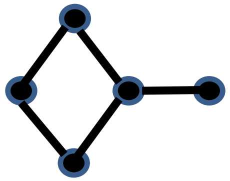
⇒ a contradiction

$G := (P_6$, )-free

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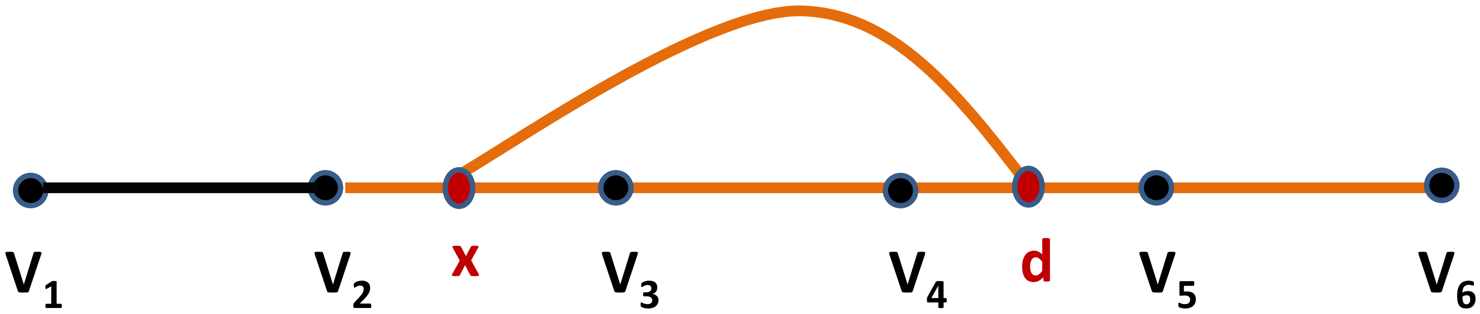

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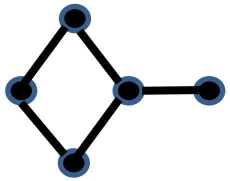
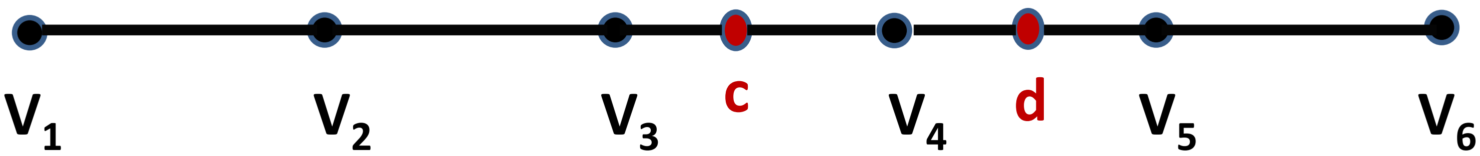
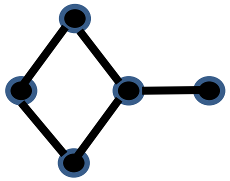
G:= (P_6, )-free

Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$



Then $cd \in E$

$G := (P_6$,  $)$-free

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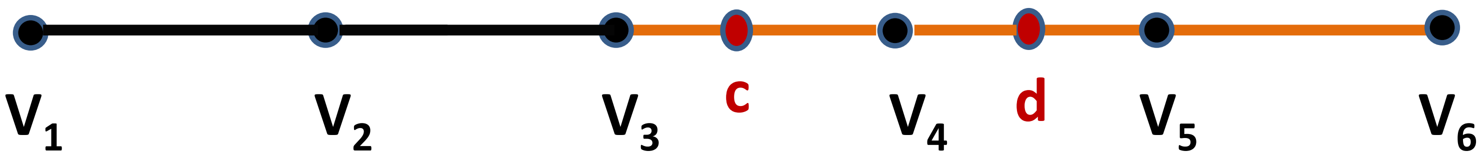

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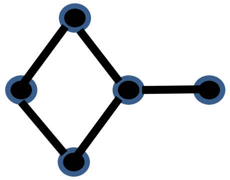
G:= (P_6, )-free

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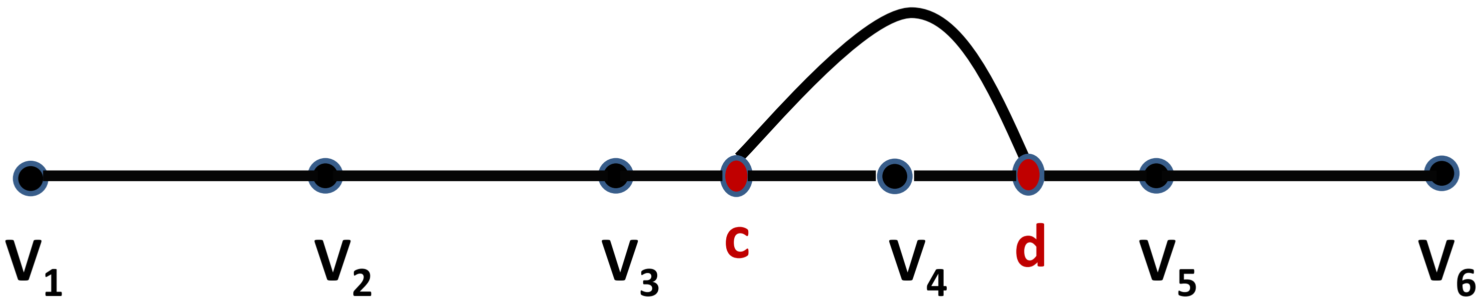

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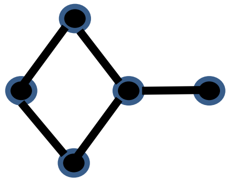
G:= (P_6, )-free

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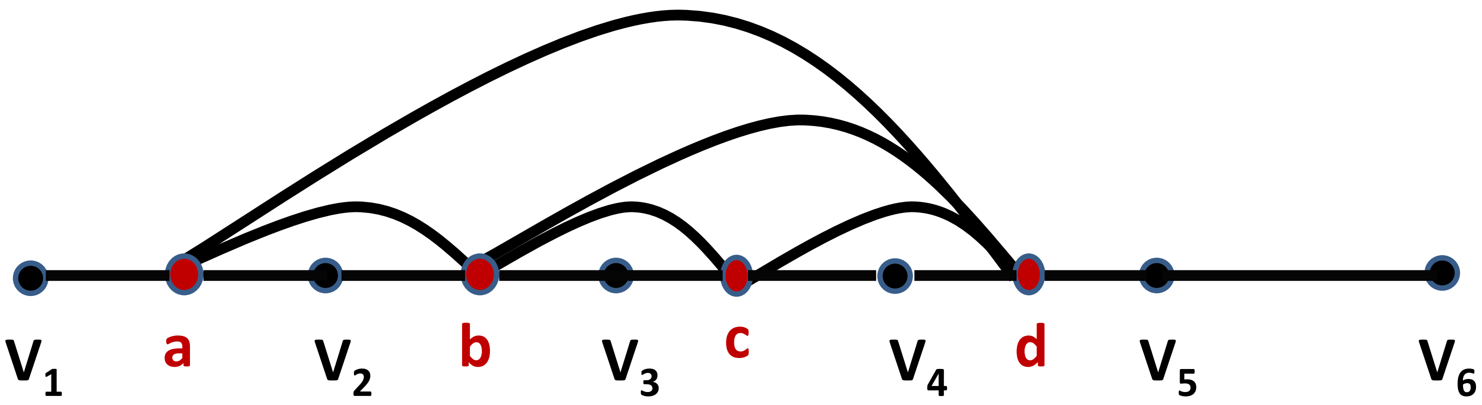
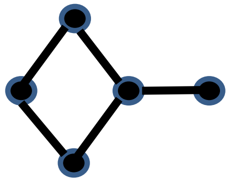


$G := (P_6,$  $)$-free

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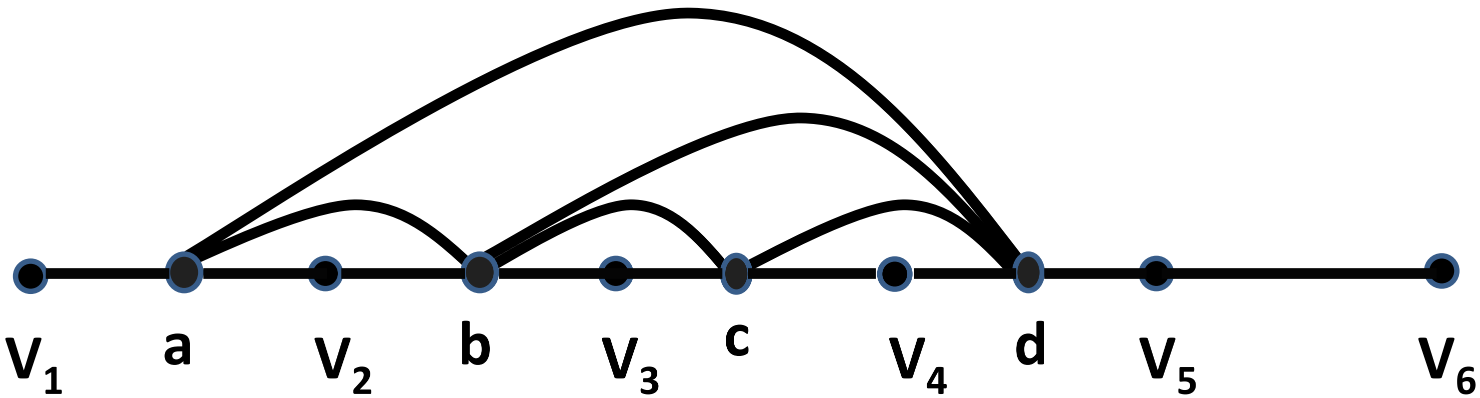
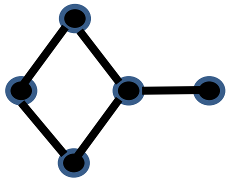


Partial Structure of G

G:= (P_6 , )-free

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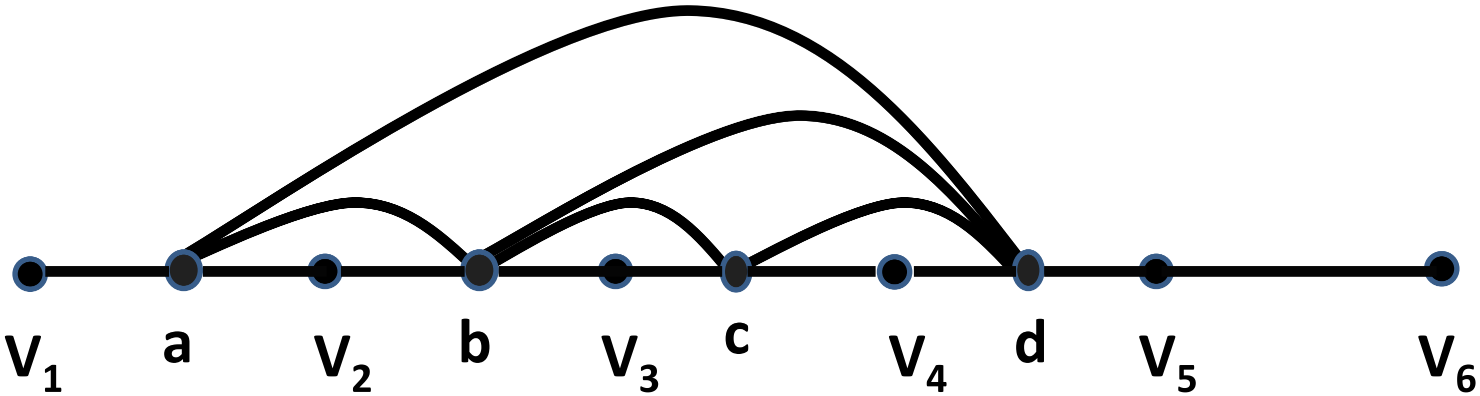
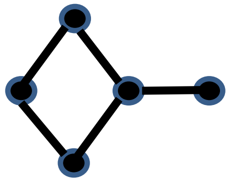


Claim 1: a, v_2, b $\notin$ D

G:= (P_6, )-free

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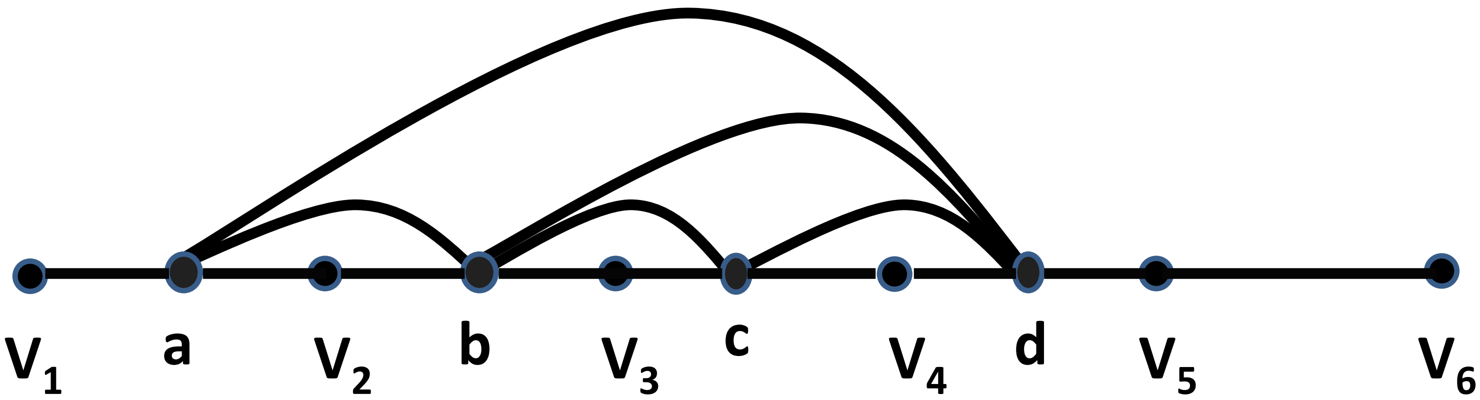
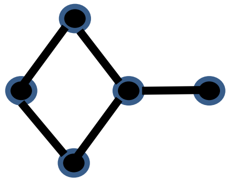


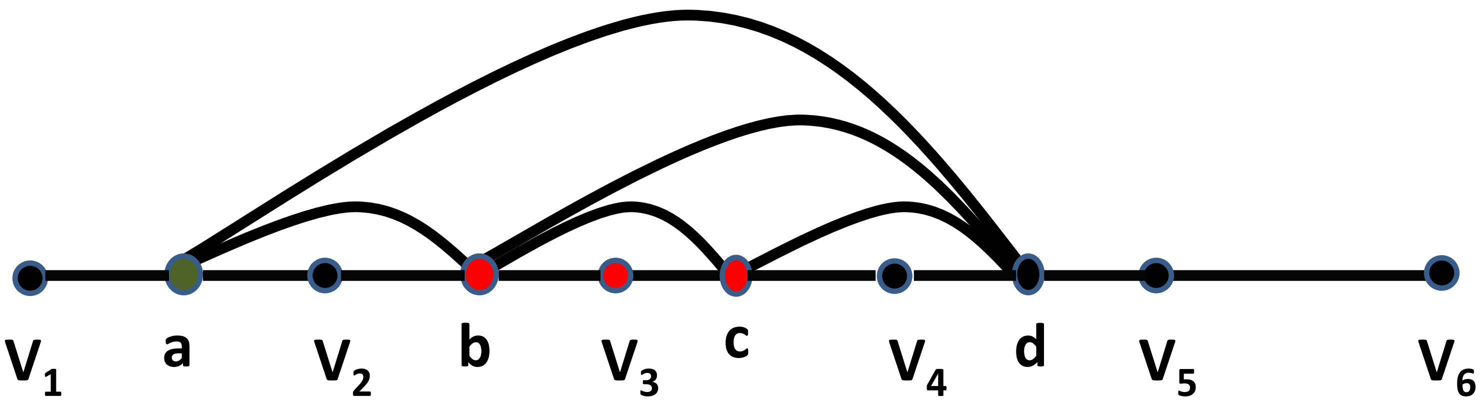
Claim 1: a, v_2, b $\notin$ D

Assume the contrary

G:= (P_6 , )-free

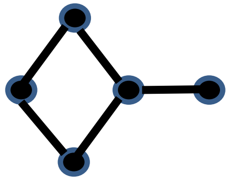
Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$



Suppose a $\in$ D

Then b, v_3 , c $\notin$ D

G:= (P_6 , )-free

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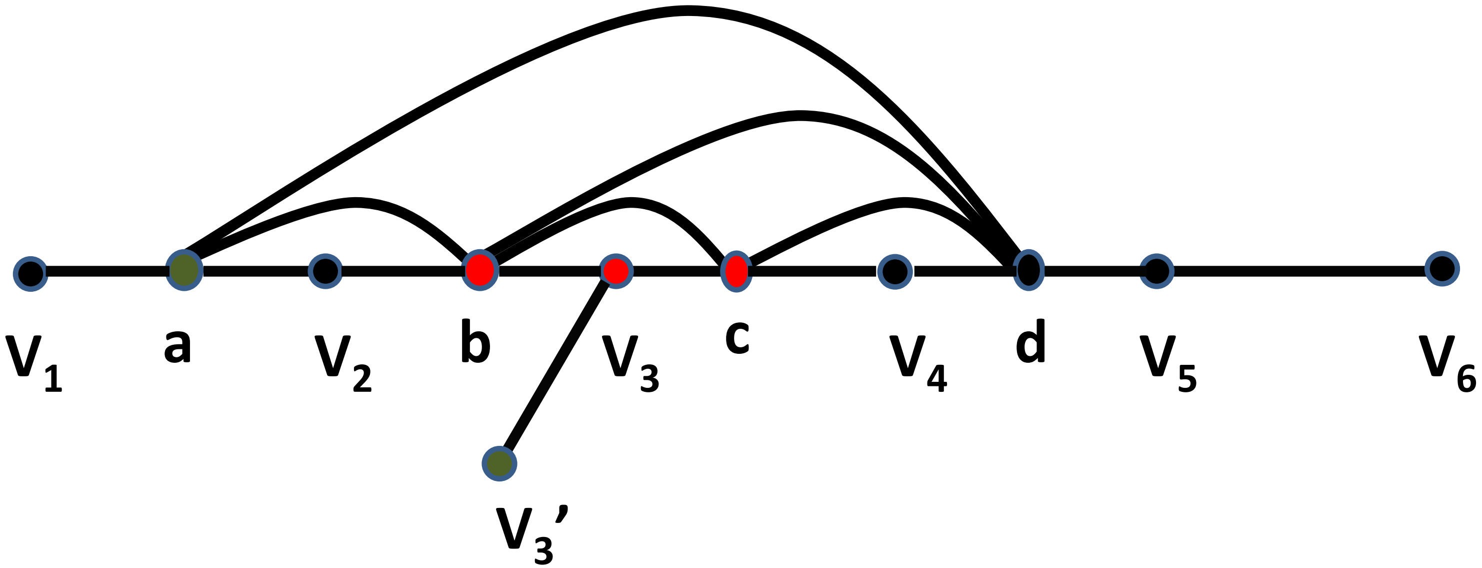
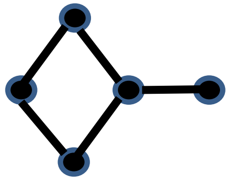
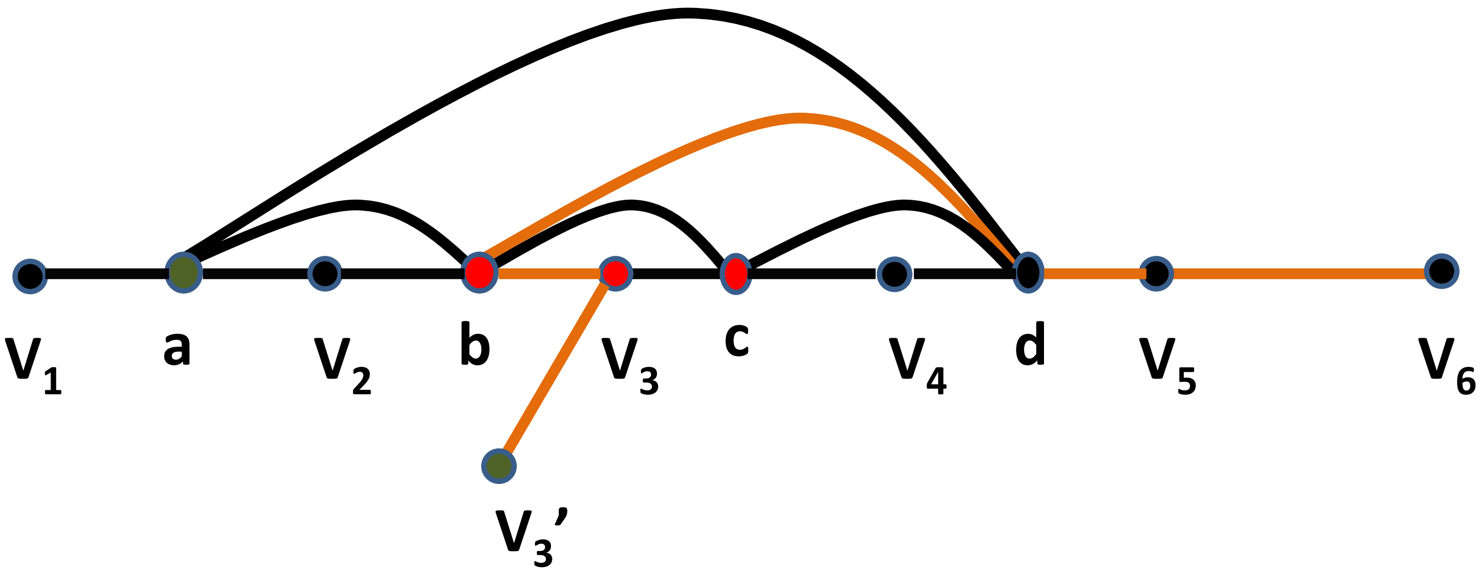
Case 1.2: $dist_G(v_3, v_4) = 2$

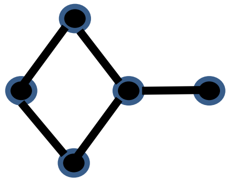


G:= (P_6 , )-free

Case 1: $dist_G(v_5, v_6) = 1$

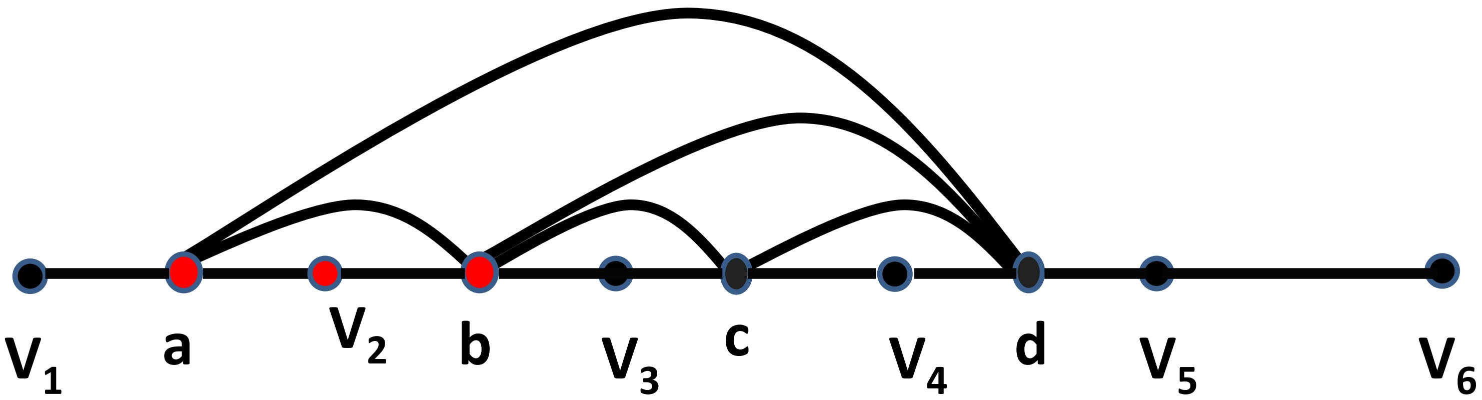
Case 1.2: $dist_G(v_3, v_4) = 2$



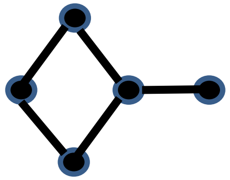
$G := (P_6,$  $)$-free

Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$



So, $a, v_2, b \notin D$

G:= (P_6 , )-free

Case 1: $dist_G(v_5, v_6) = 1$

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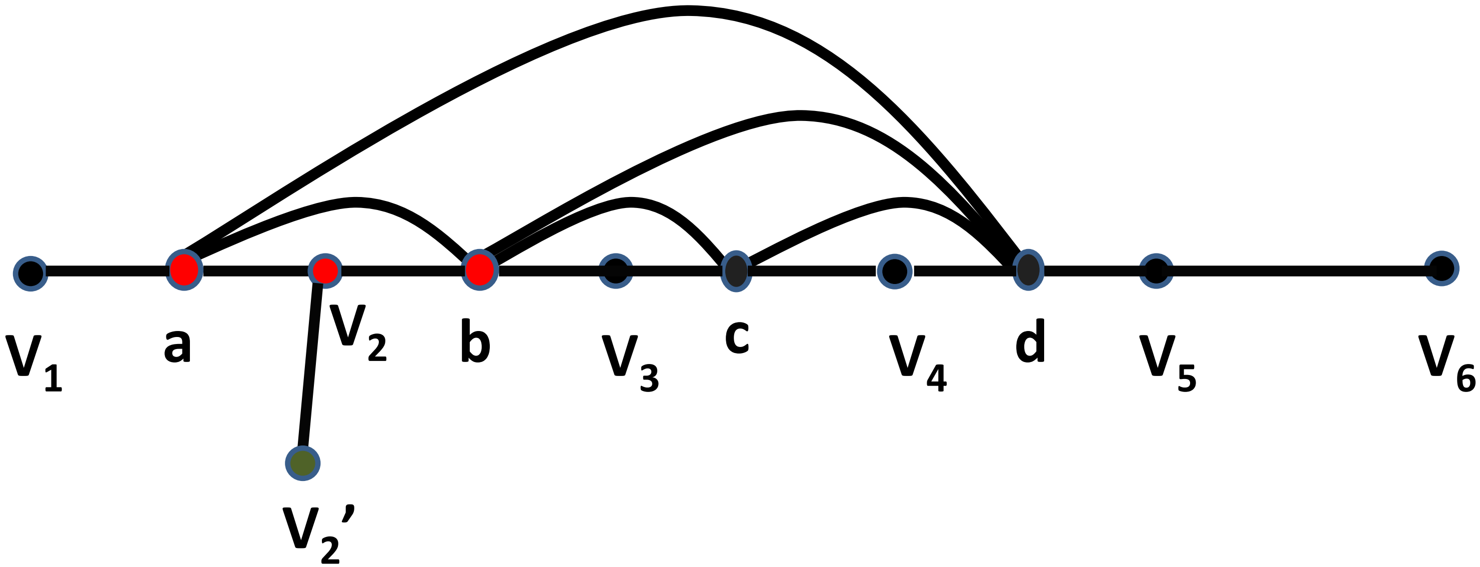
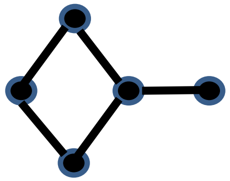


$G := (P_6$, )-free

Case 1: $\text{dist}_G(v_5, v_6) = 1$

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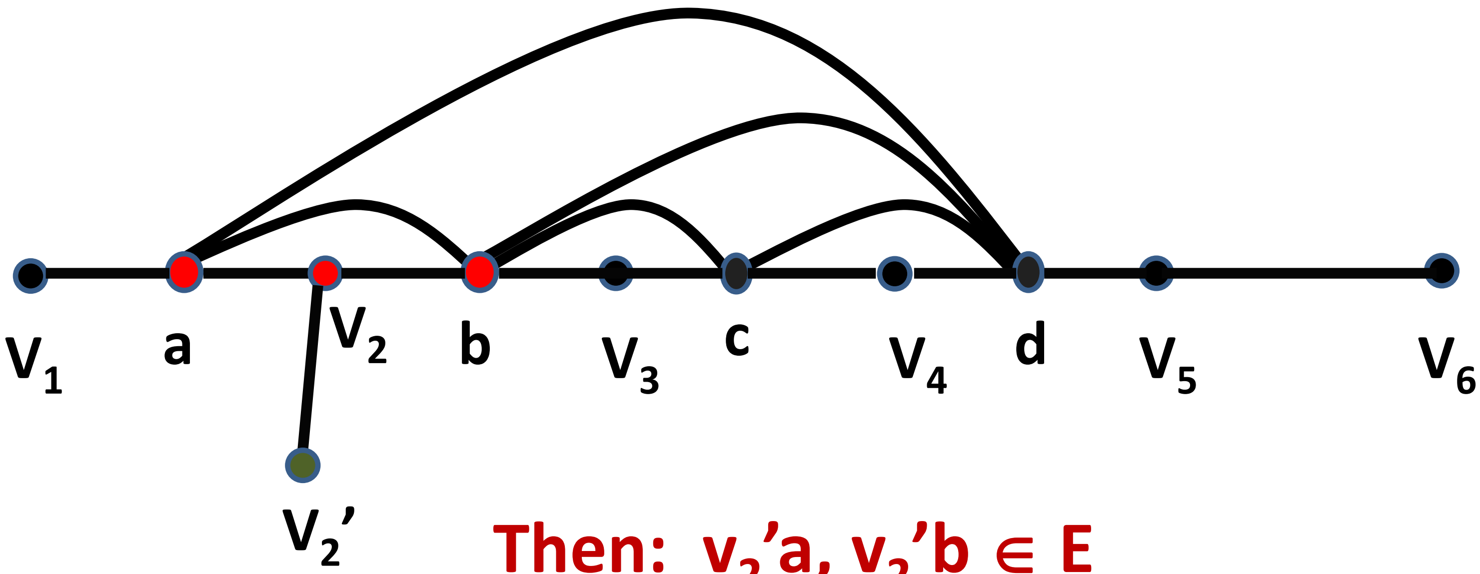
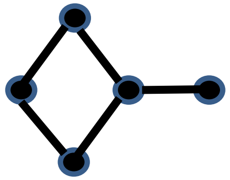
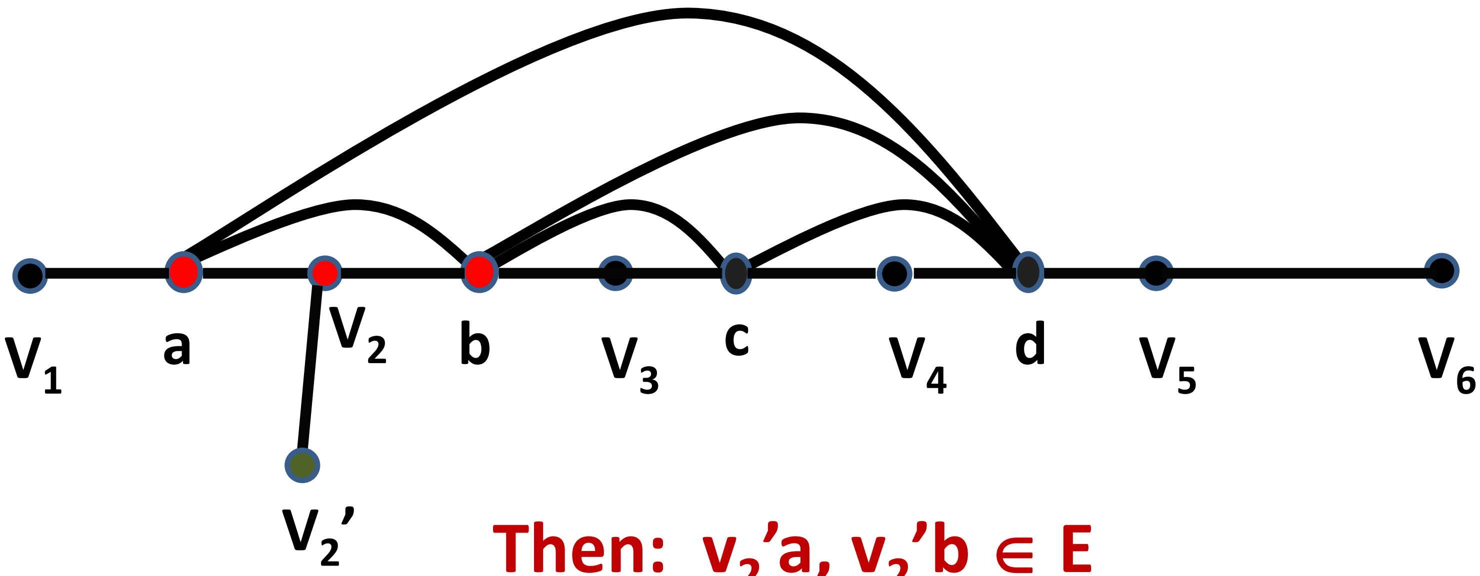


Then: $v_2'a, v_2'b \in E$

G:= (P_6 , )-free

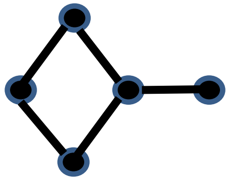
Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$



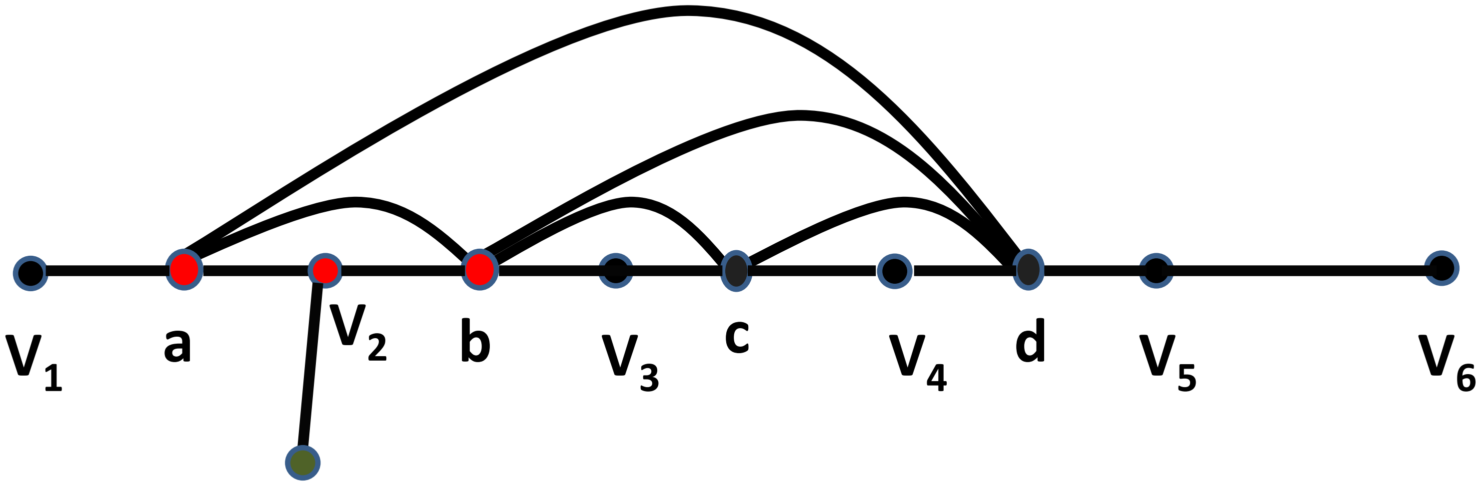
Then: $v_2'a, v_2'b \in E$

Assume the contrary

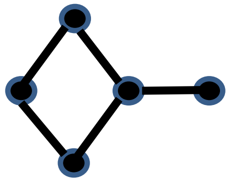
G:= (P_6, )-free

Case 1: $\text{dist}_G(v_5, v_6) = 1$

Case 1.2: $\text{dist}_G(v_3, v_4) = 2$

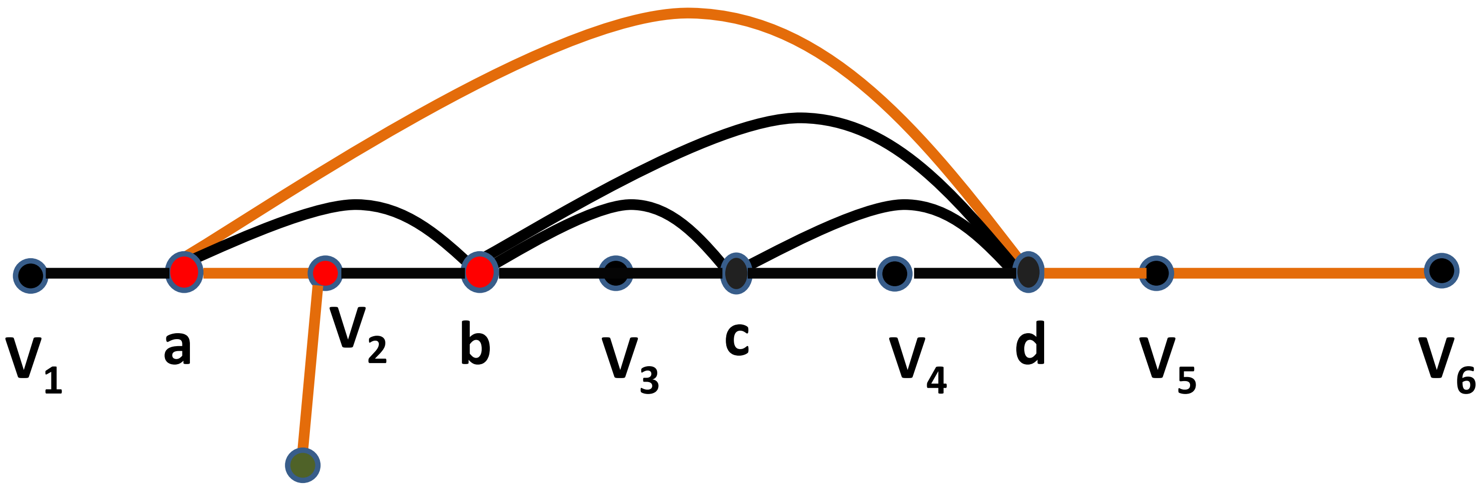


Suppose $v_2'a \notin E$

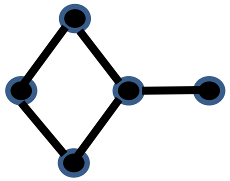
G:= (P_6 , )-free

Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$

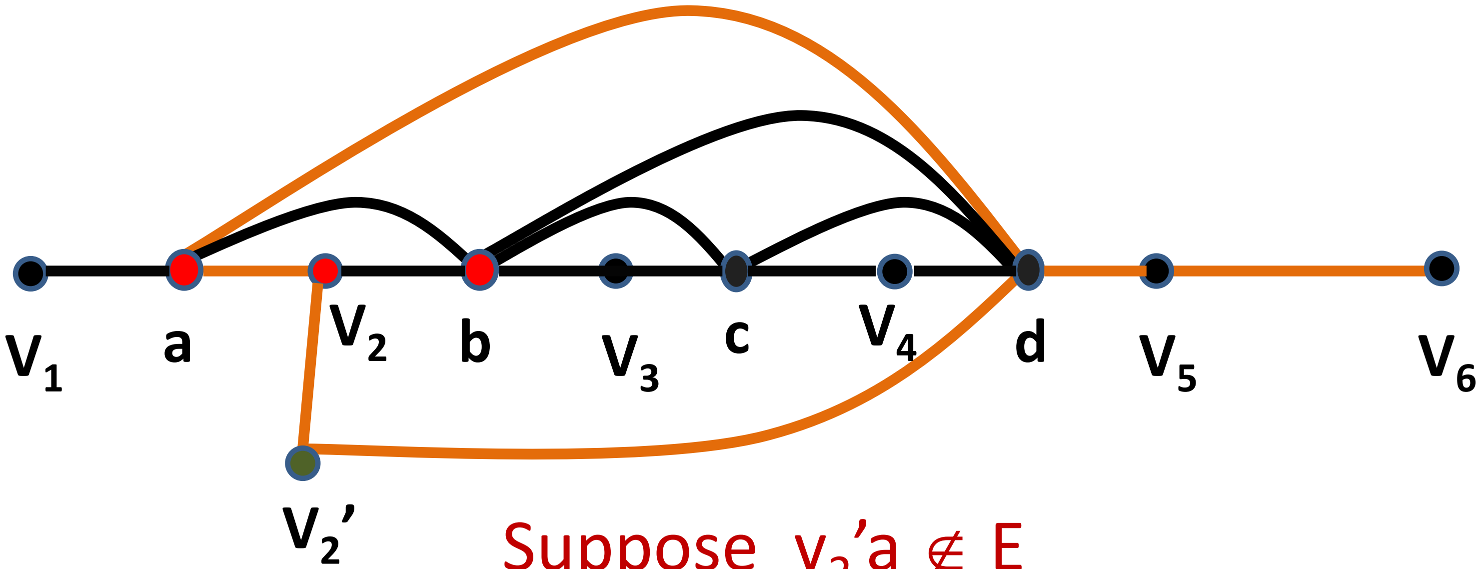


Suppose $v_2'a \notin E$

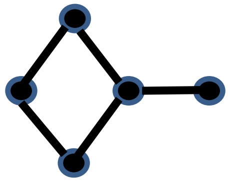
G:= (P_6 , )-free

Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$

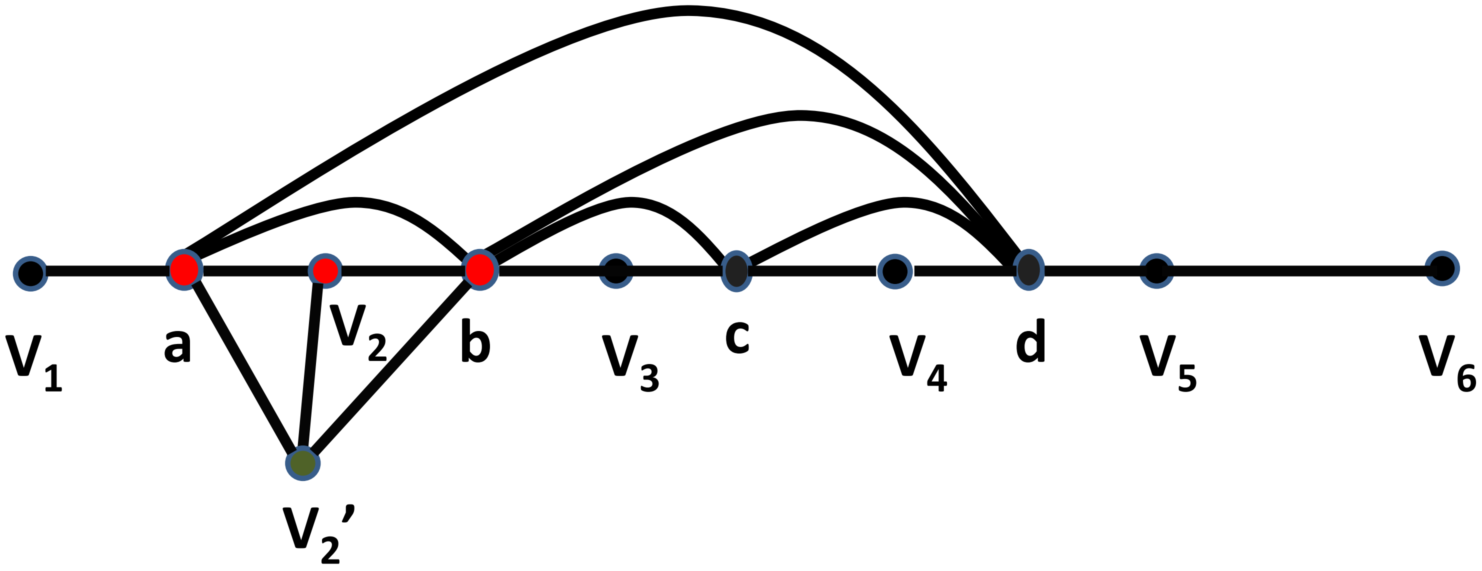


Suppose $v_2'a \notin E$

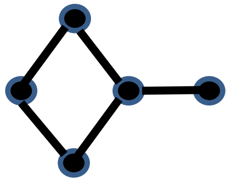
$G := (P_6$,  $)$-free

Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$

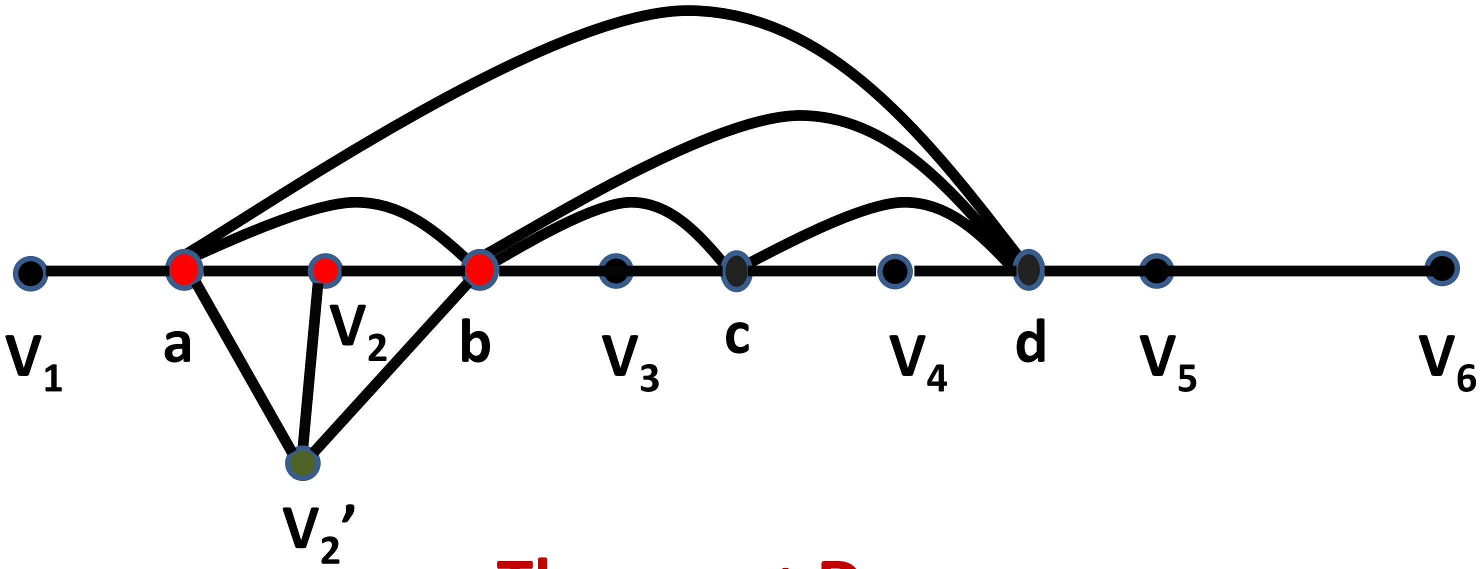


So, $v_2'a, v_2'b \in E$

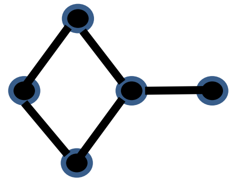
G:= (P_6, )-free

Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$

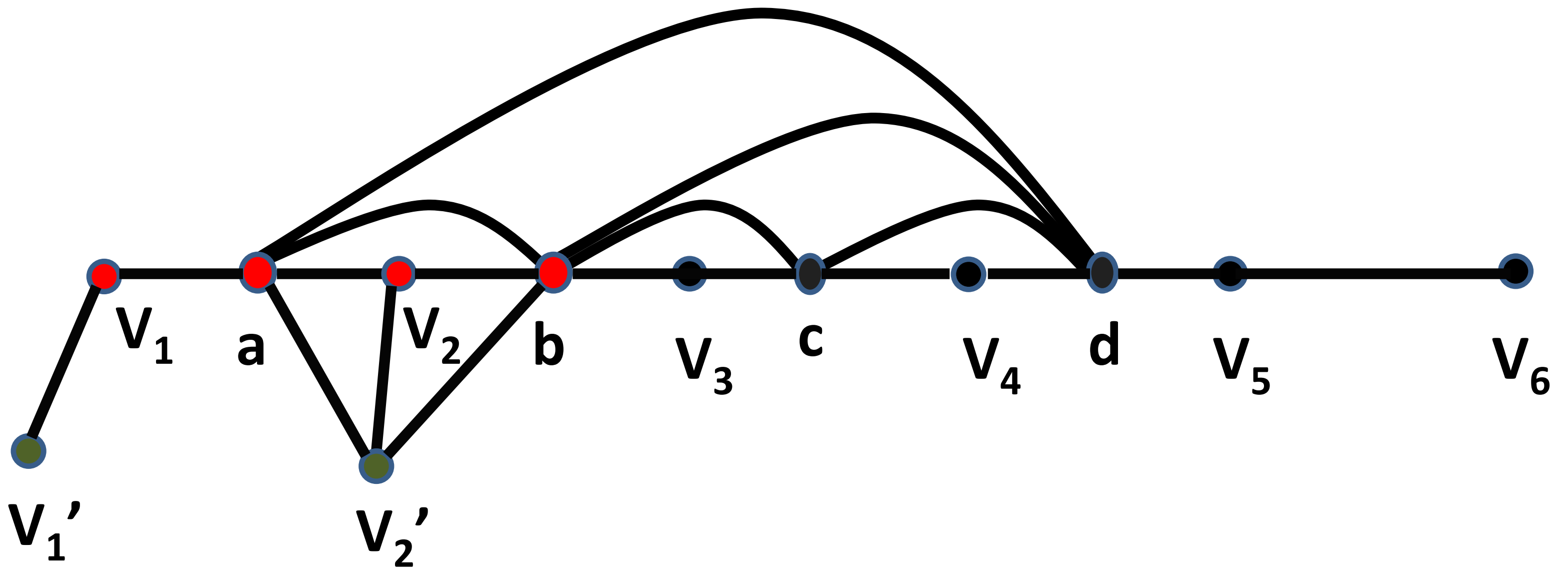


Then $v_1 \notin D$

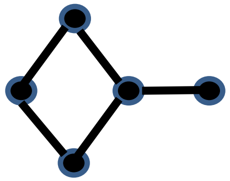
$G := (P_6,$  $)$-free

Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$

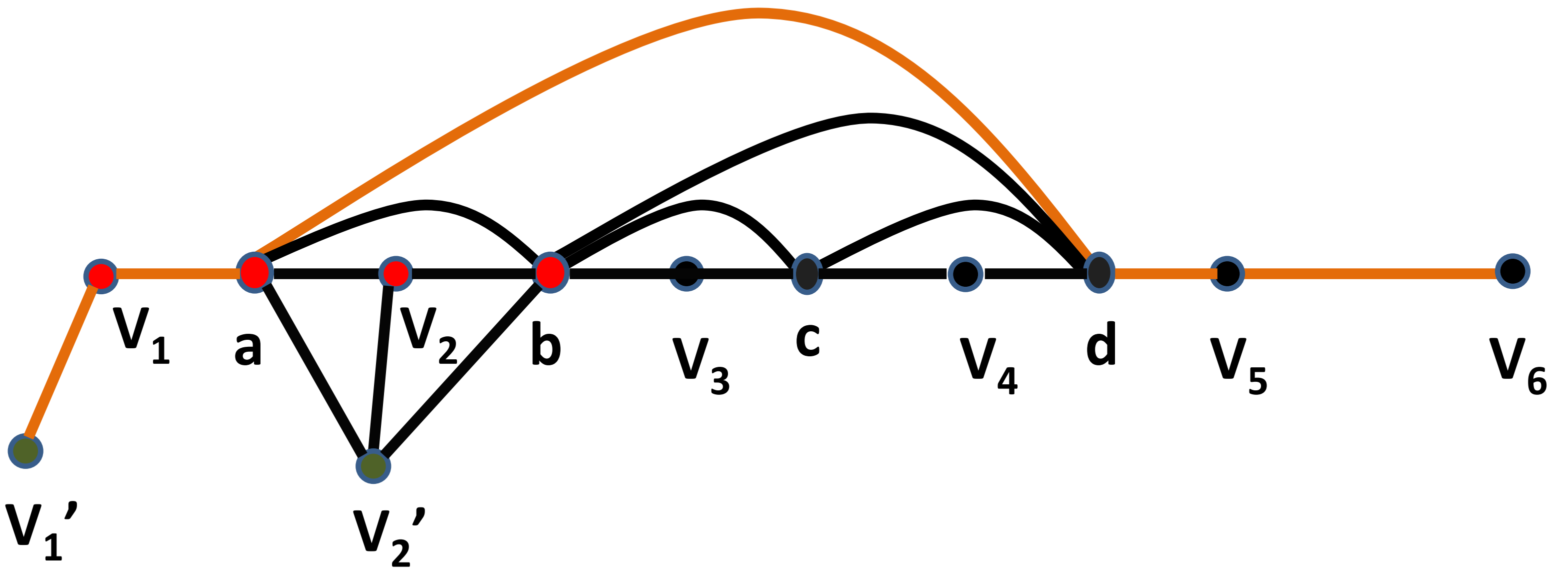


Claim 2: $v_1' \neq v_2'$

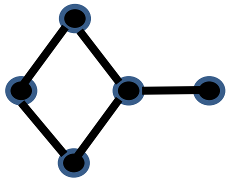
G:= (P_6 , )-free

Case 1: $dist_G(v_5, v_6) = 1$

Case 1.2: $dist_G(v_3, v_4) = 2$

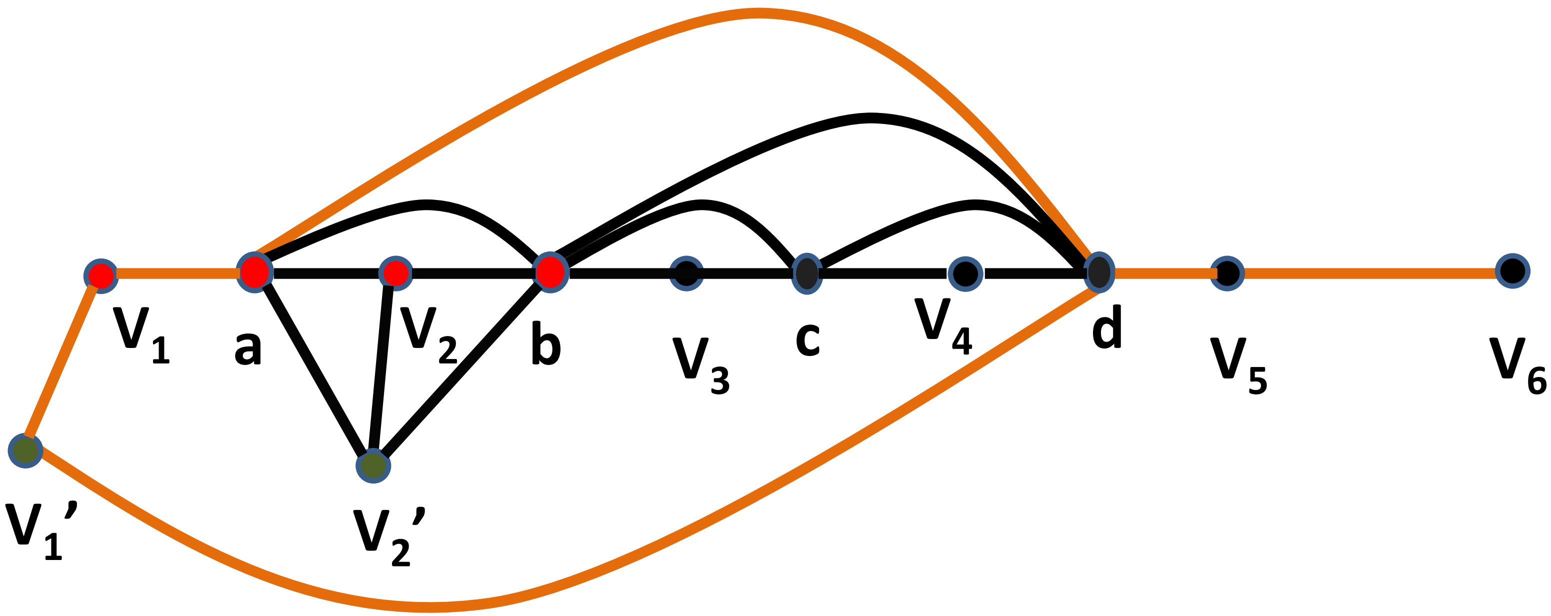


a contradiction

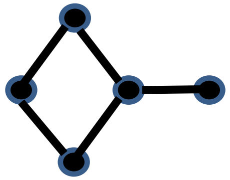
$G := (P_6$,  $)$-free

Case 1: $dist_G(v_5, v_6) = 1$

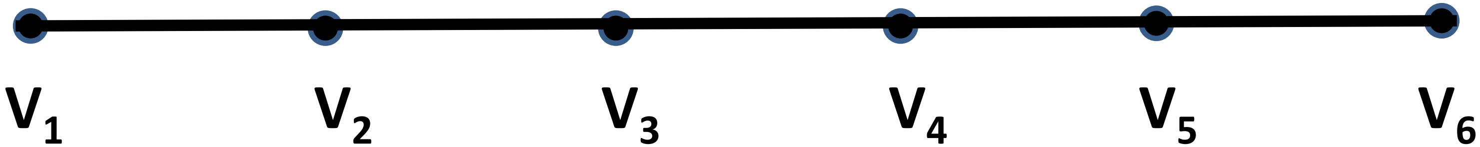
Case 1.2: $dist_G(v_3, v_4) = 2$

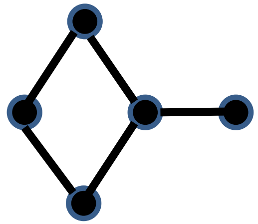


a contradiction

G:= (P_6 , )-free

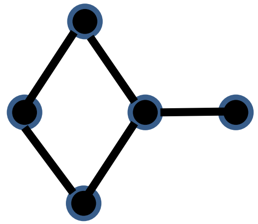
Case 2: $dist_G(v_1, v_2) = 2 = dist_G(v_5, v_6)$



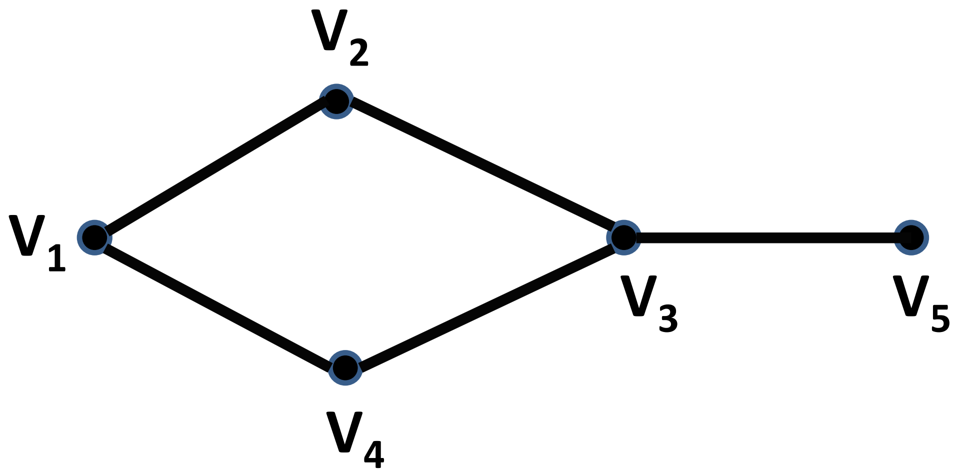
$G := (P_6$,  $)$-free

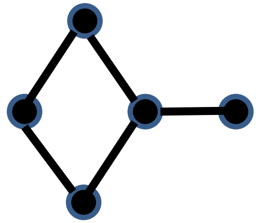
Suppose that G contains an EDS, say D

Claim : G^2 is banner-free

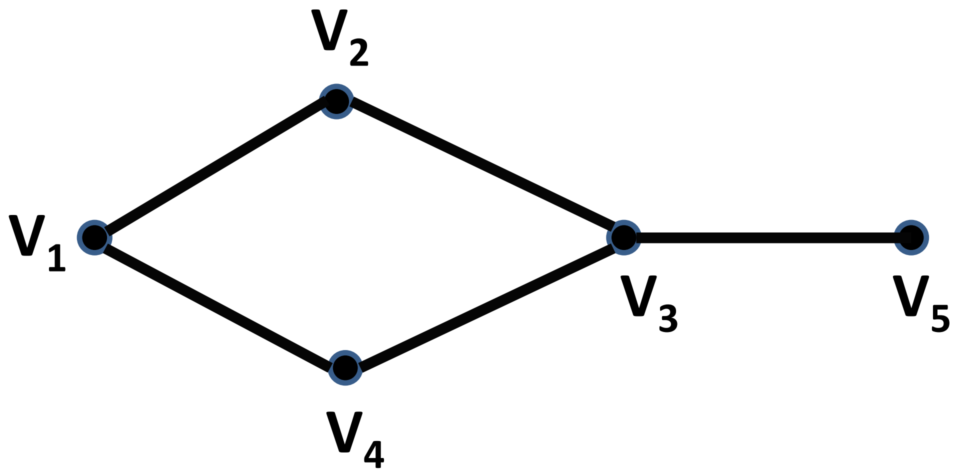
$G :=$ (P_6, )-free

Assume the contrary that G^2 contains an induced banner



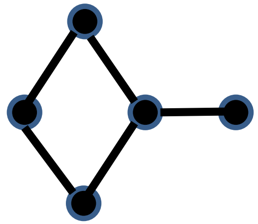
$G := (P_6,$  $)$-free

Assume the contrary that G^2 contains an induced banner

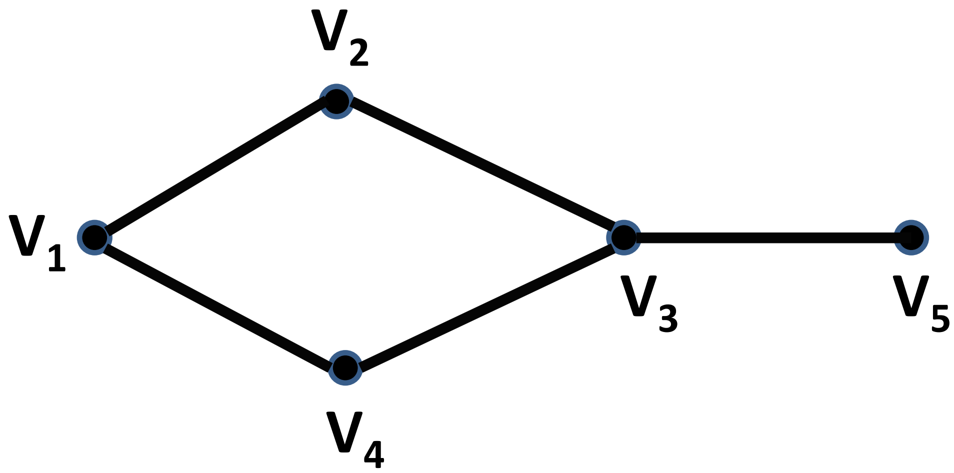


$dist_G(v_1, v_2) \leq 2$, $dist_G(v_2, v_3) \leq 2$, $dist_G(v_3, v_4) \leq 2$

$dist_G(v_1, v_4) \leq 2$, $dist_G(v_3, v_5) \leq 2$

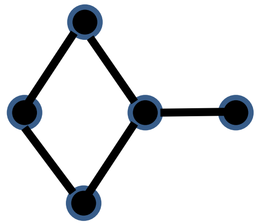
$G := (P_6,$  $)$-free

Assume the contrary that G^2 contains an induced banner

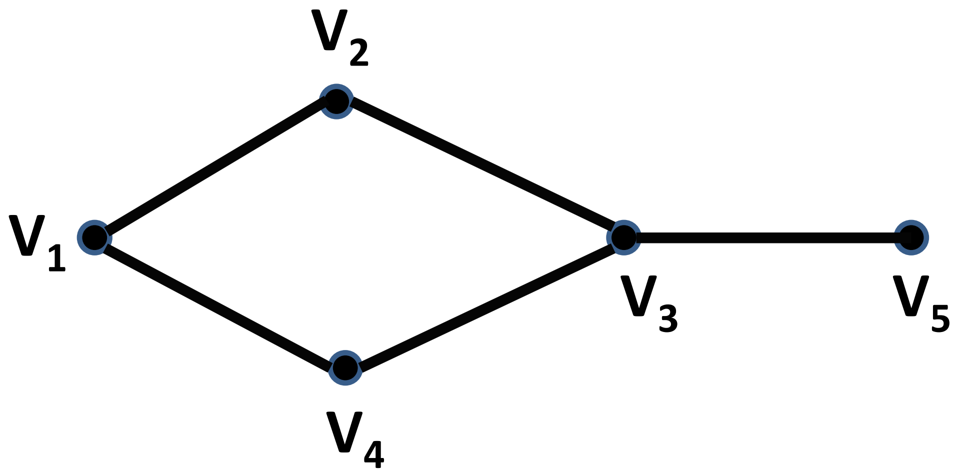


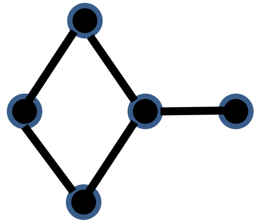
$dist_G(v_1, v_3) \geq 3$, $dist_G(v_1, v_5) \geq 3$, $dist_G(v_2, v_4) \geq 3$

$dist_G(v_2, v_5) \geq 3$, $dist_G(v_4, v_5) \geq 3$

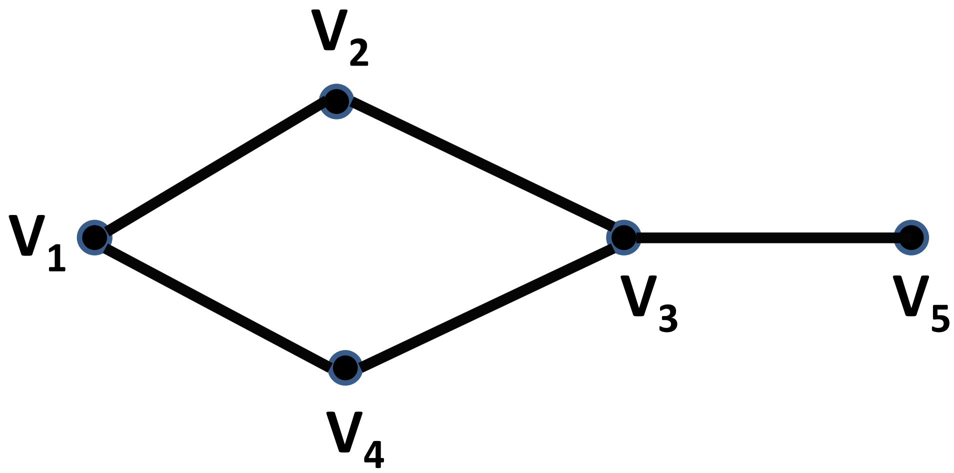
$G := (P_6,$  $)$-free

Case 1: $dist_G(v_3, v_5) = 1$



$G :=$ (P_6, )-free

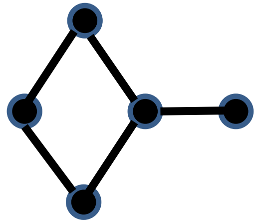
Case 1: $dist_G(v_3, v_5) = 1$



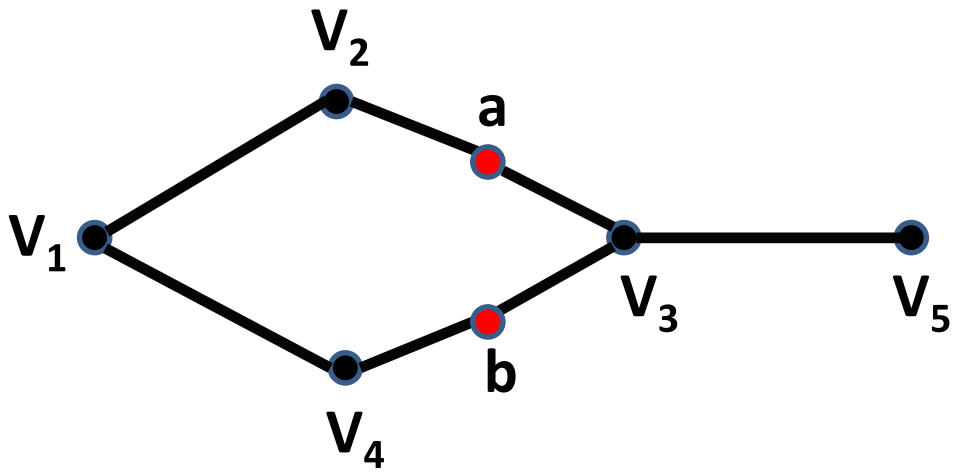
$dist_G(v_2, v_5) \geq 3$, $dist_G(v_4, v_5) \geq 3$

⬇

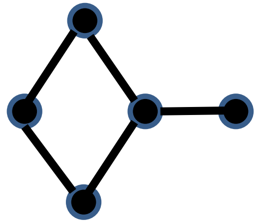
$dist_G(v_2, v_3) = 2$, $dist_G(v_4, v_3) = 2$

G:= (P_6, )-free

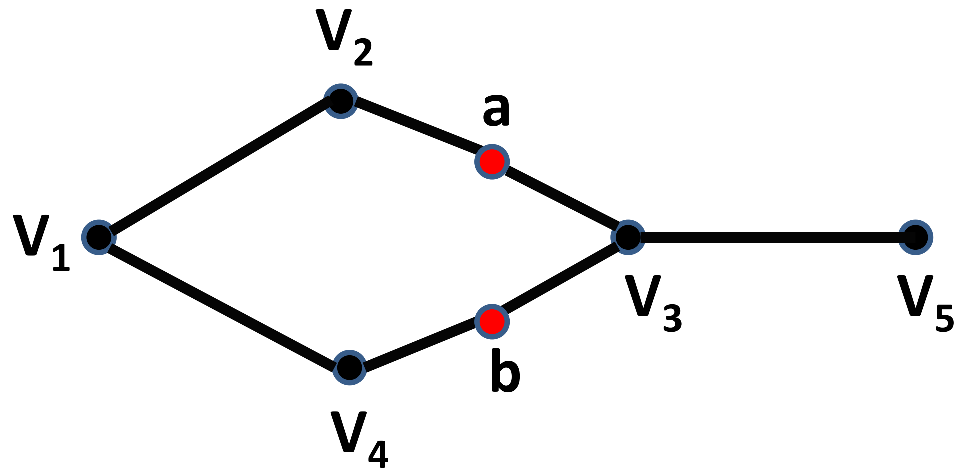
Case 1: $dist_G(v_3, v_5) = 1$



$dist_G(v_2, v_3) = 2$, $dist_G(v_4, v_3) = 2$

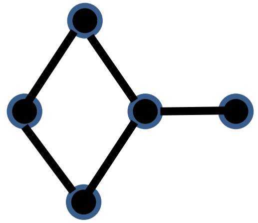
G:= (P_6, )-free

Case 1: $dist_G(v_3, v_5) = 1$

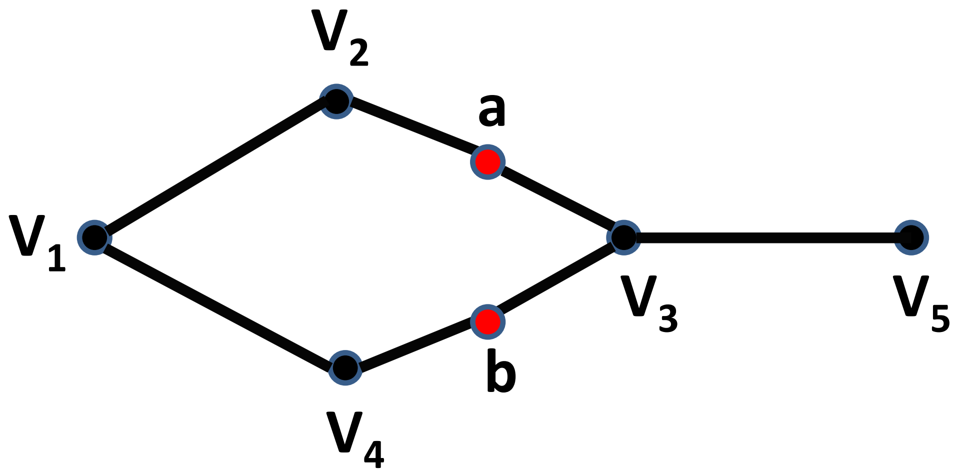


$dist_G(v_2, v_4) \geq 3$

$dist_G(v_1, v_4) = 2$ or $dist_G(v_1, v_2) = 2$ or both

G:= (P_6 , )-free

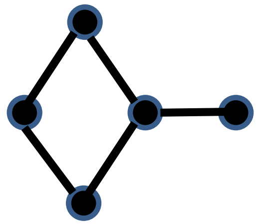
Case 1: $dist_G(v_3, v_5) = 1$



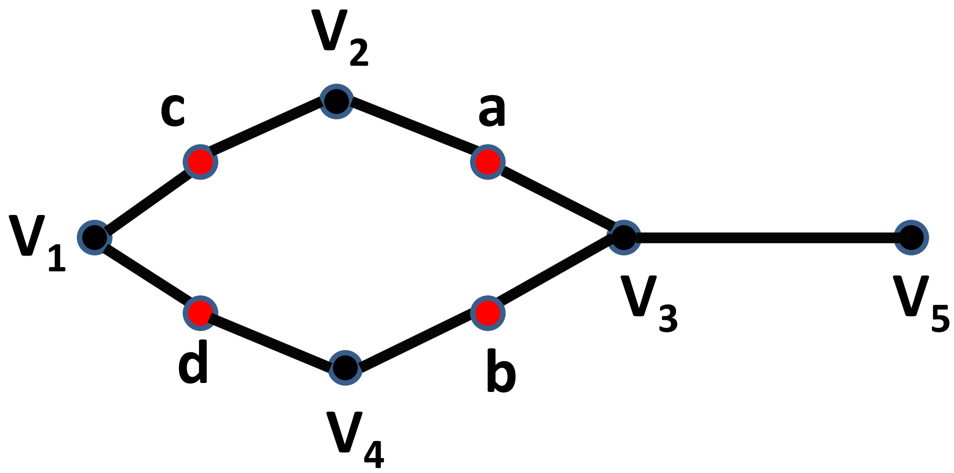
$dist_G(v_2, v_4) \geq 3$

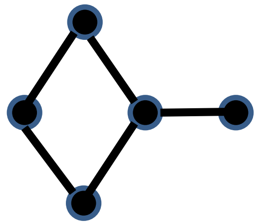
⬇

$dist_G(v_1, v_4) = 2 = dist_G(v_1, v_2)$

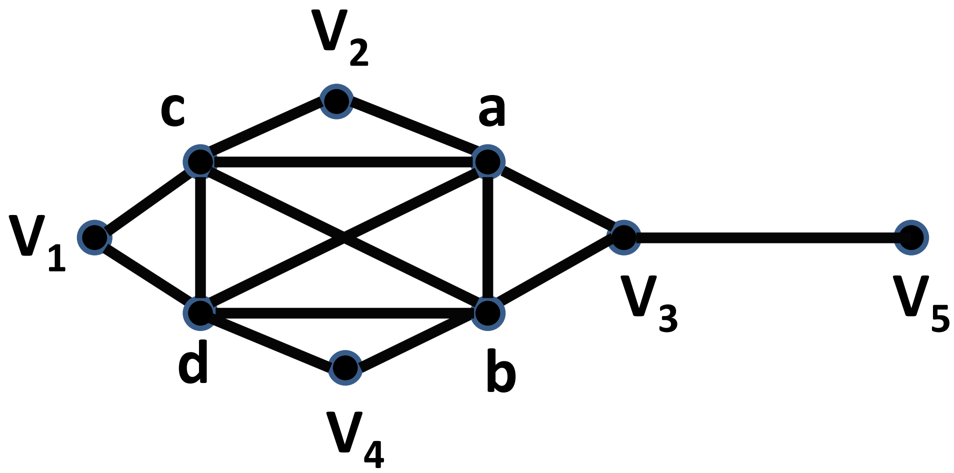
$G := (P_6,$  $)$-free

Case 1: $dist_G(v_3, v_5) = 1$

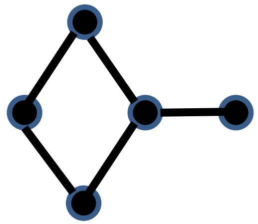


G:= (P_6, )-free

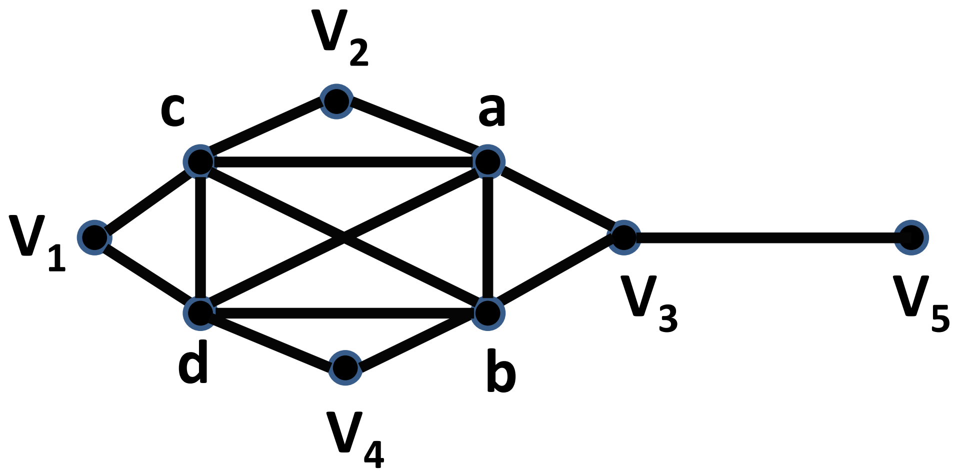
Case 1: $dist_G(v_3, v_5) = 1$



Partial structure of G

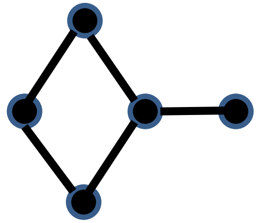
$G := (P_6$,  $)$-free

Case 1: $dist_G(v_3, v_5) = 1$

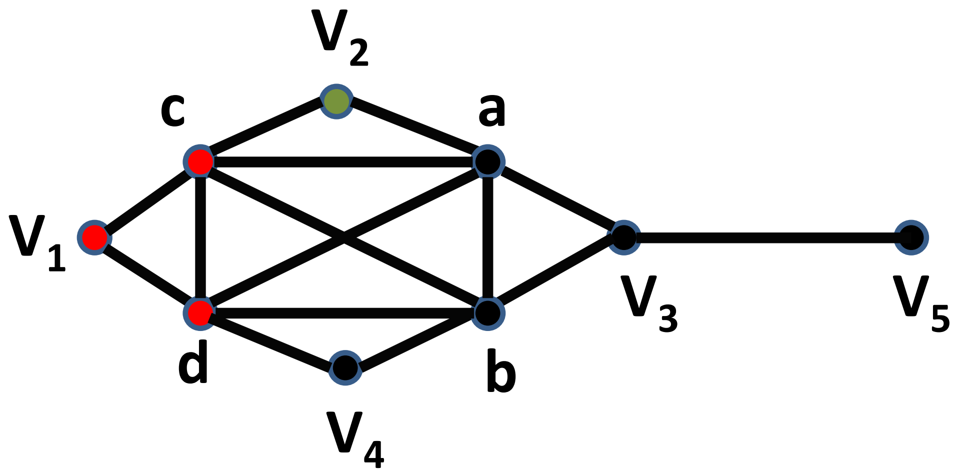


Claim 1: $v_2, v_4, a, b, c, d \notin D$

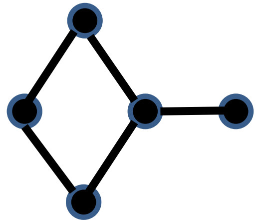
Assume the contrary

G:= (P_6, )-free

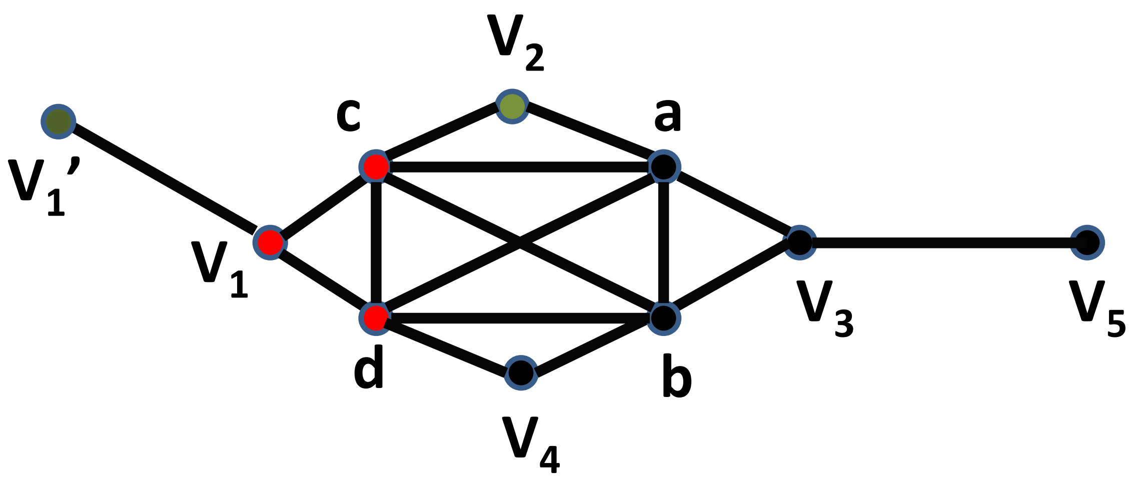
Case 1: $dist_G(v_3, v_5) = 1$



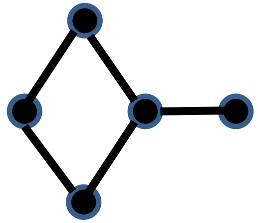
Suppose $v_2 \in D$

G:= (P_6, )-free

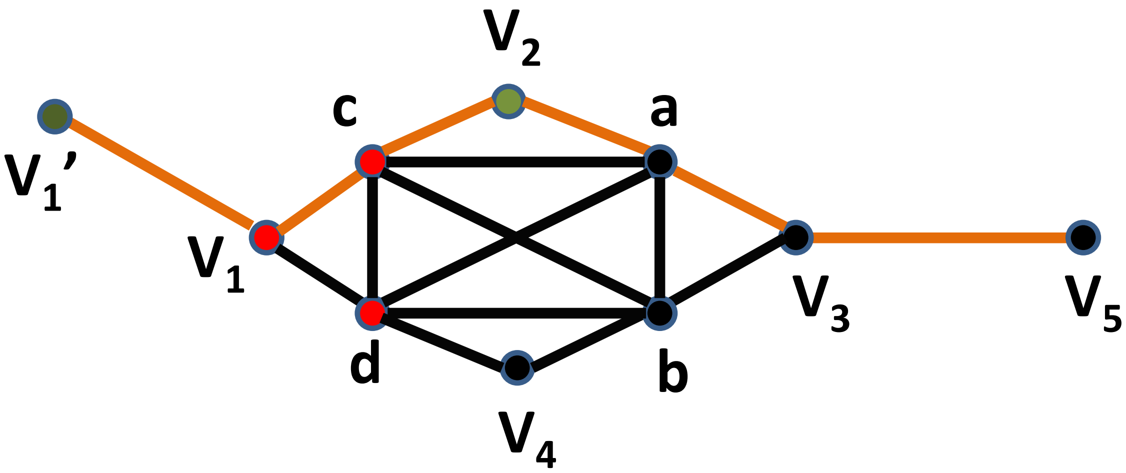
Case 1: $dist_G(v_3, v_5) = 1$



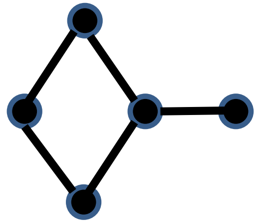
Suppose $v_2 \in D$

G:= (P_6, )-free

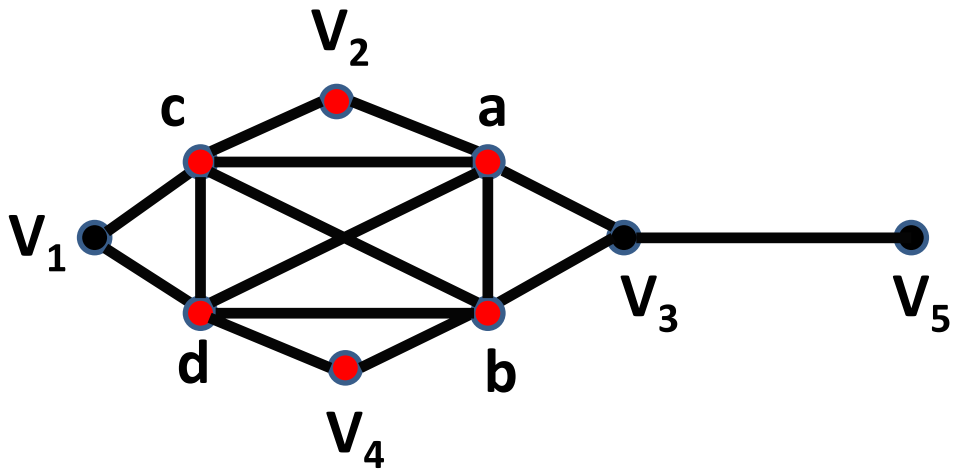
Case 1: $dist_G(v_3, v_5) = 1$

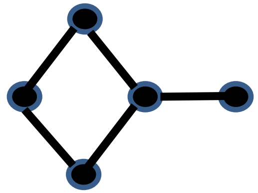


Suppose $v_2 \in D$

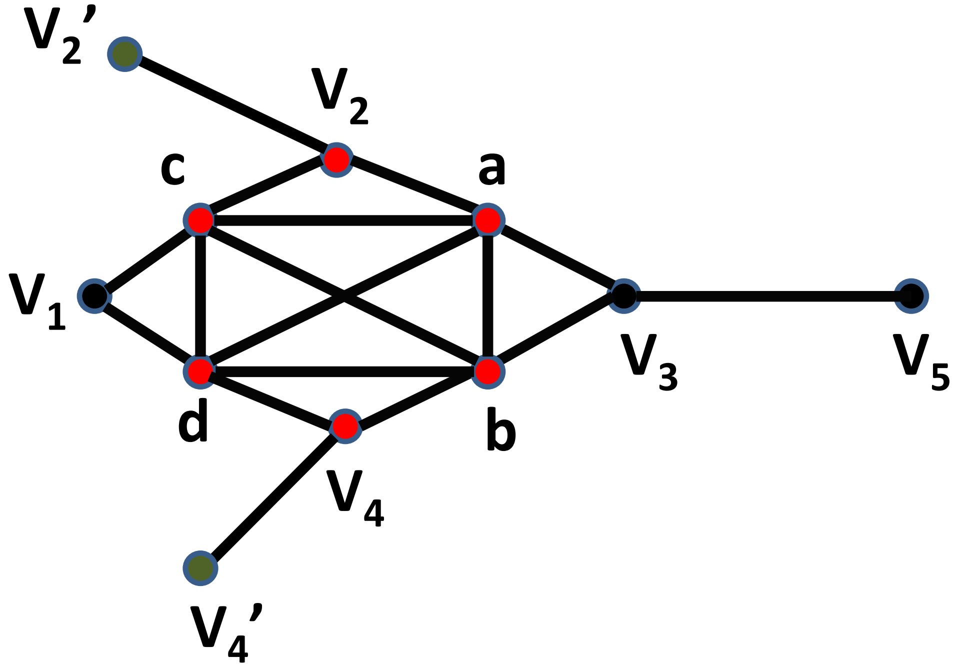
$G := (P_6,$  $)$-free

Case 1: $dist_G(v_3, v_5) = 1$

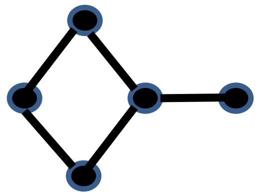


$G := (P_6,$  $)$-free

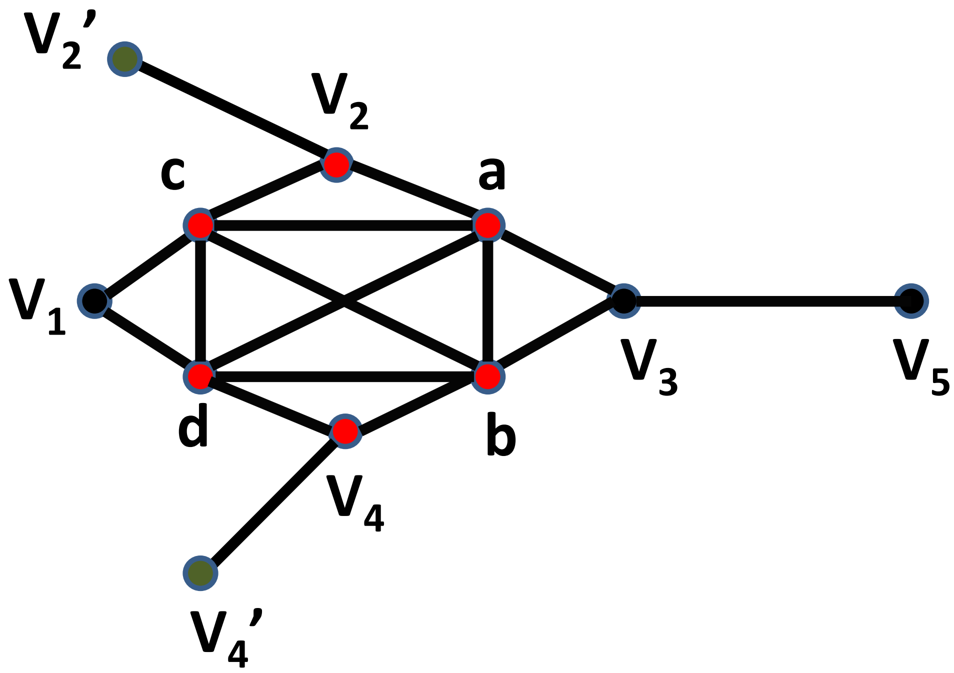
Case 1: $dist_G(v_3, v_5) = 1$



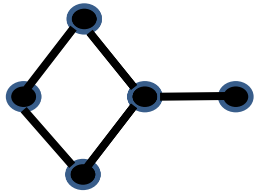
$v_2' \neq v_4'$

G:= (P_6, )-free

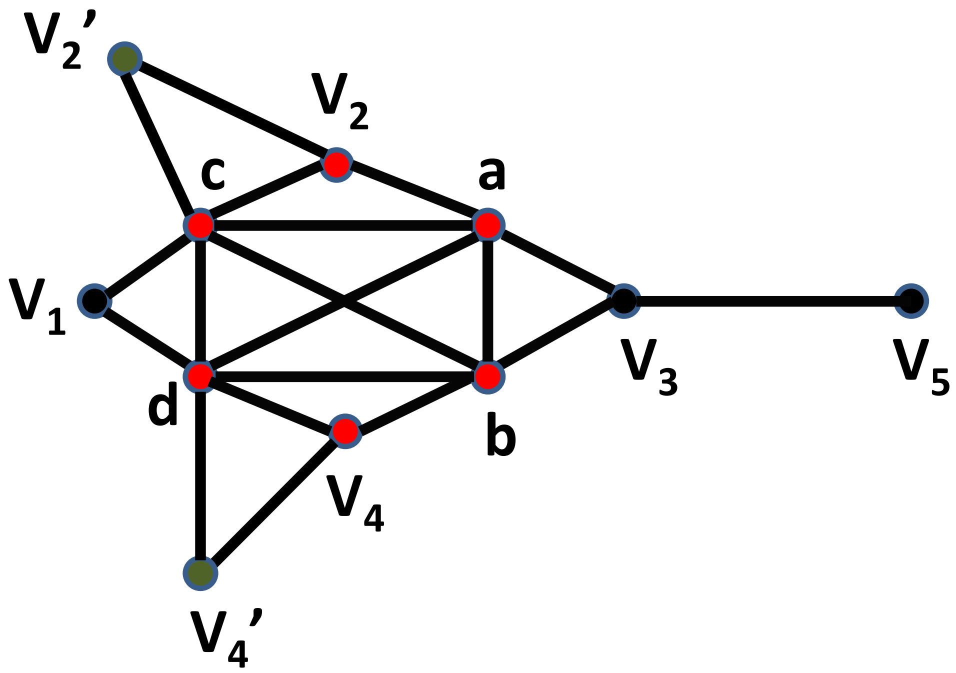
Case 1: $dist_G(v_3, v_5) = 1$

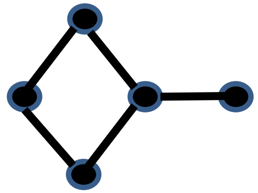


Then v_2’c , v_4’d∈E

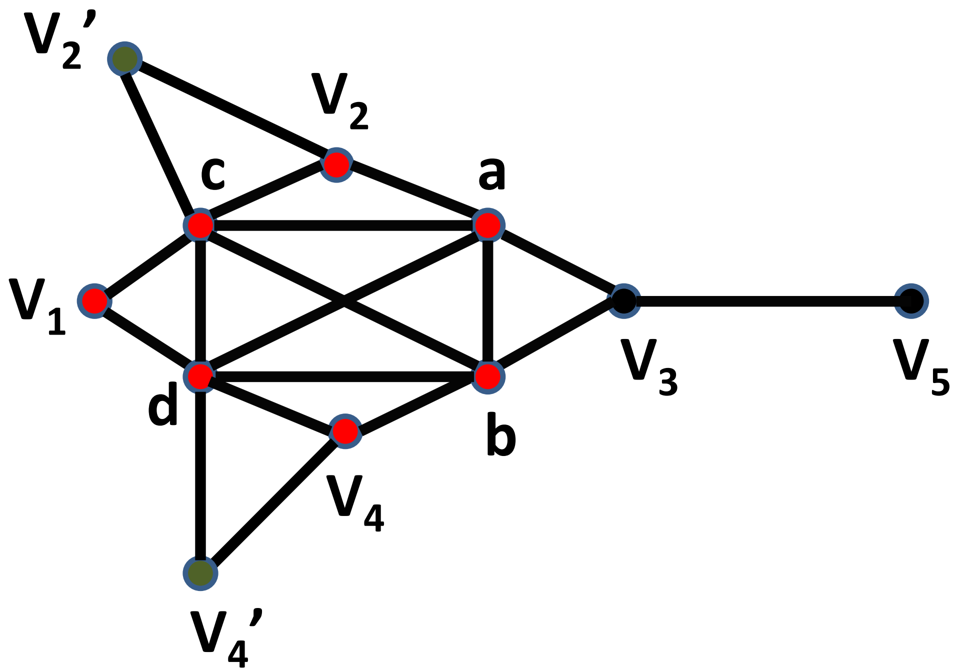
$G := (P_6,$  $)$-free

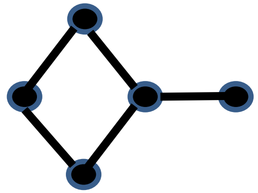
Case 1: $\text{dist}_G(v_3, v_5) = 1$



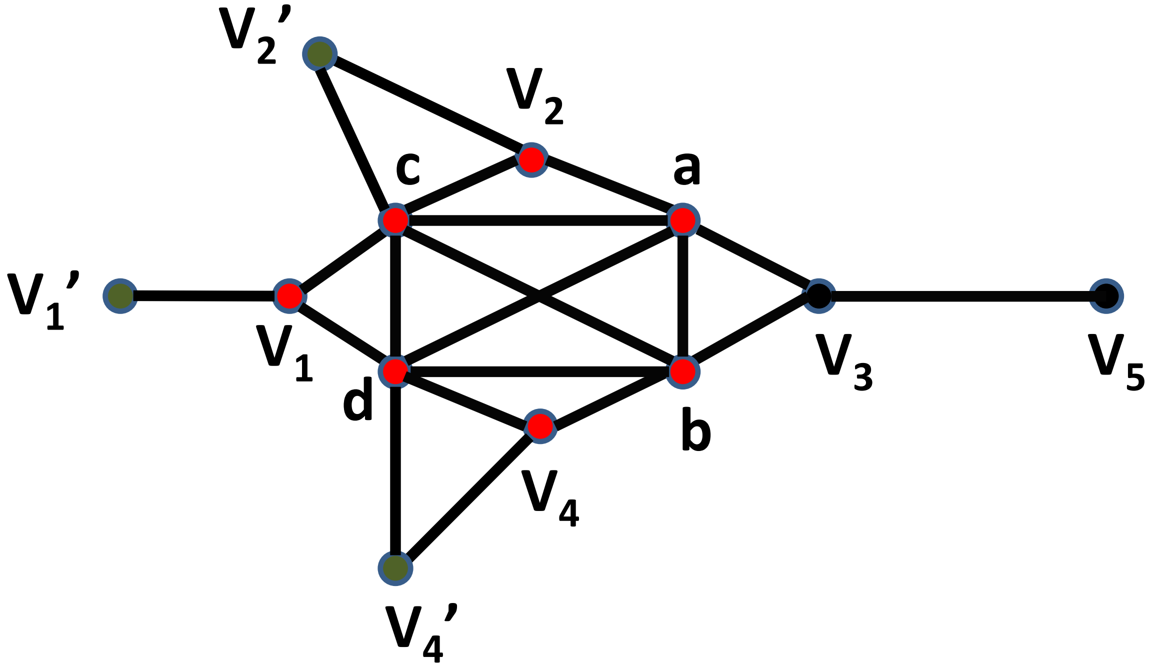
G:= (P_6, )-free

Case 1: $dist_G(v_3, v_5) = 1$

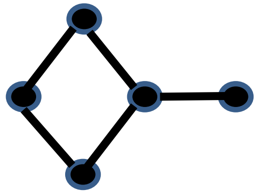


$G := (P_6,$  $)$-free

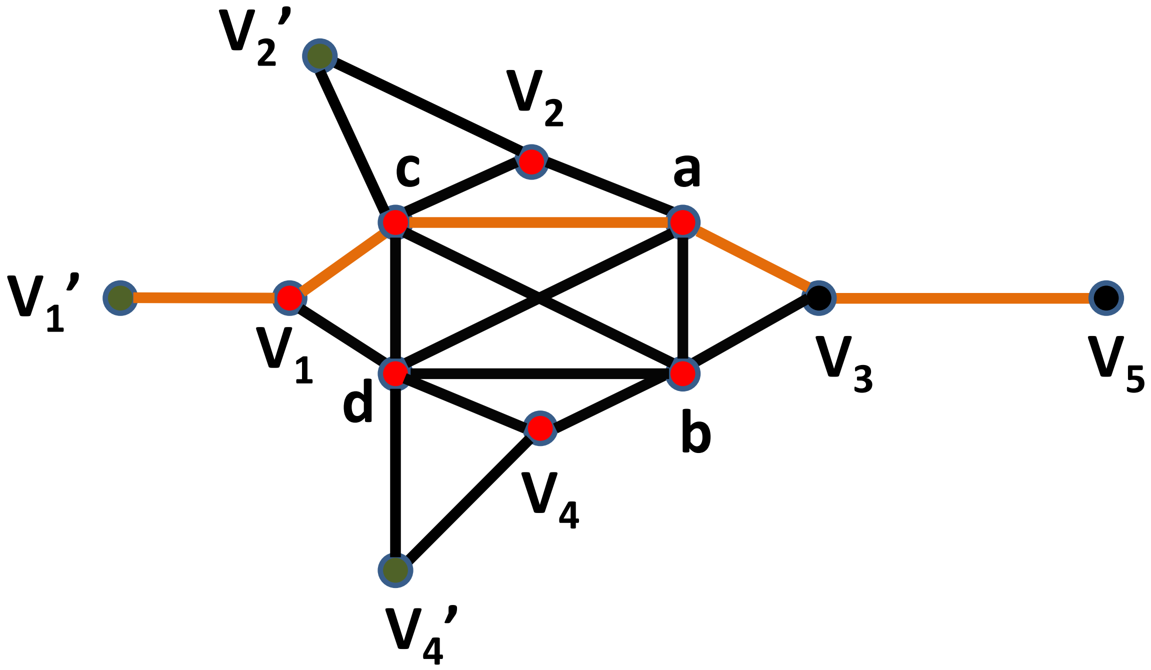
Case 1: $dist_G(v_3, v_5) = 1$



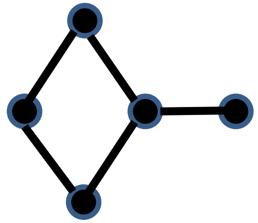
$v_1' \neq v_2', v_4'$

G:= (P_6, )-free

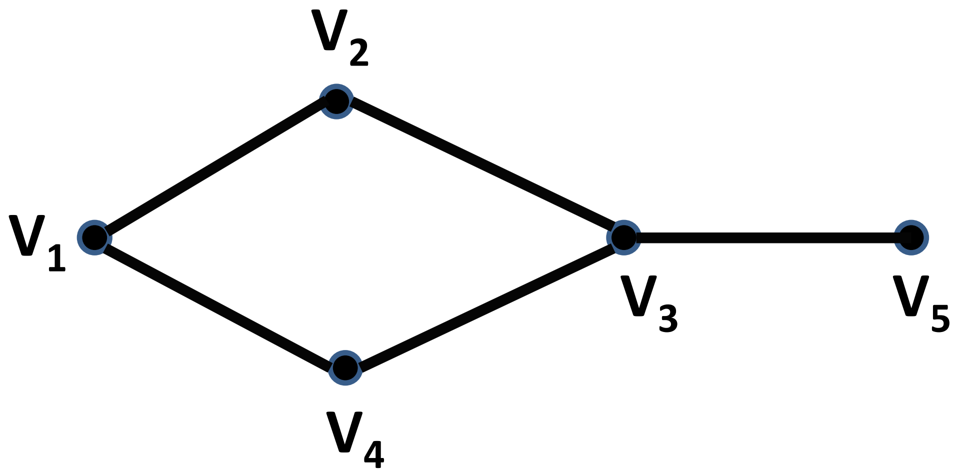
Case 1: $dist_G(v_3, v_5) = 1$

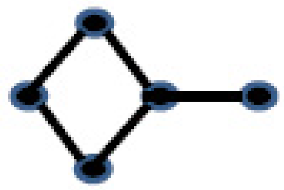


$v_1' \neq v_2', v_4'$

$G := (P_6,$  $)$-free

Case 2: $dist_G(v_3, v_5) = 2$



(P_6, )-free Graphs

WED can be solved in polynomial time

- Let G be a (P_6, banner)-free graph. If G has an EDS, then G^2 is (P_6, banner)-free.
- Karthick [15]: MWIS can be solved in polynomial time for (P_6, banner)-free graphs.
- So, the result follows by Theorem 1.

Conclusion

- The complexity of WED for P_6-free graphs is unknown.
- The complexity of MWIS for P_6-free graphs is unknown.
- WED and MWIS remains NP-complete for banner-free graphs [24, 27].
- Showed that WED can be solved in polynomial time for (P_6, banner)-free graphs.
- Banner-free graphs include some well studied classes of graphs such as $K_{1,3}$-free graphs, C_4-free graphs, and P_4-free graphs.

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THANK YOU